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Amplitude characteristics of excitation of stimulated photon echo by noise and coherent pulses

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***Abstract*. The algorithm of excitation of a stimulated photon echo by two incoherent white noise Gaussian pulses and delta-like light pulses is studied. The average complex envelope of the echo is determined by solving Bloch equations. The results obtained can be used in spectroscopy and in the analysis of nonlinear characteristics of excitation in photon processors based on the saturation effect. The results can be also generalised to the analogous regime of excitation of a spin echo.**

Keywords: *photon echo, stochastic Bloch equations, nonlinear systems, correlation processing, spin echo.*

1. Introduction

Studies of a photon echo have entered the stage of manufacturing of the first optical processors for optical data processing [1]. Processors based on a photon echo can be used as fast memory devices providing repeated data readout [2], for data storage (in cumulative regime) [3], and for the signal delay with time inversion and for the self-convolution of optical signals [4]. Along with the abovementio-ned algorithms, such processors can also perform other integral transformations of optical signals, in particular, the correlation processing of determinate signals [5, 6].

Of great interest is also the processing of random signals based on excitation of a photon echo using incoherent noise pulses [7]. In spectroscopy, this allows one to use long incoherent lower power pulses instead of high-power short coherent excitation pulses. For signal processors operating in the optical range, the possibility opens up for performing the correlation analysis of random signals. Note also that the white Gaussian noise is a convenient model for analysing nonlinear properties of systems [8].

A photon echo was first observed, using incoherent optical sources instead of traditional coherent laser pulses, in 1984 [9]. The photon echo formed by two incoherent pulses was called incoherent. Unlike coherent sources, which produce pulses of duration τ with the spectral width $\Delta f = 1/\tau$, the spectral width of incoherent pulses in independent of the pulse duration.

In Ref. [7], the algorithm is described for excitation of an accumulated photon echo by two incoherent pulses from a single source and by one short coherent pulse. It was shown that the shape of the accumulated echo corresponds to the correlation function of the electric field of the incoherent source. The correlation interval of the electric field is inversely proportional to the width of the source spectrum and determines the time resolution in photon echo experiments. This parameter plays an important role in the measurements of relaxation times. The time resolution achieved in first experiments was 220 fs. Later [10], a better time resolution of 80 fs was achieved due to the use of a light emitting diode as a radiation source.

2. Algorithm for photon echo excitation

The aim of this paper is to study the algorithm for excitation of a stimulated photon echo using two noise incoherent pulses and one short coherent pulse. Fig. 1 shows the time diagram of envelopes of the excitation and stimulated photon echo pulses. The first excitation pulse $\sigma_1 s(t)$ and the third excitation pulse $\sigma_3 s(t - t_3)$ are realisations of the white Gaussian noise of duration τ, which are obtained from the same source by introducing the delay t_3. Note that the dimensionality of the function $\sigma_1 s(t) = \gamma E(t)$ is radian per second (γ is gyroelectric ratio and E is the electric field strength).

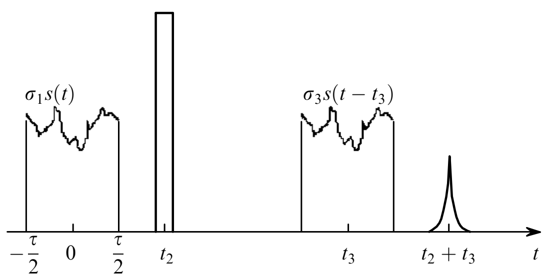

Figure 1. Time diagram of envelopes of the excitation and stimulated photon echo pulses.

The second excitation pulse represents a coherent pulse at frequency ω_0, which coincides with the central frequency of the inhomogeneous absorption line $g(\omega)$. The duration of this pulse satisfies the condition $\tau_\delta \ll 1/(2\Delta\omega_g)$, where $2\Delta\omega_g$

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is the width of the inhomogeneous line. We will call the pulse satisfying this condition a delta-like pulse in a sense that its spectral density is almost constant within the frequency interval covered by the inhomogeneous line and its properties are similar to those of the Dirac delta function. The amplitude E_δ and the duration τ_δ of this pulse determine the area of its envelope $\alpha = E_\delta \tau_\delta$.

