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On the propagation velocity of a wave packet in an amplifying medium

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***Abstract*. It is shown that the delay time of a weak signal propagating in an amplifying medium on the wings of the spectral amplification line may be shorter than the time of propagation of the signal with the velocity of light in vacuum. It is found that in this case, the time dependence of the signal is exactly 'reconstructed' at the point of detection, and the detection of the signal continues even if it is abruptly terminated at the point of transmission. It is also shown that using the complex time of group delay of the signal, it is possible to improve the accuracy of the results in the first order of dispersion theory within this approximation.**

Keywords: *velocity of light, velocity of signal, complex group velocity.*

It is well known (see, for example, Secs. 83, 84 in Ref. [1], Sec. 2.6 in Ref. [2], Sec. 16.5 in Ref. [3], Ch. 8 in Ref. [4], and Sec. 8 in Ref. [5]) that a wave packet with a smoothly varying envelope (i.e., with a sufficiently narrow frequency spectrum) propagates in a dispersion medium without absorption with the so-called group velocity

$$v_{\text{gr}} = \left(\frac{\mathrm{d}k}{\mathrm{d}\omega}\right)^{-1}, \tag{1}$$

where ω is the wave frequency and $k(\omega)$ is its wave number.

In the transparency region of a medium in thermodynamic equilibrium, the group velocity of an electromagnetic wave is always smaller than the velocity of light and coincides with the propagation velocity of the wave energy [1] and the velocity of the complex envelope of the narrowband signal as a whole in the first order of the classical theory of dispersion (in both space and time coordinates).

In the case of thermodynamically nonequilibrium media or absorbing media, it is usually assumed (see, for example, Refs [1, 3, 5]) that it is impossible to introduce the concept of group velocity. This point of view is supported by the fact that the group velocity defined by the conventional relation (1) in nonconservative media is no longer equal to the rate of the wave-energy transfer; in addition, this velocity turns out to be a complex quantity and often higher than the velocity of light, which casts a doubt on the authenticity of the results predicted by expression (1).

This work aims at analysing the peculiarities in the propagation of a signal (wave packet) just in the case when the group velocity of light defined by expression (1) is complex valued and exceeds the velocity of light.

Note first of all that neither of the above circumstances can be regarded as an evidence of inapplicability of the concept of group velocity as the velocity of the complex envelope of the signal. Indeed, the propagation of the maximum of the wave packet faster than light in a nonlinearly amplifying medium has been investigated both theoretically and experimentally in a series of publications (see Refs [6, 7] and the literature therein). The propagation of a wave packet in a nonlinear medium with the gain saturation at a velocity faster than light was observed due to a predominant amplification of the leading edge of the signal as compared to its trailing edge.

The propagation of the maximum of a wave packet with supraluminal velocity obviously does not contradict to the postulate of the special theory of relativity concerning the limiting nature of the velocity of light in vacuum for the signal propagation because this postulate refers by no means to the propagation velocity of an arbitrary 'determinate' wave whose time dependence can be reconstructed completely from any its fragment, but only to the propagation velocity of the signal capable of carrying information i.e., to the propagation velocity of discontinuities in the signal or its envelope (or the corresponding derivatives of any order)[1]. The propagation velocity of these discontinuities is determined by the refractive index of the medium for an infinitely large frequency of the wave (see, for example, Ref. [1]) and is equal to the velocity of light in vacuum. Strictly speaking, this means that the information transfer with the help of electromagnetic waves in any medium occurs precisely with the velocity of light in vacuum and has nothing to do with the group velocity of the wave (irrespective of whether the group velocity is higher or lower than the velocity of light in vacuum).

Note that the application of the complex group velocity (and the complex delay time) of signals whose envelope is an analytic function[2] (which can be continued analytically to

[1]In Refs [6, 7], such discontinuities are referred to as the 'marks applied to a pulse'.
[2]Generally speaking, the term 'group velocity' can be applied only to such signals because discontinuities always propagate at the velocity of light in vacuum (see, for example, Ref. [3]) rather than with the group velocity.

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the complex plane) is quite natural and does not lead to any (even apparent) contradictions to known facts. The use of the complex group velocity for describing the propagation of pulses in an amplifying medium is discussed in Refs [8, 9] A similar approach in the 3D problem was applied, for example, in Ref. [10].

To illustrate the above considerations, we consider the propagation of a signal $E(z,t)$ with the carrier frequency ω_1 and the complex envelope $A(z,t)$ [1–5] in a homogeneous isotropic medium along the z-axis. Let the signal frequency ω_1 be close to the frequency ω_0 of one of spectral lines of the medium. Assuming that the signal is narrow-band (the spectral width of the signal is smaller than the carrier frequency ω_1), we can write the following obvious relations:

$$E(z,t) = A(z,t)\exp(-\mathrm{i}\omega_1 t) + A^*(z,t)\exp(\mathrm{i}\omega_1 t),$$
$$A(z,\Delta\Omega) = E(z,\omega), \tag{2}$$

