

# Influence of spatial mode structure on the spectrum evolution in tunable lasers

Yu N Parkhomenko, O V Anisimova

**Abstract.** The effect of spatial mode structure on nonlinear processes in the active medium of lasers with elements having an angular dispersion is investigated. It is shown that the ‘spatially inhomogeneous broadening’ of the spectrum may result in a change in its shape and its broadening. The algorithm for synthesising a continuous spectrum in resonators with a controllable structure of the spectral function is modified.

**Keywords:** dispersive resonator, active medium, nonlinear processes, spectrum evolution

## 1. Introduction

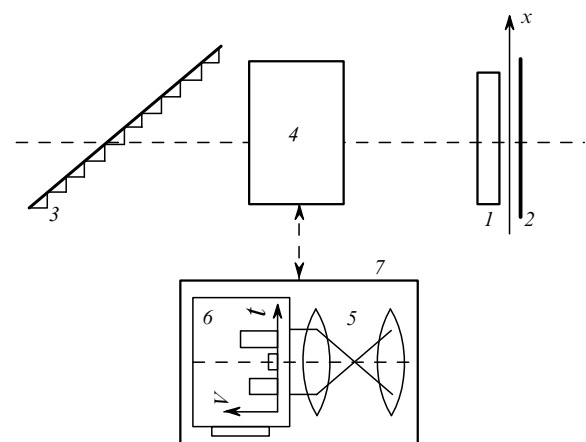
At present, tunable lasers serve as a widely accepted tool in many scientific and technological applications (e.g., in spectroscopy, communications, metrology and isotope separation). Some of the applications require a single-frequency lasing line which can be obtained by means of elements that considerably increase the losses (Fabry–Perot interferometer or grazing incidence grating [1]). However, lasers with a selection of radiation by means of more economical and simpler elements (gratings, prisms and, less frequently, interference polarisation filters [1]) with a linewidth of  $\sim 1 - 10$  pm, which is sufficient for most practical applications, are employed more frequently.

It can be assumed that most of the mechanisms governing the selection of radiation and the formation of its spectrum in tunable lasers have been established. However, a number of questions still remain to be answered. One of them concerns the effect of the active medium on the lasing spectrum and its evolution. It is assumed that in case of a homogeneous broadening of the emission spectrum of the active medium (only such media are efficient in tunable lasers), a narrowing of the spectrum is proportional to the square root of the number of round trips of radiation in the cavity. Such a narrowing does not depend on the nature of the filter (see, for example, Ref. [2]).

However, lasers in which mode selection is provided by the angular dispersion (in gratings and prisms) have a distinguishing feature related to the specific nature of the selection mechanism: a change in the wavelength within the lasing linewidth is accompanied by a displacement of the field in the transverse direction in the resonator and, in particular, in the active medium. No significance has been attached to this circumstance so far, and this prompted us to study the effect of this peculiarity on the emission spectrum of lasers with a simple lasing line and with a synthesised complex spectrum [3]. The results of these investigations are presented in this work, where it is shown that under certain conditions, such a peculiarity can basically change the laws governing the spectrum evolution.

## 2. Model and basic equations

At the stage of nonlinear generation, modes with different transverse distributions (and even ‘longitudinal’ modes) will ‘saturate’ the active medium in different ways, which is equivalent to its ‘spatially inhomogeneous broadening’. Consider a travelling-wave laser or a linear laser (Fig. 1) in which the length  $\Delta z$  of a four-level active medium (1) and its separation from mirror (2) (in the case of a linear laser) are much smaller than the length  $l$  of the resonator (this situation is typical of dye and other lasers). The medium (1) is assumed to be a thin layer brought in coincidence



**Figure 1.** Optical scheme of a tunable laser: (1) active medium, (2) mirror, (3) diffraction grating, (4, 7) optical systems, (5) phase modulator, (6) amplitude modulator.

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with the mirror (2) in a linear resonator. This simplifies the procedure of derivation of equations and makes it possible to take into account the above-mentioned nonlinear effects while preserving all the properties of a dispersive resonator and the mechanism of mode selection in it (an increase in losses upon a deviation from its tuning wavelength  $\lambda_0$  by  $\Delta\lambda$ ).