We will analyse the photon echo using optical Bloch equations [11]. Formally, the solution of these equations in the coordinate system, which is rotated around the longitudinal axis with the frequency ω_0 coinciding with the central frequency of the inhomogeneous absorption line $g(\omega)$, can be represented in the form [5, 6]

$$\boldsymbol{P}(t,\Omega) = \boldsymbol{A}(t,t_0,\Omega)\boldsymbol{P}(t_0,\Omega) + \boldsymbol{P}_\mathrm{i}(t,t_0,\Omega),$$

$$\boldsymbol{P} = \begin{pmatrix} \tilde{p} \\ \tilde{p}^* \\ p_z \end{pmatrix}, \tag{1}$$

where $\boldsymbol{P}$ is the pseudopolarisation vector with complex transverse components $\tilde{p}$ and $\tilde{p}^*$ and the longitudinal component p_z; $\Omega = \omega - \omega_0$ is the detuning of the transition frequency ω relative to ω_0; $\boldsymbol{A}(t,t_0,\Omega)$ is the transition matrix of the system state; t_0 is the initial instant of time for which the initial vector $\boldsymbol{P}(t_0,\Omega)$ is specified; $\boldsymbol{P}_\mathrm{i}(t,t_0,\Omega)$ is the vector taking into account the inhomogeneity of the system of differential Bloch equations.

If the duration of excitation pulses $\tau \ll T_1$, where T_1 is the longitudinal relaxation time, the second term in (1) can be neglected and then the complex transverse component of the pseudopolarisation vector, which corresponds to the stimulated echo, can be represented in the form [6]

$$\tilde{p}_\mathrm{s}(t,\Omega) = p_0 a_{13}^{(3)}(\Omega) a_{32}^{(2)}(\Omega) a_{23}^{(1)}(\Omega) k_\mathrm{r}(t) \exp[\mathrm{i}\Omega(t - t_2 - t_3)],$$

$$k_\mathrm{r}(t) = \exp\left(-\frac{t_3 - t_2}{T_1} - \frac{t - t_3 + t_2}{T_2}\right), \tag{2}$$

where p_0 is the modulus of the vector $\boldsymbol{P}$ at the instant of time $t = -\tau/2$ preceding the onset of the first excitation pulse; T_2 is the transverse relaxation time; $a_{kl}^{(n)}$ is a matrix element of the transition matrix $\boldsymbol{A}$ for the nth excitation pulse.

A matrix element of the transition matrix for the second (delta-like) excitation pulse of interest to us has the form [6]

$$a_{32}^{(2)}(\Omega) = \frac{\mathrm{i}}{2}\sin\alpha\exp(\mathrm{i}\varphi_\delta), \tag{3}$$

where φ_δ is the initial phase of the delta-like pulse.

Under the initial conditions

$$\boldsymbol{P}(t_0,\Omega) = \begin{pmatrix} 0 \\ 0 \\ p_0 \end{pmatrix} \tag{4}$$

the matrix elements $a_{13}^{(3)}$ and $a_{23}^{(1)}$ determine the corresponding components of the pseudopolarisation vector at the instant of pulse termination:

$$\tilde{p}^*(\Omega) = p_0 a_{23}^{(1)}(\Omega),\ \tilde{p}(\Omega) = p_0 a_{13}^{(3)}(\Omega). \tag{5}$$

3. Mathematical expectation of the complex envelope of a stimulated echo

Because the first and third excitation pulses are random processes, the function $\tilde{p}_\mathrm{s}(t,\Omega)$ also a random function of time. Let us calculate the mathematical expectation of this function. For this purpose, we will use the relation

$$\langle a_{23}^{(1)}(\Omega) a_{13}^{(3)}(\Omega)\rangle = \frac{\langle \tilde{p}^*(\Omega,\sigma_1)\tilde{p}(\Omega,\sigma_3)\rangle}{p_0^2}. \tag{6}$$

which follows from expression (5). To find $\langle \tilde{p}(\Omega,\sigma_3)\tilde{p}^*(\Omega,\sigma_1)\rangle$ upon excitation by white Gaussian noise, we consider stochastic Bloch equations in a coordinate system at rest using the interpretation of Stratonovich [12]:

$$\mathrm{d}\boldsymbol{P} = (\boldsymbol{C}\boldsymbol{P} + \boldsymbol{c})\mathrm{d}t + \boldsymbol{D}\boldsymbol{P}\mathrm{d}W(t), \tag{7}$$

