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Theory of differential and integral scattering of laser radiation by a dielectric surface taking a defect layer into account

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Abstract. **The differential and integral light scattering by dielectric surfaces is studied theoretically taking a thin near-surface defect layer into account. The expressions for the intensities of differential and total integral scattering are found by the Green function method. Conditions are found under which scattering by the defect layer can be neglected compared to scattering by the surface roughness. A method is proposed to separate the scattering from the surface roughness and the defect layer. Estimates are made of the typical changes in the permittivity and scattering intensity which correspond to the defect layer related to the higher concentration of structural defects in the near-surface region.**

Keywords: *differential scattering, total integrated scattering, defect layer*

1. Introduction

The methods of differential (angle-resolved) scattering (ARS) and total integral scattering (TIS) are widely used for the metrological study of optical surfaces [1]. These methods involve measurements of the integral or angular powers of radiation scattered by a surface, which is normalised to the power of specularly scattered radiation. The surface scattering is caused by the nonuniformity of optical properties of the surface and the near-surface region, such as the surface roughness and the defect layer, which appear after precision surface processing.

The interpretation of experimental results on angle-resolved and total integral scattering requires the development of a complete physicomathematical model of light scattering by an optical surface with the inclusion of all the underlying effects. Adequately studied to date is only the scattering by a rough surface [2–4]. The issues related to other possible mechanisms of surface scattering, in particular, to the nonuniformity of optical properties of the near-surface region, are poorly known. As a consequence, in the analysis of experimental data it is commonly assumed that roughness is the only source of surface scattering; no estimates of the accuracy of this approximation are made in doing this.

Therefore, the absence of a well-developed theory of surface scattering with the inclusion of both roughness and the near-surface defect layer does not allow one to interpret reliably experimental results obtained by the angle-resolved and integral scattering methods, which involve measurements of the scattered radiation intensity. Moreover, the development of such a theory would enable determining the differences in scattering characteristics related to roughness and the defect layer and thereby proposing the method to determine the parameters of roughness as well as the defect layer with the use of the above methods. Note that the task of determining the parameters of near-surface defect and disturbed layers is topical in several problems of laser physics, such as improvement of the optical surface resistance to laser radiation, frequency lock-in in the cavities of laser gyroscopes, etc.

In this paper, we consider the problem of elastic surface scattering related to the existence of a thin (compared to the wavelength) near-surface layer with small fluctuations of the permittivity or the index of refraction. The model under consideration is an approximation for the description of nonuniformities of the optical properties of the near-surface layer related to the high concentration of structural defects of different type (for instance, vacancies, dislocations, vacancy pores, interstitial atoms, etc.) and also to defect concentration fluctuations (a near-surface layer of this kind will be referred to as a defect layer).

The defect layer appears mainly due to high stresses arising in the precision surface processing, for instance, upon grinding and polishing, resulting in a drastic lowering of the production threshold of structural defects. In addition, the existence of a free surface leads to the appearance of several surface defects (for instance, transition layer, surface electron or Tamm states), which may also cause changes in the optical properties in the near-surface region.

We studied the scattering problem, by the Green function method which was earlier applied to describe the scattering by rough surfaces [3]. The dependences of angle-resolved and total integral scattering on the parameters of a defect layer were determined. We considered the conditions for smallness of the intensity of radiation scattered by the defect layer compared to the intensity of radiation arising from roughness and also the possibility to distinguish the scattering arising from the above mechanisms. A simple model was considered, which describes the effect of structural

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defects on the optical properties and makes it possible to estimate the characteristic parameters of the defect layer and the corresponding scattering related to the high concentration of structural defects. Coefficients of total integral scattering from a defect layer and roughness, as well as the Mandel'shtam – Brillouin scattering intensities for high-precision quartz substrates, are compared.