We expand the field  $E(x, t)$  in the plane of the medium (1) in dispersive resonator modes  $\{u_{nj}(x)\}$ , i.e., in eigenfunctions of the equation

$$\Lambda u(x) = \int_{-\infty}^{\infty} K(x, x_1)u(x_1)dx_1. \quad (1)$$

The kernel  $K(x, x_1)$  (whose form will be discussed below) characterises field transformations in linear elements, including the diaphragms. The operator  $\Lambda$  has eigenvalues  $A_{nj} = \exp(-\gamma_{nj})$ , where  $\gamma_{nj}$  defines the loss logarithm and the phase shift of modes;  $n = 0, 1, 2 \dots$  is the transverse mode index; and  $j = m - m_0$  is counted relative to the longitudinal index  $m_0$  for the mode with  $\Delta\lambda_0 = 0$ . The spectral function  $G(\lambda_0 + \Delta\lambda) = |A_{0j}(\Delta\lambda_j)|^2 / |A_{00}(0)|^2$  of the dispersive resonator [3] defines its selectivity. In this case, we obtain

$$E(x, t) = \sum_j \exp(i\omega_j t) \sum_n E_{nj}(t)u_{nj}(x), \quad (2)$$

where  $E_{nj}(t)$  are the time-dependent expansion coefficients taking into account the variations related to the difference in frequencies ( $\omega_j$ ) of longitudinal ( $n = 0$ ) and transverse modes. After a round trip of radiation through the resonator, we obtain

$$E_{kj}(t + \tau_c) = \sum_n E_{nj}(t)F_{knj}(t), \quad (3)$$

where

$$F_{knj}(t) = \int dx_1 \int dx_2 K(x_1, x_2) \exp[2sN_2(x_2, t)\Delta z] \times v_{kj}(x_1)u_{nj}(x_2); \quad (4)$$

$v_{kj}(x)$  are the eigenfunctions of an equation conjugate to (1) with an orthogonality  $\int v_{kj}(x)u_{nj}(x)dx = \delta_{kn}$  [in a linear resonator,  $v_{kj}(x) = u_{kj}(x)$ ];  $\tau_c$  is the round-trip transit time in the cavity;  $N_2(x, t)$  is the population of the upper laser level; and  $s = s(\lambda_0)$  is the cross section for active centres.

We determine the increment in  $E_{kj}$  per round trip and divide it by  $\tau_c$ . Going from finite differences to derivatives and using the traditional approximation of smallness of the changes during a round trip (for series expansion of the exponent and for other applications), we arrive at the following equation for  $E_{kj}$ :

$$\frac{dE_{kj}}{dt} = \frac{\sum_n E_{nj}(t)2s\Delta z \int N_2(x, t)u_{nj}(x)v_{kj}(x)dx - \gamma_{kj}E_{kj}(t)}{\tau_c}. \quad (5)$$

The constitutive equations have the conventional form and are presented below.

### 3. Evolution of the spectrum of a single line with a simple regular structure

The set of equations (5) can be simplified by using additional approximations that have been verified experimentally and theoretically, in particular, for tunable lasers with angular dispersion. Consider the case of 'transmission' of a resonator with a parabolic or Gaussian law, when the selection within the lasing line is associated with the displace-

ment of the field to lower reflectivity regions during misalignment of the system by varying  $\lambda$  [4]. In this case, which is encountered in many lasers (with longitudinal pumping, with a profiled reflection coefficient of the mirrors for improving mode composition [5–7], etc.), only longitudinal modes are involved in lasing for a quite long duration of the linear stage and for the diaphragm parameter  $\beta = [\pi w^2/(\lambda l)]^{1/2} < 10$ , where  $2w$  is the width of the diaphragm (for  $\beta > 10$ , resonators with angular dispersion reduce the selectivity), while mode fields and their losses are approximated by analytic expressions. We shall use these approximations for simplifying the equations.

Suppose that the laser contains a diffraction grating (3), and the system (4) consists of Gaussian optics elements that provide the generation of a single line. The kernel of Eqn (1) with the reference plane on mirror (2) has the form [8, 9]

$$K(\xi_1, \xi) = \left(-\frac{i}{2\pi}\right)^{1/2} \exp\left\{i\left[f(\xi_1 + \xi) + \frac{g\xi_1^2 + g\xi^2 - 2\xi_1\xi}{2}\right]\right\},$$