$$\boldsymbol{C} = \begin{pmatrix} -T_2^{-1} & \omega & 0 \\ -\omega & -T_2^{-1} - \sigma^2/2 & 0 \\ 0 & 0 & -T_1^{-1} - \sigma^2/2 \end{pmatrix},$$

$$\boldsymbol{D} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma \\ 0 & -\sigma & 0 \end{pmatrix}, \boldsymbol{c} = \begin{pmatrix} 0 \\ 0 \\ p_0/T_1 \end{pmatrix},$$

where $\boldsymbol{P}$ is the vector of the system state with the Cartesian coordinates p_1, p_2, p_3, which is a pseudopolarisation vector; $p_1 = 0$, $p_2 = 0$, and $p_3 = p_0$ are the initial conditions; $\mathrm{d}W(t) = s(t)\mathrm{d}t$; $W(t)$ is the Wiener process; $\sigma s(t)$ is the function describing the input action; and σ is a dimensionless coefficient.

We assume that $s(t)$ is the white Gaussian noise with the spectral power density $N_0 = 1$ rad^2 s^{-1}. Then, the spectral power density of the process described by the function $\sigma s(t)$ is $N = \sigma^2 N_0$.

Consider two pseudopolarisation vectors $\boldsymbol{P}(\sigma_1)$ and $\boldsymbol{P}(\sigma_3)$, which correspond to the same transition at the frequency ω, but excited by noise pulses with different intensities σ_1 and σ_3. Let us introduce the vector $\boldsymbol{X}$ with the components

$$x_1 = p_1(\sigma_1), x_2 = p_2(\sigma_1), x_3 = p_3(\sigma_1), x_4 = p_1(\sigma_3),$$

$$x_5 = p_2(\sigma_3), x_6 = p_3(\sigma_3). \tag{8}$$

Let us define a covariance matrix $\boldsymbol{K}_{\boldsymbol{X}}(t) = \langle \boldsymbol{X}(t)\boldsymbol{X}^\mathrm{t}(t)\rangle$, where $\boldsymbol{X}^\mathrm{t}$ is a transposed vector. According to the Ito theorem [13], if there are d stochastic processes $x_i(t)$ specified by their stochastic differentials

$$\mathrm{d}x_i = f_i\,\mathrm{d}t + G_i\,\mathrm{d}W \tag{9}$$

and produced by the same Wiener process $W(t)$, then the stochastic process $\boldsymbol{Y}(t) = \boldsymbol{u}(t, x_1(t), \ldots, x_d(t))$ has the stochastic differential

$$\mathrm{d}\boldsymbol{Y}(t) = \left(\boldsymbol{u}_t + \sum_{i=1}^{d}\boldsymbol{u}_{x_i} f_i + \frac{1}{2}\sum_{i=1}^{d}\sum_{j=1}^{d}\boldsymbol{u}_{x_i x_j} G_i G_j\right)\mathrm{d}t + \sum_{i=1}^{d}\boldsymbol{u}_{x_i} G_i \mathrm{d}W(t), \tag{10}$$

where $\boldsymbol{u}_t = \partial\boldsymbol{u}/\partial t$; $\boldsymbol{u}_{x_i} = \partial\boldsymbol{u}/\partial x_i$; $\boldsymbol{u}_{x_i x_j} = \partial^2\boldsymbol{u}/\partial x_i \partial x_j$.

In the case under study, $\boldsymbol{u}(t, x_1(t), \ldots, x_d(t)) = \boldsymbol{X}(t)\boldsymbol{X}^{\mathrm{t}}(t)$, and stochastic processes x_i in (8) and (9) are specified by stochastic Bloch equations (7) for the parameter σ that is equal to σ_1 or σ_3.