2. Formulation of the problem and basic equations

Consider a plane monochromatic wave incident from vacuum on a plane surface of an optical dielectric at an angle θ_0 (the z-axis is directed normally to the dielectric surface, the plane of incidence coincides with the xz plane, Fig. 1):

$$\boldsymbol{E}(\boldsymbol{R}) = \boldsymbol{E}_0 \exp(-\mathrm{i}\omega t + \mathrm{i}\boldsymbol{k}^{(0)}\boldsymbol{R}). \tag{1}$$

We will use the following model of the permittivity $\varepsilon(\omega)$. The dielectric is assumed to be described by a constant isotropic real permittivity. In a thin near-surface layer of thickness d there exist small isotropic fluctuations of the permittivity:

$$\varepsilon(\boldsymbol{R}) = \varepsilon^{(0)} - \Delta\varepsilon(x,y)\exp\frac{z}{d}, \quad -d < z < 0, \qquad \varepsilon(\boldsymbol{R}) = \varepsilon^{(0)}, \quad z < -d, \tag{2}$$

$$\frac{\Delta\varepsilon}{\varepsilon^{(0)}} \ll 1, \quad \frac{d}{\lambda} \ll 1. \tag{3}$$

Here, $\Delta\varepsilon(x,y)$ is a random function, which describes the permittivity fluctuations.

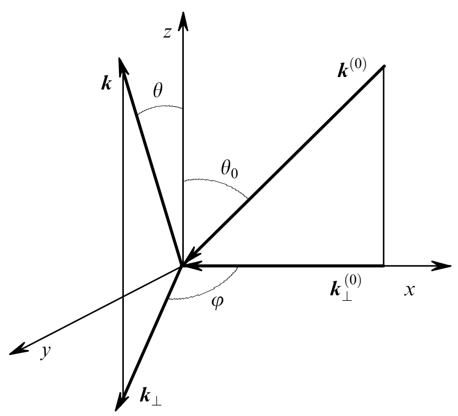

Figure 1. Geometry of the angles.

The stationary problem of elastic surface scattering involves the solution of the wave equation

$$\hat{\Delta}\boldsymbol{E}(\boldsymbol{R}) + \varepsilon^{(0)}\frac{\omega^2}{c^2}\boldsymbol{E}(\boldsymbol{R}) = -\Delta\varepsilon(\boldsymbol{R})\frac{\omega^2}{c^2}\boldsymbol{E}(\boldsymbol{R}) + \frac{1}{\varepsilon^{(0)2}}[\boldsymbol{E}(\boldsymbol{R})\operatorname{grad}\Delta\varepsilon(\boldsymbol{R})]\operatorname{grad}\Delta\varepsilon(\boldsymbol{R}), \tag{4}$$

which follows from the Maxwell equations, with the boundary conditions

$$E_{\mathrm{n}1} = \varepsilon E_{\mathrm{n}2}, \quad E_{\mathrm{t}1} = E_{\mathrm{t}2}, \quad z = 0, \tag{5}$$

where the indices 1 and 2 refer to vacuum and the dielectric, respectively; n and t are the normal and tangential components of the field vectors.

We will assume that the condition

$$\frac{\Delta\varepsilon}{d^2}\sin\theta_0 \ll \varepsilon^{(0)2}\frac{\omega^2}{c^2}, \tag{6}$$

for the smallness of $\Delta\varepsilon$ is fulfilled, which means that the gradient term in the wave equation (4) can be neglected. Note that the condition (6) imposes a limitation on the gradient in the longitudinal direction perpendicular to the surface. The gradient in the transverse direction along the surface is far smaller than the longitudinal one because in the scattering problem, fluctuations of ε over distances longer than or comparable to λ are important, whereas $d \ll \lambda$ is the case of thin layers under study. The condition (6) is fulfilled, for instance, for a defect layer of thickness $d = 100$ Å and $\Delta\varepsilon = 0.01$ for $\lambda = 0.63$ μm.

Therefore, when the condition (6) is fulfilled, the wave equation takes the form

$$\hat{\Delta}\boldsymbol{E}(\boldsymbol{R}) + \varepsilon^{(0)}\frac{\omega^2}{c^2}\boldsymbol{E}(\boldsymbol{R}) = -\Delta\varepsilon(\boldsymbol{R})\frac{\omega^2}{c^2}\boldsymbol{E}(\boldsymbol{R}). \tag{7}$$

When the smallness conditions (3) are fulfilled, the right-hand side of Eqn (7) can be considered as a perturbation of the Fresnel problem

$$\hat{\Delta}\boldsymbol{E} + \varepsilon^{(0)}\frac{\omega^2}{c^2}\boldsymbol{E} = 0, \tag{8}$$

whose solutions are well-known and describe the incident, specularly reflected, and refracted waves.

Therefore, to solve the problem (7), we can use the perturbation theory. To find the scattered radiation intensity in the first order of the perturbation theory, we will use the Green function method and the expansion in terms of the spatial Fourier integrals developed for the scattering by a rough surface in Ref. [3].