where  $\xi = x[2\pi/(\lambda B_s)]^{1/2}$  is a dimensionless coordinate;  $f = (2\pi B_s/\lambda)^{1/2} D_a \Delta\lambda/2$ ;  $g = A_s$ ;  $A_s = 1 + 2CB$ ;  $B_s = 2BD$ ;  $A, B, C, D$  are the elements of the resultant resonator matrix including the Gaussian diaphragm;  $D_a$  is the angular dispersion of the grating (3). The solutions of Eqn (1) for the field  $u_{0j}$  of longitudinal modes and their eigenvalues  $A_{0j}$  have the form

$$u_{0j} = \sqrt{\frac{2\sqrt{g^2 - 1}}{i\pi}} \exp\left[i\sqrt{g^2 - 1} \frac{(\xi + \Theta_j)^2}{2}\right],$$

$$A_{0j} = M^{1/2} \exp\left(\frac{-if_j^2}{g - 1}\right),$$

where  $M = g + (g^2 - 1)^{1/2}$ ;  $f_j = (2\pi B_s/\lambda_j)^{1/2} D_a \Delta\lambda_j/2$ . The 'tunings'  $\Delta\lambda_j$  in the expression for  $f_j$  are chosen from the condition of reproducibility of mode fields, while the signs of radicals are chosen from the condition of a decrease in the field at infinity. The complex field displacement (of its amplitude and phase) in the transverse direction is determined by the expression  $\Theta_j = f_j/(g - 1)$ . The spectral function  $G(\lambda)$  in this case is a Gaussian with the half-width  $\sigma_0$ .

We will determine  $N_2(x, t)$  from the rate equation, which is conventionally used for many types of tunable lasers (dye solution lasers neglecting triplet transitions, electron–photon transition lasers, etc.), taking into account the parametric dependence on  $x$ , as in Ref. [10], and going over to the multimode case, as in Ref. [11] (assuming the modes to be statistically independent). The equation for the number  $q_j(t)$  of photons is obtained by differentiating the relation  $q_j(t) \sim |E|^2$  and taking Eqn (5) into account:

$$\frac{dq_j(t)}{dt} = 2q_j(t) \left[ \frac{s\Delta z}{\tau_c} \int N_2(x, t)F_j^q(x)dx - \frac{\text{Re } \gamma_{0j}}{\tau_c} \right], \quad (6)$$

$$\frac{dN_2(x, t)}{dt} = W(t)[N_T - N_2(x, t)] - UN_2(x, t) \sum_j F_j^N(x)q_j(t) - \frac{N_2(x, t)}{\tau}, \quad (7)$$

where  $q_j$  is the number of the  $j$ th mode photons;  $U = sc\Delta z/(V_a l)$  is the Einstein coefficient for induced

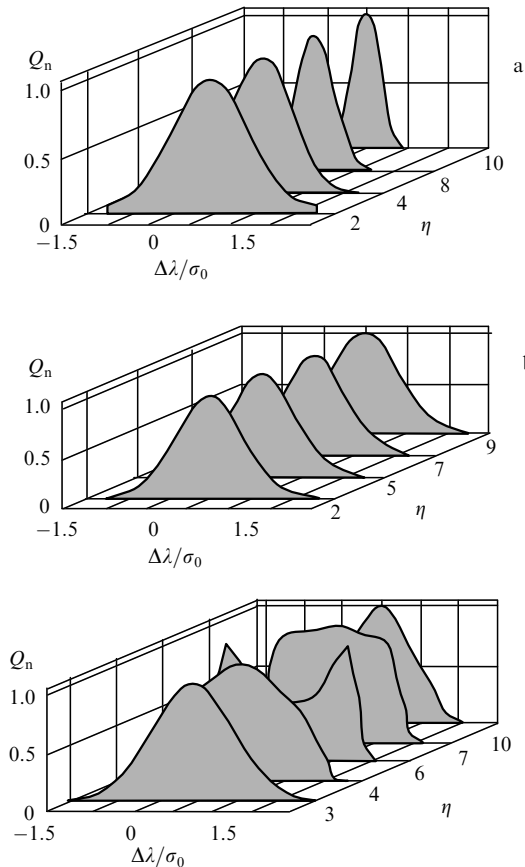
transitions;  $c$  is the velocity of light;  $V_a$  is mode volume in the active medium;  $\tau$  is the spontaneous emission lifetime;  $\text{Re}\gamma_{0j}$  are the losses in the  $j$ th mode (both total and selective);  $W(t)$  is the pump power;  $N_T$  is the number of active centres.