The elements of the covariance matrix $\boldsymbol{K_X}(t)$ of interest are determined by averaging the differential equations obtained from (10):

$$\frac{\mathrm{d}\langle x_1x_4\rangle}{\mathrm{d}t} = -\frac{2}{T_2}\langle x_1x_4\rangle + \omega\langle x_2x_4\rangle + \omega\langle x_1x_5\rangle,$$

$$\frac{\mathrm{d}\langle x_1x_5\rangle}{\mathrm{d}t} = -\left(\frac{2}{T_2}+\frac{\sigma_3^2}{2}\right)\langle x_1x_5\rangle + \omega\langle x_2x_5\rangle - \omega\langle x_1x_4\rangle,$$

$$\frac{\mathrm{d}\langle x_2x_4\rangle}{\mathrm{d}t} = -\omega\langle x_1x_4\rangle - \left(\frac{2}{T_2}+\frac{\sigma_1^2}{2}\right)\langle x_2x_4\rangle + \omega\langle x_2x_5\rangle,$$

$$\frac{\mathrm{d}\langle x_2x_5\rangle}{\mathrm{d}t} = -\omega\langle x_1x_5\rangle - \omega\langle x_2x_4\rangle \quad (11)$$

$$-\left(\frac{2}{T_2}+\frac{\sigma_1^2+\sigma_3^2}{2}\right)\langle x_2x_5\rangle + \sigma_1\sigma_3\langle x_3x_6\rangle,$$

$$\frac{\mathrm{d}\langle x_3x_6\rangle}{\mathrm{d}t} = -\left(\frac{2}{T_1}+\frac{\sigma_1^2+\sigma_3^2}{2}\right)\langle x_3x_6\rangle + \sigma_1\sigma_3\langle x_2x_5\rangle$$

$$+\frac{p_0}{T_1}[\langle x_3(t)\rangle + \langle x_6(t)\rangle].$$

To find (6), we pass to the coordinate system, which rotates around the longitudinal axis with the frequency ω_0:

$$\tilde{p}^*(\Omega, \sigma_1) = (x_1 + \mathrm{i}x_2)\exp(\mathrm{i}\omega_0 t),$$

$$\tilde{p}(\Omega, \sigma_3) = (x_4 - x_5)\exp(-\mathrm{i}\omega_0 t).$$

Let us introduce the notation

$$\tilde{y} = \langle \tilde{p}^*(\Omega, \sigma_1)\tilde{p}(\Omega, \sigma_3)\rangle,$$

$$\tilde{y}^* = \langle \tilde{p}(\Omega, \sigma_1)\tilde{p}^*(\Omega, \sigma_3)\rangle,$$

$$y_z = \langle x_3(\Omega, \sigma_1)x_6(\Omega, \sigma_3)\rangle.$$

Then, the system of differential equations (11) in the coordinate system at rest will correspond to the system of differential equations in the rotating coordinate system

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y} = -\left(\frac{2}{T_2}+a\right)\tilde{y} + by_z,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}^* = -\left(\frac{2}{T_2}+a\right)\tilde{y}^* + by_z, \quad (12)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}y_z = \frac{b}{4}\tilde{y} + \frac{b}{4}\tilde{y}^* - \left(\frac{2}{T_1}+2a\right)y_z + \frac{p_0}{T_1}[\langle x_3(t)\rangle + \langle x_6(t)\rangle],$$

$$a = \frac{\sigma_1^2+\sigma_3^2}{4},\ b = \sigma_1\sigma_3.$$

The pulse duration in devices for signal processing usually satisfies the condition $\tau \ll T_1,\ T_2$, which allows one to ignore relaxation processes during pulsed excitation. In this case, under the initial conditions (4), the solution of (12) for the component of interest to us at the instant of pulse termination has the form

$$\tilde{y} = p_0^2\frac{b}{(a^2+2b^2)^{1/2}}[\exp(r_2\tau) - \exp(r_3\tau)],$$
$$r_{2,3} = -\frac{3}{2}a \pm \frac{1}{2}(a^2+2b^2)^{1/2}. \quad (13)$$

Using expressions (2), (3), (6), and (13), we can find the mathematical expectation for the complex envelope $\langle \tilde{p}_{\mathrm{s}}(t)\rangle$ of the stimulated echo. In this case, to take into account contributions from all components of the inhomogeneously broadened system, we should perform the weighted integration over the frequencies of all transitions:

$$\langle \tilde{p}_{\mathrm{s}}(t)\rangle = 2\pi\mathrm{i}p_0\sin\alpha\exp(\mathrm{i}\varphi_\delta)k_{\mathrm{r}}(t)A_{\mathrm{s}}G(t - t_2 - t_3), \quad (14)$$

$$A_{\mathrm{s}} = \frac{b}{2(a^2+2b^2)^{1/2}}[\exp(r_2\tau) - \exp(r_3\tau)], \quad (15)$$

$$G(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(\Omega)\exp(\mathrm{i}\Omega t)\mathrm{d}\Omega.$$