3. Results of calculations

Let P_{ik} be the intensity of radiation scattered in the direction corresponding to the scattering angles θ, φ. Normalisation was performed to the incident radiation intensity I_0 and a unit solid angle Ω, the subscripts $i,k = \mathrm{s}$ or p denote the polarisation of the incident and scattered fields, respectively. After calculations, we obtain

$$P_{ik} = \frac{1}{I_0}\frac{\mathrm{d}I_{ik}}{\mathrm{d}\Omega} = \frac{\omega^4}{\pi^2 c^4}\, g\left(\left|\boldsymbol{k}_\perp - \boldsymbol{k}_\perp^{(0)}\right|\right) F_{ik}(\theta,\varphi,\theta_0,\varepsilon), \tag{9}$$

where

$$g\left(\left|\boldsymbol{k}_\perp - \boldsymbol{k}_\perp^{(0)}\right|\right) = \int \exp\left[-\mathrm{i}\left(\boldsymbol{k}_\perp - \boldsymbol{k}_\perp^{(0)}\right)\boldsymbol{r}\right]\Psi(r)\mathrm{d}S \tag{10}$$

is the spectral power density (SPD) function of the permittivity fluctuations in the defect layer, which is a spatial two-dimensional Fourier transform of the correlation function of the permittivity fluctuations

$$\Psi(r) = \Psi(|\boldsymbol{r}' - \boldsymbol{r}''|) = \langle \Delta\varepsilon(\boldsymbol{r}')\Delta\varepsilon(\boldsymbol{r}'')\rangle. \tag{11}$$

The averaging is performed over a cylindrical volume with thickness d and a base area equal to the irradiated surface area;

$$\boldsymbol{k}_\perp = \{k_x, k_y, 0\} = \frac{2\pi}{\lambda}\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, 0\},$$
$$\boldsymbol{k}_\perp^{(0)} = \{k_x^{(0)}, 0, 0\} = \frac{2\pi}{\lambda}\{\sin\theta_0, 0, 0\} \tag{12}$$

are the wave vector components of the scattered and incident waves perpendicular to the normal to the surface. The expressions for the angular functions $F_{ik}(\theta, \varphi, \theta_0, \varepsilon)$ are given in the Appendix.

The total integral intensity, i.e., the intensity of radiation scattered in the upper half-space, is derived by integrating the corresponding expressions for angle-resolved scattering over all possible scattering and azimuth angles:

$$P_{\mathrm{TIS}i}(\theta_0) = \int_0^{\pi/2} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\varphi [P_{si}(\theta, \varphi, \theta_0)$$
$$+ P_{pi}(\theta, \varphi, \theta_0). \tag{13}$$

The correlation function of permittivity fluctuations in the defect layer is assumed to be Gaussian,

$$\Psi_g(r) = \Delta\bar{\varepsilon}^2 d^2 \exp\left(-\frac{r^2}{l^2}\right), \tag{14}$$

with the SPD function

$$g_g\left(|\boldsymbol{k}_\perp - \boldsymbol{k}_\perp^{(0)}|\right) = \pi\Delta\bar{\varepsilon}^2 d^2 l^2 \exp\left(-\frac{|\boldsymbol{k}_\perp - \boldsymbol{k}_\perp^{(0)}|^2 l^2}{4}\right), \tag{15}$$

where l is the correlation length corresponding to the characteristic transverse dimensions of permittivity fluctuations.

In addition, we neglect the weak dependence of the angular functions on the scattering angles. Then, for correlation lengths longer than the wavelength – the most important case from the practical viewpoint – we obtain the following expression for P_{TIS} for normal incidence ($\theta_0 = 0$, with normalisation performed to the intensity of specularly reflected radiation):

$$P_{\mathrm{TIS}} = \left(\frac{4\pi d}{\lambda}\frac{\Delta\bar{\varepsilon}}{\varepsilon - 1}\right)^2, \quad l \geqslant \lambda. \tag{16}$$

One can see that this expression will correspond to the expression for P_{TIS} for roughness if the rms roughness σ is replaced by $d\Delta\bar{\varepsilon}(\varepsilon - 1)^{-1}$. Therefore, when the condition

$$\frac{\Delta\bar{\varepsilon}\, d}{\varepsilon - 1} < \sigma \tag{17}$$

is fulfilled, the scattering by the defect layer is weak compared to that from roughness.