It should be emphasised that preservation of the properties of dispersive resonator led to different ‘coupling’ coefficients in nonlinear terms of the equations for the photon numbers and for inverse population. For a linear laser (see Fig. 1), we can write

$$F_j^q(x) = 2\text{Re}[u_{0j}(x)u_{0j}(x)], \quad F_j^N(x) = u_{0j}(x)u_{0j}^*(x). \quad (8)$$

Thus, Eqns (6)–(8) differ not only in their ‘physical’ content, but also in the structure as compared to the case of a spectrally inhomogeneous medium [10].

The set of equations (6)–(8) was analysed numerically for characteristic parameters of laser-pumped pulsed dye lasers (e.g., rhodamine 6G laser, for which  $s = 1.3 \times 10^{-16} \text{ cm}^2$ ,  $\tau \simeq 4 \text{ ns}$  [11], etc.). The parameter  $\beta$ , the maximum excess  $\rho = W_{\text{max}}/W_{\text{th}}$  in pumping power over the threshold value, and the number of modes (from 20 to 60) were varied. The initial (seed) mode intensities at the threshold were estimated from the typical experimental mean power  $P(\rho)$  of superluminescence noise, which was of about 0.05–0.2 the power of laser radiation for  $\rho \approx 5$  and had a spectral density  $p(\lambda) = hY(\lambda, N_2)$ , where  $Y(\lambda, N_2) = \exp[2s(\lambda)N_2\Delta z] - 1$ ;  $h = P(\rho) / \int d\lambda Y(\lambda, N_2)$  is a parameter depending on the system geometry [12]. The power per mode near the threshold was  $P_{j\text{th}} \approx (h/\rho)Y(\lambda_0, N_{2\text{th}})\delta\lambda/\chi^2$ , where  $\chi$  is the ratio of the

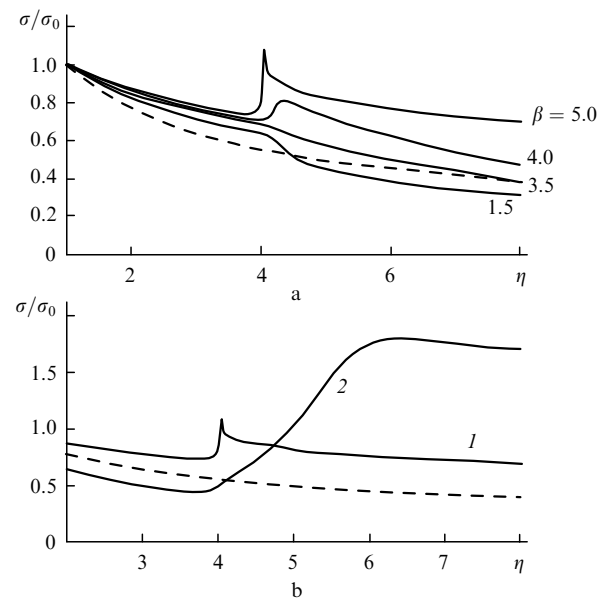


**Figure 2.** Dependences of the laser spectrum  $Q_n$  on time ( $\eta = t/\tau_c$ ) for the diaphragm parameter  $\beta = 4$  for  $\rho = W_{\text{max}}/W_{\text{th}} = 5$  (a), 10 (b), and 15 (c).

divergences of noise and laser radiation ( $\sim 5 - 20$  in our case);  $\delta\lambda$  is the spectral width, which was varied from the mode separation  $\lambda_0^2/(2l)$  to the limiting width  $\lambda_0^2/(2cT)$  ( $T$  is the mean duration of the lasing pulse). At the nonlinear stage of the processes considered below, no changes were observed due to a variation of the above-mentioned parameters, mode numbers, or the shape of the leading edge of the pump pulse (as long as the duration of the linear stage is kept long enough to ensure longitudinal mode separation).

The solution of the set of equations (6)–(8) was represented in the form  $Q(\lambda, t) = Q_n(\lambda, t)I(t)$ , where  $Q(\lambda, t) = \sum_j q_j(t)$ ;  $Q_n(\lambda, t)$  is the normalised part of the solution ( $\int Q_n(\lambda, t)d\lambda = 1$ ) reflecting the change in the shape of the spectrum;  $I(t)$  is the intensity of the spectrum. The results of calculations of the function  $Q_n(\lambda, t)$  and its half-width  $\sigma$  are presented in Figs. 2 and 3a (the leading edge of the pump pulse is a segment of a Gaussian). The inclusion of the variation of the spatial structure leads to a considerable rearrangement of the spectral dynamics over the pulse duration for  $\beta \geq 3.5$  and  $\rho > \rho_0$  ( $\rho_0$  depends on  $\beta$  and decreases with increasing  $\beta$ ). Fig. 2 shows typical dependences of  $Q_n(\lambda, t)$  on  $\eta = t/\tau_c$  for the case  $\beta = 4$ .