4. Discussion of results

The dependence of the average amplitude of the stimulated echo on the parameters σ_1, σ_3, and τ of noise pulses is determined by the function A_{s}. Fig. 2 shows the dependences $A_{\mathrm{s}}(\sigma_3)$ for $N_0 = 1$ rad^2 s^{-1}, $\tau = 10$ ns, and different σ_1. These dependences are nonlinear because of the saturation of the level populations. As the parameter σ_3 increases, the echo amplitude first increases linearly, then, its increase slows down, the amplitude reaches its maximum and begins to decrease.

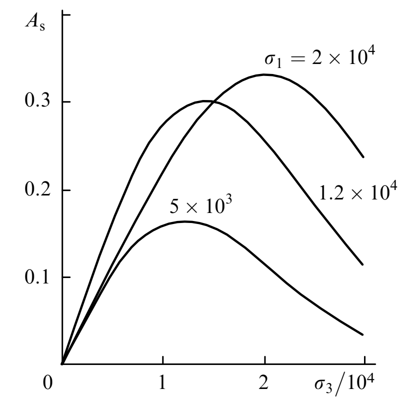

Figure 2. Dependences of the average amplitude of a stimulated echo on σ_3 for different σ_1, $N_0 = 1$ rad^2 s^{-1} and $\tau = 10$ ns.

In the linear regime for the third pulse for $\sigma_3 = \mathrm{const} \leqslant 5 \times 10^3$, the echo amplitude first linearly increases with increasing parameter σ_1 (the curve for $\sigma_1 = 5 \times 10^3$), then, it reaches the maximum (the curve for $\sigma_1 = 1.2 \times 10^4$) and begins to fall (the curve for $\sigma_1 = 2 \times 10^4$). As a whole, the echo amplitude behaves symmetrically with respect to parameters σ_1 and σ_3. Note also that the maximum of the echo amplitude $A_{\mathrm{s}}(\sigma_3)$ for $\sigma_1 = \mathrm{const}$ shifts to a greater values of the parameter σ_3 with increasing σ_1.

If $\sigma_1 = \sigma_3 = \sigma$, then the average echo amplitude is described by the expression

$$A_s = \frac{1}{3}\left[1 - \exp\left(-\frac{3}{2}\sigma^2\tau\right)\right], \quad (16)$$

which follows from (13) and (15).

This function is presented in Fig. 3. The average amplitude of the stimulated echo tends to 1/3 with increasing σ^2.

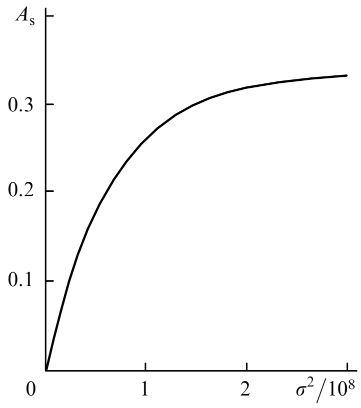

Figure 3. Dependence of the average amplitude of a stimulated echo on σ^2 for $\sigma_1 = \sigma_2 = \sigma$, $N_0 = 1$ rad^2 s^{-1} and $\tau = 10$ ns.

The average amplitude of the stimulated echo in the algorithm with noise incoherent pulses can be compared with the stimulated-echo amplitude in the algorithm with coherent delta-like pulses [6]

$$A_s = \frac{1}{2}\sin\alpha_1 \sin\alpha_2 \sin\alpha_3, \quad (17)$$

where α_i is the area of the envelope of the ith pulse. In the second case, the maximum amplitude equal to 1/2 is achieved for $\alpha_1 = \alpha_2 = \alpha_3 = \pi/2$.