4. Discussion of results

4.1 Possibility of separating the scattering from roughness and the defect layer

Expression (9) describes the scattering caused by a defect layer. This expression is of the same form as the expression for scattering by roughness (see Ref. [4]), i.e., the scattered radiation intensity is determined by the product of the frequency raised to the fourth power, the SPD function, and the angular function. A comparison of the angular functions corresponding to scattering by the defect layer with those corresponding to scattering by roughness shows that they coincide for all types of scattering, except the pp-type. For pp-type scattering by roughness, the intensity of radiation scattered in the plane of incidence ($\varphi = 0$) vanishes for a scattering angle θ' determined by the condition

$$\sin^2\theta' = \frac{\varepsilon(\varepsilon - \sin^2\theta_0)}{\varepsilon - \sin^2\theta_0(1 - \varepsilon^2)}. \tag{18}$$

This minimum is observed only for angles of incidence $\theta_0 > 40°$. However, when the scattering arises from the defect layer, the scattered radiation intensity does not vanish at any scattering angle. Fig. 2 shows the angular functions $F_{\mathrm{pp}}^{\mathrm{r}}(\theta)$ fnd $F_{\mathrm{pp}}^{\mathrm{b}}(\theta)$, which correspond to the pp-type scattering by roughness and the defect layer in the plane of incidence ($\varphi = 0$), for an angle of incidence $\theta_0 = 45°$.

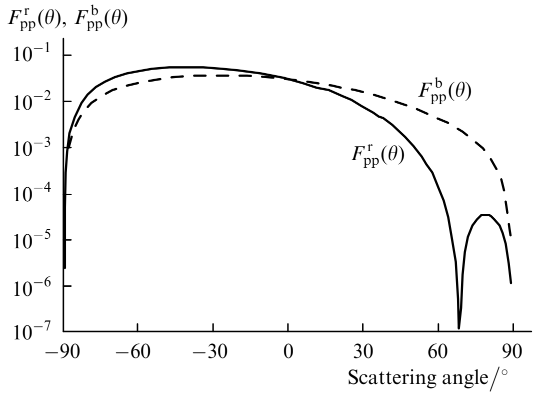

Figure 2. Angular functions $F_{\mathrm{pp}}^{\mathrm{r}}(\theta)$ and $F_{\mathrm{pp}}^{\mathrm{b}}(\theta)$, corresponding to the pp-type scattering by roughness (r) and the defect layer (b) in the plane of incidence ($\varphi = 0$), as functions of the scattering angle for an angle of incidence $\theta_0 = 45°$.

This difference in the dependences allows us to propose the following method of separating the scattering from roughness and from the defect layer, which is based on the use of radiation of different polarisation and involves a simultaneous determination of the corresponding SPD functions:

(1) Two measurements of angle-resolved scattering indicatrix are accomplished in the plane of incidence ($\varphi = 0$) for s- [($I_s(\theta)$)] and p-polarised [($I_p(\theta)$)] incident radiation for an angle of incidence $\theta_0 = 40 - 55°$.

(2) The SPD functions of roughness $g_r(k)$ and the defect layer $g_b(k)$ are found as follows:

$$g_{\rm r}(k) = \frac{1}{F_{\rm bs}(\theta)} \frac{I_{\rm s}(\theta)F_{\rm bp}(\theta) - I_{\rm p}(\theta)F_{\rm rs}(\theta)}{F_{\rm bp}(\theta) - F_{\rm rp}(\theta)},$$
$$g_{\rm b}(k) = \frac{1}{F_{\rm bs}(\theta)} \frac{I_{\rm p}(\theta)F_{\rm bs}(\theta) - I_{\rm s}(\theta)F_{\rm rp}(\theta)}{F_{\rm bp}(\theta) - F_{\rm rp}(\theta)}, \tag{19}$$

where $F_{\rm rs}(\theta)$, $F_{\rm rp}(\theta)$, $F_{\rm bs}(\theta)$, and $F_{\rm bp}(\theta)$ are the angular functions corresponding to s- and p-polarised incident radiation scattered by roughness (r) and the defect layer (b).