For small  $\rho$  ( $\rho = 5$ , Fig. 2a), the evolution of  $Q_n(\lambda, t)$  is similar to the case considered in Ref. [2], i.e., it gradually narrows down, but its shape is preserved. Upon an increase in  $\rho$  ( $\rho = 15$ , Fig. 2c), the  $Q_n(\lambda, t)$  spectrum narrows down only during the first few round trips of the radiation in the resonator ( $\eta \leq 3$ ), after which the spectrum is gradually transformed ( $\eta = 4 - 10$ ), its wings are raised, and it is transformed successively into a rectangular and double-humped shape followed by the same transformations in the reverse order to a Gaussian and a subsequent narrowing of the spectrum. Samples of these phases, constituting the first cycle of the nonlinear dynamics of the spectrum, are shown in Fig. 2c. The cycle is repeated upon an increase in the pump duration: after a secondary narrowing ( $\eta = 14 - 26$ ),



**Figure 3.** Dependence of the current ratio  $\sigma/\sigma_0$  on time ( $\eta = t/\tau_c$ ) calculated for  $\rho = 15$  and different  $\beta$  (a), as well as a comparison of the dependences obtained from Eqns (6)–(8) (curve 1) and those borrowed from [10] (curve 2) for  $\rho = 15$ ,  $\beta = 5$  (b). The dashed curves show the dependence of  $\sigma/\sigma_0$  on  $\eta$  in accordance with Ref. [2].

the spectrum is repeatedly transformed to a double-humped curve ( $\eta = 28 - 30$ ), then into a Gaussian, and so on.

The time dependence of the spectral width (width of the function  $Q_n(\lambda, t)$  on  $\eta = t/\tau_c$  in Fig. 3a) obtained by us differs from that obtained in Ref. [2] (dashed curve). Note the appearance of jumps upon the transition to the nonlinear stage and slowing down of the spectrum narrowing. These effects are explained by the fact that at the linear stage of the spectrum narrowing, peripheral regions are formed in the active medium, which are weaker occupied by mode fields than the other regions. For  $\beta \geq 3.5$ , these regions are large enough to accommodate the modes of new spectral peaks. For  $\rho > \rho_0$ , the degree of 'saturation' at the centre is so high that amplification at the nonlinear stage dominates at the periphery and lasing is observed, which changes the shape of the spectrum. For  $\rho = \rho_0$  (Fig. 2b), the dynamic equilibrium may be observed between the processes at the periphery and at the centre (the width and shape of the spectrum remain virtually unchanged).

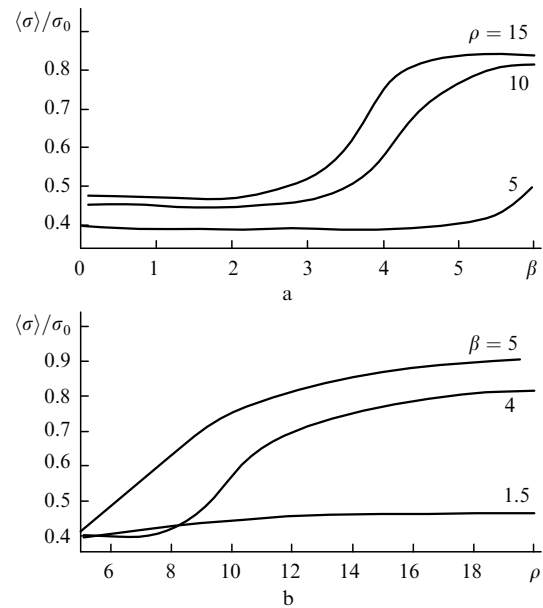
The radiation emitted by the laser in the case considered by us differs in principle from the radiation emitted by a laser with spectrally inhomogeneous broadening of the laser transition [10] in the following aspects (in spite of the similarity of these effects): (1) the transition remains homogeneously broadened, and the role of an inhomogeneous broadening is played by the regular variation of field distribution within the lasing line; (2) a change in the 'coupling' terms (8) leads to a qualitative and quantitative rearrangement of the dynamics of the emergence of these phases, which is confirmed by a comparative analysis of solutions of two types of equations carried out by us. Fig. 3b illustrates an example of such a situation, in which the passage to Eqns (6)–(8) (curve 1) is accompanied by a significant variation of the shape of the curve and by the emergence of a jump during a transition to the nonlinear stage.