The results obtained for the white noise can be generalised to the case of coloured noise with the spectral power density $N(\Omega)$. The inhomogeneously broadened system consists of a number of parallel narrow-band channels with close frequencies, which form homogeneous lines. The spectral width of such a narrow-band channel excited by the white noise with the spectral power density N_0 is determined by the natural width and the broadening caused by the noise field and is described by the relation [14]

$$\Delta\Omega_0 = \frac{2}{T_2} + \frac{N_0}{2}. \quad (18)$$

If the spectral width of the function $N(\Omega)$ equal to $\Delta\Omega_N$ satisfies the condition $\Delta\Omega_N \gg \Delta\Omega_0$, then such a coloured noise can be treated as a white noise with respect to the narrow-band channel with frequency Ω and its spectral power density can be considered constant and determined from the condition $N_0 = N(\Omega)$, Then, the coefficients a and b will be determined in terms of spectral power densities of the first ($N_1(\Omega)$) and third ($N_3(\Omega)$) excitation pulses:

$$a(\Omega) = \frac{N_1(\Omega) + N_3(\Omega)}{4}, \quad b(\Omega) = [N_1(\Omega)N_3(\Omega)]^{1/2}. \quad (19)$$

The complex envelope of the stimulated echo will be determined by the expressions

$$\langle\tilde{p}_s(t)\rangle = 2\pi i p_0 \sin\alpha \exp(i\varphi_\delta) k_r(t) U_s(t - t_2 - t_3),$$
$$U_s(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(\Omega) A_s(\Omega) \exp(i\Omega t) d\Omega. \quad (20)$$

In the linear regime for $\sigma_1 = \sigma_2 = \sigma$, it follows from (16) that

$$A_s(\Omega) \approx \frac{1}{2} N_0(\Omega)\tau. \quad (21)$$

In this case, if the spectral power density of the noise is concentrated within the frequency interval limited by the inhomogeneous line and the line itself satisfies the condition $g(\Omega) \approx g_0 = \text{const}$ within this interval, then, according to (20) and (21), the function $U_s(t)$ will have the form of the correlation function of the noise, which is related to $N(\Omega)$ by the inverse Fourier transform.

For the determinate signal $s(t)$ with the corresponding spectral density $S(i\Omega)$, we have $A_s(\Omega) \approx |S(i\Omega)|^2/2$ in the linear regime, which also corresponds to the autocorrelation function of this signal.

5. Conclusions

The results obtained above allow one to optimise the parameters of incoherent pulses for achieving the maximum amplitude of a stimulated echo. It is also possible to analyse nonlinear distortions and to determine the upper bound of the dynamic range of the processor and the upper bound of its linear interval in the correlation-processing regime. In addition, note the possibility of analysis of the analogous algorithm for signal processing in spin processors.

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References

1. Samartsev V V *Izv. Ross. Akad. Nauk, Ser. Fiz.* **63** 835 (1999)
2. Zuikov V A, Kalachev A A, Nefed'ev L A, Samartsev V V *Kvantovaya Elektron.* **23** 273 (1996) [*Quantum Electron.* **26** 267 1996)]
3. Khasanov O Kh, Smirnova T V *Kvantovaya Elektron.* **23** 447 (1996) [*Quantum Electron.* **26** 437 (1996)]
4. Man'kov V Yu, Parkhomenko A Yu, Sazonov S V *Kvantovaya Elektron.* **24** 934 (1997) [*Quantum Electron.* **27** 908 (1997)]
5. Ustinov V B, Kovalevskii M M, Baruzdin S A *Izv. Akad. Nauk SSSR, Ser. Fiz.* **50** 1459 (1986)
6. Baruzdin S A, Egorov Yu V, Kalinikos B A, et al. *Funktsional'nye Ustroistva Obrabotki Signalov (Osnovy Teorii i Algoritmy)* (Functional Devices for Signal Processing (Theoretical Foundations and Algorithms) (Moscow: Radio i Svyaz', 1997)
7. Asaka S, Nakatsuka N, Fujiwara M, Matsuoka M *Phys. Rev. A* **29** 2286 (1984)
8. Baruzdin S A *Opt. Spektrosk.* **85** 634 (1998)
9. Beach R, Hartmann S R *Phys. Rev. Lett.* **53** 663 (1984)
10. Nakatsuka H, Wakamiya A, Abedin K M, Hattori T *Opt. Lett.* **18** 832 (1993)
11. Manykin E A, Samartsev V V *Opticheskaya ekho-spektroskopiya* (Optical Echo Spectroscopy) (Moscow: Nauka, 1984)
12. Baruzdin S A *Zh. Tekh. Fiz.* **69** 65 (1999)
13. Bartholdi E, Wokaun A, Ernst R R *Chem. Phys.* **18** 57 (1976)
14. Knight W R, Keiser R *J. Magn. Reson.* **48** 293 (1982)