The weakness of scattering by the defect layer compared to that by the roughness can also be determined in the following way. In the pp-type scattering indicatrix recorded for an angle of incidence of $40-45^\circ$, we find a scattering angle θ' defined by condition (18), at which scattering from the roughness does not occur. If the indicatrix does exhibit a minimum of the order of the measurement error, the scattering from the defect layer is weak compared to that from the roughness. If the minimum is not observed, the scatterings from the layer and the roughness are comparable. In this case, to make a correct measurement of the roughness characteristics, the scattering from the defect layer should be subtracted according to expressions (19).

As an example, Fig. 3 shows typical indicatrices of the ss- and pp-type angle-resolved scattering measured on a polished quartz substrate for an angle of incidence of $\sim 45^\circ$ (the $41-49^\circ$ angle range corresponds to irradiation by specularly reflected radiation). Also plotted are the curves which correspond to the theoretical approximation of the experimental data employing expressions for the scattering from a roughness with an exponential statistics [$g(k) = 2\pi\sigma^2 l^2/(1+k^2 l^2)^{3/2}$, $\sigma \approx 5$ nm, $l = 0.7$ μm]. One can see that a well-defined minimum is indeed observed for the pp-type scattering and a scattering angle of $\sim 70^\circ$ corresponding to expression (18). Furthermore, the experimental data are in good agreement with the theoretical curves corresponding to the scattering from roughness. Hence, the scattering from the defect layer is much weaker than that from the roughness for substrates of this type.

Therefore, measurements of the scattering parameters for different polarisations of the incident radiation allow separating the contributions to scattering associated with roughness and the defect layer.

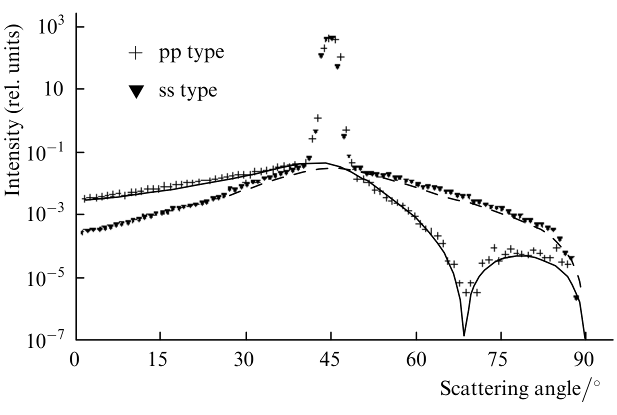

Figure 3. Indicatrices of the ss- and pp-type angle-resolved scattering from a polished quartz substrate (the normalisation was made to the intensity of specularly reflected radiation), along with theoretical approximations (curves) which make use of expressions for the roughness-induced scattering and an exponential statistics ($\sigma \approx 5$ nm, $l = 0.7$ μm).

4.2 Estimate of the characteristic parameters of the defect layer

We assume that, the defect layer is associated with the high concentration of structural defects. Consider the following simplified model of the influence of defects on the permittivity in this layer. Let us assume that the influence of a defect involves an isotropic deformation of the medium in some region $V_{\rm d}$ around the defect, this deformation being described by the simplified dependence:

$$\varDelta(R) = \varDelta_0 \exp\left(-\frac{R-a}{a_{\rm d}}\right), \quad R \geqslant a, \tag{20}$$

where $\varDelta_0$ is the displacement of the atom nearest to the defect; $a_{\rm d}$ is the characteristic radius of the region of the defect influence; and a is the average interatomic distance, with

$$\frac{\varDelta_0}{a} \ll 1, \quad \frac{a}{a_{\rm d}} \ll 1, \quad a_{\rm d} \ll \lambda\,. \tag{21}$$

By using the expression for elastooptical effect, we can estimate the average change ε in the region $V_{\rm d}$ of the defect influence (this approach corresponds to the traditional technique employed in Ref. [5]):

$$\Delta\bar{\varepsilon}_{\rm d} \sim 6\,\frac{\varDelta_0}{a_{\rm d}}p\,, \tag{22}$$

where p is the elastooptical constant.

We determine the change of ε in the region with the point defect concentration $N_{\rm d}(R)$ from the formula for the permittivity of a mixture [5]:

$$\Delta\varepsilon_{\rm db}(R) = \frac{4\pi}{3}\,a_{\rm d}^3 \Delta\varepsilon_{\rm d} N_{\rm d}(R)\,. \tag{23}$$

The average change in the region with the defect density $N_{\rm d}$ is

$$\Delta\bar{\varepsilon}_{\rm db} = \frac{4\pi}{3}\,a_{\rm d}^3 \Delta\bar{\varepsilon}_{\rm d} N_{\rm d}\,. \tag{24}$$

Note that $\Delta\bar{\varepsilon}_{\rm d}$ is the average change of ε in the region of the defect influence and $\Delta\bar{\varepsilon}_{\rm db}$ is the average change of the layer with the defect concentration $N_{\rm d}$.