The average parameters of the output laser spectrum are determined by the function  $Q(\lambda, t)$ . Fig. 4 shows the dependences of the width of this function (spectral width) on parameters  $\beta$  and  $\rho$ , while the corresponding dependences of its form (spectral shape) averaged over the first cycle are shown in Fig. 5. For  $\beta > 3.5$ , an increase in the pump energy (Fig. 4b) leads to a strong broadening of the spectrum due to nonlinear processes. Fig. 5a shows the transformation  $Q(\lambda) = \langle Q(\lambda, t) \rangle$  with increasing  $\beta$  for a fixed pump energy ( $\rho = 15$ ) through a series of intermediate phases, while Fig. 5b shows another aspect of this process ( $\rho$  changes and  $\beta = 4$ ) associated with an increase in the effect of peripheral regions (for comparison, the function  $G(\lambda)$  is shown by the dashed curve).

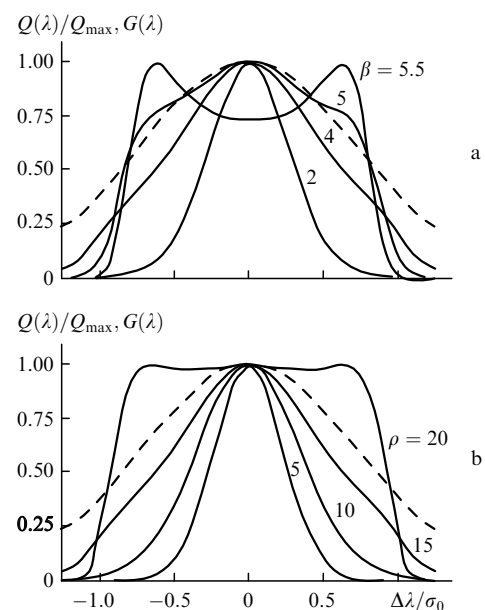
Note that specific laser parameters were chosen for the convenience of presenting the results, which are also obtained for other parameters (other lasers) with increasing the lasing duration by several orders of magnitude.

#### 4. Effect of the active medium in a laser with spectral synthesis

Another class of problems that can be studied by using Eqn (5) involves the evolution of more complex spectra. We will consider an important laser for practical applications with a rearrangement (synthesis) of the spectral line [3], whose analysis will be carried out by using the results and solutions obtained in the previous section. The method used for this analysis [3] is based on controlling the shape of the



**Figure 4.** Dependences of the ratio  $\langle \sigma \rangle / \sigma_0$  averaged over the first cycle of spectral dynamics on  $\beta$  for different values of  $\rho$  (a) and on  $\rho$  for different values of  $\beta$  (b).



**Figure 5.** Spectra averaged over the first cycle of spectral dynamics for (a) different values of  $\beta$  for  $\rho = 15$  and (b) different values of  $\rho$  for  $\beta = 4$  (the dashed curve is the spectral function  $G(\lambda)$  of a dispersive resonator with a half-width  $\sigma_0$ ).

function  $G(\lambda)$  during a rearrangement of the transverse spatial distribution of the amplitude and phase of the transmission coefficient. The varying phase, controlled by the modulator (5) in the system (7) (Fig. 1), 'tunes' the partial regions to different wavelengths, and their contribution to  $G(\lambda)$  is controlled by the amplitude established during the rearrangement of the shape of the acoustic wave  $V(t)$  in the acousto-optical modulator (6).

For a discrete distribution of  $a$ , the resonator can be represented as a system of partial resonators with a composite function  $G(\lambda) = \sum a_r G_r(\lambda - \lambda_r)$  (whose form is presented in

Ref. [3]), where  $G_r(\lambda)$  is the spectral function of the partial resonator  $r$ ;  $\lambda_r$  is its tuning wavelength; and  $a_r$  is the amplitude of its contribution.