Then, it is necessary to find the correlation function and the SPD of the defect layer. For correlation lengths of the order of λ, the correlation function is, as follows from the smallness condition of the region of the defect influence (21), related only to the fluctuations of the defect density:

$$\varPsi(\boldsymbol{r}') = \Delta\bar{\varepsilon}_{\rm db}^2\,\frac{\langle n_{\rm d}(\boldsymbol{r}) n(\boldsymbol{r}+\boldsymbol{r}')\rangle}{n_{\rm d}^2}\,,\ r' \geqslant \lambda\,, \tag{25}$$

where $n_{\rm d}(\boldsymbol{r})$ is the number of point defects under a unit surface area, i.e., $n_{\rm d}(\boldsymbol{r}) = N_{\rm d}(\boldsymbol{R})d$, $n_{\rm d} = N_{\rm d}d$.

Two variants of the defect distribution are possible.

(1) The random (normal) distribution. In this case, the variance of the number of defects in a circle of radius L may be assumed to be equal to the number $N_{\rm d}L^2 d$ of defects in this circle. The correlation function which corresponds to random fluctuations of the point defect density can be written in the form (for definiteness, the random distribution is assumed to be Gaussian):

Table 1. Characteristic magnitudes and scattered radiation intensities associated with the defect layer, roughness, and MBS.

Effect	$\Delta\bar{\varepsilon}_{db}$	Estimate of $\Delta\bar{\varepsilon}_{db}$	I_{TIS}	Estimate of I_{TIS}	Parameters of the estimates
Roughness	–	–	$\left(\frac{4\pi\sigma}{\lambda}\right)^2$	4×10^{-4}	$\sigma = 10\,\text{Å}$
Random distribution of structural defects	$\Delta\bar{\varepsilon}_d a_d^3 N_d$	10^{-5}	$\frac{d}{N_d}\left(\frac{4\pi}{\lambda L}\frac{\Delta\varepsilon_{db}}{\varepsilon-1}\right)^2$	4×10^{-13}	$\Delta\bar{\varepsilon}_d \sim 10^{-3}$, $L = 1$ μm, $N_d = 10^{15}$ cm^{-3}
Regular distribution of structural defects	$\Delta\bar{\varepsilon}_d a_d^3 N_d$	10^{-3}	$\left(\frac{4\pi d}{\lambda}\frac{\Delta\varepsilon_{db}}{\varepsilon-1}\right)^2$	4×10^{-8}	$\Delta\bar{\varepsilon}_d \sim 10^{-3}$, $N_d = 10^{17}$ cm^{-3}
MBS coefficient	–	–	$\frac{p^2kT}{64\rho u^2}D$	10^{-10}	$p = p_{11} \approx 0.13$, $u \approx 5\times10^5$ cm s^{-1}, $\rho = 2.65$ g cm^{-3}, $D = 0.5$ cm

$$\Psi_c(r') = \frac{\Delta\bar{\varepsilon}_{db}^2}{L^2 N_d d}\exp\left(-\frac{r^2}{L^2}\right). \tag{26}$$

(2) The regular distribution, when the defect density is modulated to one extent or another by some mechanism. A distribution of this kind may arise, for instance, during polishing: the defect density rises sharply under the grain of a polishing powder during the contact interaction to attain a density of $\sim 10^{19}$ cm^{-3} required for the multiplication of dislocations whose motion is a plastic flow (see, for instance, Ref. [6]). In the general case, the correlation function corresponding to a regular defect density distribution is difficult to obtain. Consider a special case when the defect density at the maxima is much higher than the average defect density. In this case, the variance of the number of defects is, as follows from its definition, equal to the square of the number of defects at the density maxima, and therefore the correlation function takes the simple form:

$$\Psi_r(r) = \Delta\bar{\varepsilon}_{db}^2\exp\left(-\frac{r^2}{L^2}\right). \tag{27}$$

The SPD functions which correspond to the random $[g_c(k_\perp)]$ and regular $[g_r(k_\perp)]$ defect distributions are as follows:

$$g_c(k_\perp) = \frac{\pi\Delta\bar{\varepsilon}_{db}^2}{N_d d}\exp\left(-\frac{k_\perp^2 L^2}{4}\right), \tag{28}$$

$$g_r(k_\perp) = \pi\Delta\bar{\varepsilon}_{db}^2 L^2\exp\left(-\frac{k_\perp^2 L^2}{4}\right). \tag{29}$$

By using expressions (9) and (13), we can find the scattered radiation intensities for both methods of scattering investigation in the presence of a defect layer caused by the high concentration of structural defects.

Table 1 presents the characteristic $\Delta\bar{\varepsilon}_{bd}$ and integral scattering intensities and their estimates made for parameter values of the defect layer typical for high-precision polished quartz substrates ($a_d = 100$ Å, $\Delta_0 = 0.1$, $d = 100$ Å, $\lambda = 0.63$ μm).

For comparison, Table 1 also gives the corresponding expressions for the integral scattering by the surface roughness and the coefficient of volume Mandel'shtam – Brillouin scattering (MBS) (D is the sample thickness) [5]. One can see that only regular defect density fluctuations may be responsible for a significant scattered radiation intensity, which is nevertheless well below the intensity of radiation scattered by the roughness.

Therefore, the scattering by the roughnessis prevalent in high-precision quartz substrates. Nevertheless, this by no means implies that the scattering from a defect layer may be neglected for other types of surfaces without making estimates.

Appendix

Expressions for the angular functions

The angular functions which appear in the expressions for angle-resolved scattering from the defect layer (9) have the following form (the first and second subscripts denote the polarisation of the scattered and incident fields, respectively; normalisation is accomplished to the incident radiation intensity):

$$F_{ss} = \frac{\cos\theta_0\cos^2\theta\cos^2\varphi}{\left[\cos\theta_0 + (\varepsilon-\sin^2\theta_0)^{1/2}\right]^2\left[\cos\theta + (\varepsilon-\sin^2\theta)^{1/2}\right]^2},$$

$$F_{sp} = \frac{\cos\theta_0\cos^2\theta\sin^2\varphi(\varepsilon-\sin^2\theta_0)}{\left[\varepsilon\cos\theta_0 + (\varepsilon-\sin^2\theta_0)^{1/2}\right]^2\left[\cos\theta + (\varepsilon-\sin^2\theta)^{1/2}\right]^2},$$

$$F_{ps} = \frac{\cos\theta_0\cos^2\theta\sin^2\varphi(\varepsilon-\sin^2\theta)}{\left[\cos\theta_0 + (\varepsilon-\sin^2\theta_0)^{1/2}\right]^2\left[\varepsilon\cos\theta + (\varepsilon-\sin^2\theta)^{1/2}\right]^2},$$

$$F_{pp} = \frac{\cos\theta_0\cos^2\theta\cos^2\varphi}{\left[\varepsilon\cos\theta_0 + (\varepsilon-\sin^2\theta_0)^{1/2}\right]^2\left[\varepsilon\cos\theta + (\varepsilon-\sin^2\theta)^{1/2}\right]^2}$$
$$\times\left[(\varepsilon-\sin^2\theta_0)^{1/2}(\varepsilon-\sin^2\theta)^{1/2} - \sin\theta_0\sin\theta\right]^2.$$

When normalising to the specularly reflected radiation, the angular functions are obtained from the above functions by changing the plus signs to the minus signs in front of the $(\varepsilon-\sin^2\theta_0)^{1/2}$ factor in the denominators.

References

1. Bennett J M, Mattson L *Introduction to Surface Roughness and Scattering* (Washington, DC: Opt. Soc. Am., 1989)
2. Elson J M, Bennett J M *Opt. Eng.* **18** 116 (1979)
3. Maradudin A A, Mills D L *Phys. Rev. B: Condens. Matter* **11** 2943 (1975)
4. Azarova V V, Dmitriev V G, Lokhov Yu N, Malitskii K N *Kvantovaya Elektron.* **30** 360 (2000) [*Quantum Electron.* **30** 360 (2000)]
5. Landau L D, Lifshitz E M, Pitaevskii L P *Electrodynamics of Continuous Media* (Oxford: Pergamon Press, 1984)
6. Kittel C *Introduction to Solid State Physics* (New York: Wiley, 1976)