Let us see how the results of calculations vary when the contribution of the above processes occurring in the active medium is taken into account. In this case, a complete analysis of the system (5) or (6)–(8) requires the inclusion of a large number of modes and is virtually impossible. Hence, we use one more simplification in addition to those made above. We present the intensity of radiation in terms of the number  $q_{rj}$  of photons in the partial resonator modes, and characterise the spatial structure of these modes through the distribution of the central longitudinal mode by putting  $F_{rj}^{q_r N}(x) = F_r^{q_r N}(x)$ . Omitting simple transformations, we arrive at the following equations:

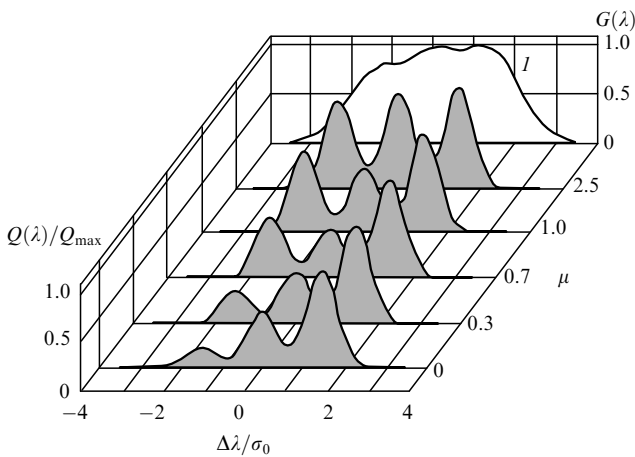
$$\frac{dN_2(x, t)}{dt} = W[N_T - N_2(x, t)] - UN_2(x, t) \sum_r F_r^N(x - x_r)q_r(t) - \frac{N_2(x, t)}{\tau}, \quad (9)$$

$$\frac{dq_r(t)}{dt} = q_r(t) \left[ UV \int_{-\infty}^{+\infty} F_r^q(x - x_r)N_2(x)dx - \sum_j \text{Re} \gamma_{rj} \Phi_r(\lambda_j - \lambda_r, t)/\tau_c \right], \quad (10)$$

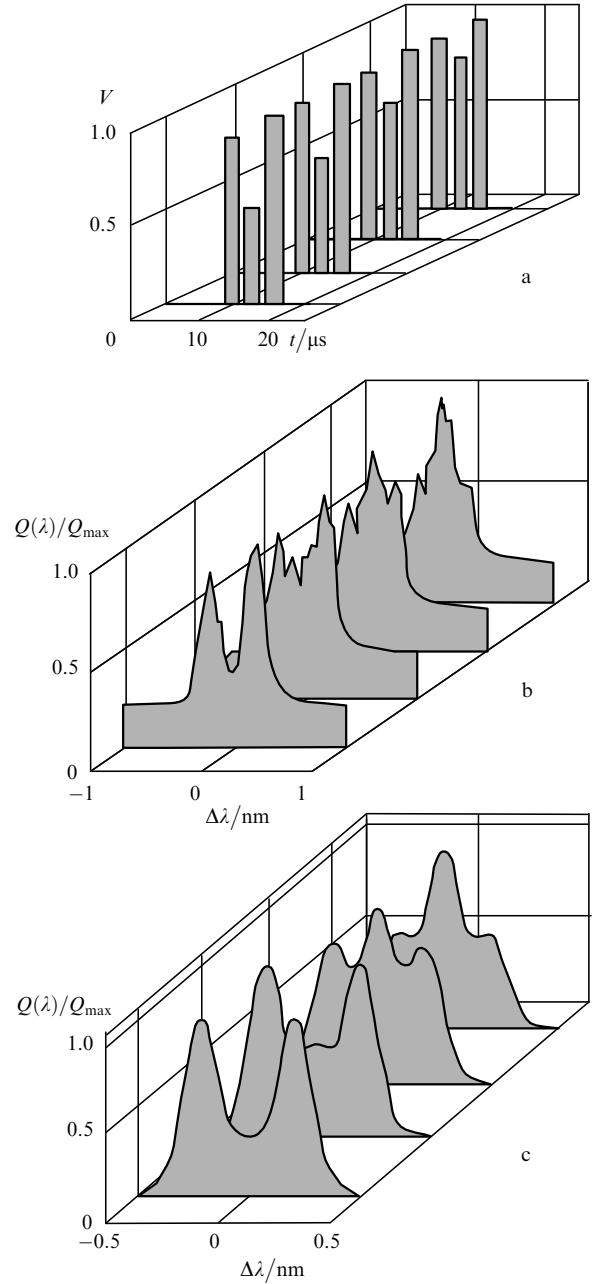
where  $q_r(t) = \sum_j q_{rj}(t)$  is the field intensity of the resonator  $r$  and  $\Phi_r(\lambda_j, t)$  ( $\sum_j \Phi_r(\lambda_j, t) = 1$ ) is a function characterising the time dependence of the field spectrum of resonator  $r$ , for which a solution of the set (6)–(8) is a sufficient approximation. Substitution of  $\Phi_r(\lambda_j, t)$  and  $F_r(x)$  into (9) and (10) leads to a transition from  $\lambda_j$  to the wavelength  $\lambda_r$  of ‘tuning’ of the partial resonator, and from  $x$  to the coordinate  $x_r$  of the resonator axis. The resulting spectrum takes into account the evolution of the spectra of partial resonators and the competition between their fields:

$$Q(\lambda, t) = \sum_r \Phi_r(\lambda, t)q_r(t).$$

A numerical analysis of the set (9), (10) was carried out for a series of functions  $G(\lambda)$  (with different asymmetries



**Figure 6.** Dependences of the shape of the average laser spectrum on the parameter  $\mu$  of ‘spatially inhomogeneous broadening’ (curve  $I$  is the spectral function  $G(\lambda)$  of the resonator).



**Figure 7.** Rearrangement of the excess: relation between the shapes of the sound waves (a), experimental (b) and theoretical (c) laser spectra.

and excesses). Fig. 6 shows the effect of the extent of ‘spatially inhomogeneous broadening’  $\mu$  [ratio of the separation between the peaks of the fields of adjacent resonators in the active medium to the half-width of the function  $F_r(x)$ ] on  $Q(\lambda)$  in lasers with the same pumping kinetics and the same asymmetric function  $G(\lambda)$  (curve  $I$ ). Upon an increase in  $\mu$ , the distribution of the mean values  $\{q_r(t) \approx Q(\lambda_r)\}$  gradually approaches the amplitude distribution  $\{a_r\}$  for the function  $G(\lambda)$  (the difference between these distributions is the largest for  $\mu = 0$ ). In this case, the spectrum becomes more and more distorted upon an increase in the duration between  $\{\lambda_r\}$  due to a competition between modes in partial resonators.

Taking these circumstances into account, we can modify the algorithm of synthesis. We shall try to establish a similarity of the spectrum not with  $G(\lambda)$ , but with an approxi-

mation of this function to the average spectrum by replacing  $G_r(\lambda - \lambda_r)$  with the average spectra of partial resonators [in the case of Gaussian functions, only the width of the function  $G_r(\lambda - \lambda_r)$  is modified] and choosing the initial amplitudes of  $\{a_r\}$  in such a way as to ensure the required relation between the average quantities  $\{\langle q_r(t) \rangle\}$ .

Fig. 7 confirms the efficiency of such an algorithm in a laser with the spatial acoustic modulator ( $\delta$ ) which provides a stepwise distribution of the amplitude transmission for the case of excess rearrangement. The experimental and theoretical average spectra are found to be in satisfactory agreement.

## 5. Conclusions

The most important results of this work, associated with the inclusion of the effect of spatial mode structure on nonlinear processes can be formulated as follows:

(1) In tunable lasers with resonators having an angular dispersion in the region of transition to the angular dispersion of the wide-aperture resonators ( $\beta > 3.5$ ), the spectrum may change its shape from single-humped to double-humped and back through intermediate phases. The dynamics of the spectral variation is considerably altered if we take into account the real normalisation of the dispersive resonator modes. This circumstance may also be important in other problems whose solution in the nonlinear region depends on ratios of mode configurations, in particular, in lasers with other types of resonators (for example, unstable resonators).

(2) The algorithm of continuous spectral synthesis, which takes into account not only the peculiarities of wave selection in dispersive resonators, but also the effect of mode competition in lasers and nonlinear 'spatially inhomogeneous broadening' in the active medium, is substantiated.

The obtained regularities have a wider significance, since they can also be responsible for other effects of practical importance (e.g., a jumpwise displacement of the line upon a shift in the spectral function tuning), including effects which are manifested in a more complex manner under the influence of nonstationary variations of the active medium parameters (thermal lens induction, transverse displacement of the excitation band upon a change in the pump laser characteristics, etc.) and interference effects in the active medium (e.g., in corundum crystals with titanium ion impurities) which change the form of the spectral function.

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