where

$$\omega = \omega_1 + \Delta\Omega, \quad \omega_1 = \omega_0 + \Omega_0;$$
$$E(z,t) = \int_{-\infty}^{+\infty} E(z,\omega)\exp(-\mathrm{i}\omega t)\,\mathrm{d}\omega,$$
$$E(z,\omega) = (2\pi)^{-1}\int_{-\infty}^{+\infty} E(z,t)\exp(\mathrm{i}\omega t)\,\mathrm{d}t$$

are the high-frequency signal and its spectrum;

$$A(z,t) = \int_{-\infty}^{+\infty} A(z,\Delta\Omega)\exp(-\mathrm{i}\Delta\Omega t)\,\mathrm{d}\Delta\Omega,$$
$$A(z,\Delta\Omega) = (2\pi)^{-1}\int_{-\infty}^{+\infty} A(z,t)\exp(\mathrm{i}\Delta\Omega t)\,\mathrm{d}t$$

are the low-frequency complex envelope of the signal and its spectrum; Ω_0 is the shift of the carrier frequency of the signal relative to the centre of the spectral line.

For definiteness, we will consider the propagation of light in a medium with the refractive index $n(\omega) = n_0 + \Delta n(\omega)$, where n_0 is the background (nonresonant) refractive index of the medium, which weakly depends on the frequency of light near ω_0 and $\Delta n(\omega)$ is a complex correction to n_0 due to the spectral amplification line with the central frequency ω_0. In this case, for the complex transfer function of a layer of substance of thickness z, we have

$$F(z,\omega) = \exp[\mathrm{i}kn(\omega)z], \tag{3}$$

where $k = \omega/c$.

By introducing the amplitude gain at the centre of the spectral line of frequency ω_0

$$\alpha_0 \equiv \mathrm{i}k_0\Delta n(\omega_0), \quad k_0 \equiv \frac{\omega_0}{c} \tag{4}$$

and the complex form factor of the line normalised to unity at the centre of the spectral line of frequency ω_0

$$g(\Omega) \equiv \mathrm{i}k\alpha_0^{-1}\Delta n(\omega_0 + \Omega) \tag{5}$$

($\Omega \equiv \omega - \omega_0$ is the detuning of the wave frequency from the central frequency of the spectral line), we can easily write the transfer function of the layer in the form

$$F(z,\omega) = \exp(\mathrm{i}kn_0 z)\exp[\xi g(\Omega)], \tag{6}$$

where $\xi \equiv \alpha_0 z$ is the optical thickness of the layer. We do not consider here the reflection of the signal from the layer boundaries, assuming that the layer is sufficiently thick so that boundary effects can be neglected. We can also assume that inversion is inhomogeneous in space; i.e., the gain $\alpha_0(z)$ gradually changes along the z-axis. In this case, the reflection is exponentially small (see, for example, Sec. 52 in Ref. [11] and Sec. 12.3 in Ref. [4]) and it is sufficient to replace $\xi = \alpha_0 z$ by $\xi = \int_0^z \alpha_0(z)\mathrm{d}z$ in our analysis without changing the remaining formulas.

In the cross section z, we have the following relation for the complex envelope of the signal:

$$A(z,t) = \int_{-\infty}^{+\infty} A^{(0)}(\Delta\Omega)F(z,\omega)\exp(-\mathrm{i}\Delta\Omega t)\,\mathrm{d}\Delta\Omega, \tag{7}$$

where $A^{(0)}(t) \equiv A(0,t)$ is the complex envelope of the signal at the initial point $z = 0$. Assuming that the signal spectrum is concentrated in the vicinity of the carrier frequency ω_1 and restricting the analysis to the linear terms in Taylor series expansion of the expression in the second exponential in relation (6) (i.e., restricting the analysis to the first order in the classical theory of dispersion [5]), we can easily obtain the following expression instead of (7):

$$A(z,t) = \exp[\mathrm{i}k_1 z + \xi g(\Omega_0)]A^{(0)}[t - \tau(z)], \tag{8}$$

where $k_1 \equiv k_0 n(\omega_1)$ and the complex delay time τ is defined as

$$\tau(z) \equiv \tau_0 + \tau_\mathrm{r} + \mathrm{i}\tau_\mathrm{i}, \quad \tau_0 \equiv \frac{z}{\upsilon_\mathrm{ph}}, \quad \upsilon_\mathrm{ph} \equiv \frac{c}{n_0},$$
$$\tau_\mathrm{r} \equiv \xi\frac{\partial \mathrm{Im}\, g(\Omega_0)}{\partial\Omega_0}, \quad \tau_\mathrm{i} \equiv -\xi\frac{\partial \mathrm{Re}\, g(\Omega_0)}{\partial\Omega_0}. \tag{9}$$

This result differs from the conventionally used version of the first order dispersion theory [5] only in that the imaginary component of the delay time of the wave packet is now taken into account. The signal delay (as expected in the linear theory) is independent both of the signal intensity and the time dependence of the signal amplitude, but depends considerably on the shift of the carrier frequency of the signal relative to the centre of the spectral line (in contrast to the nonlinear situation considered in Refs [6, 7]).

In the case of an arbitrary signal with a smooth envelope, the inclusion of the imaginary component of the delay time leads to distortion of the time dependence of the signal intensity even in the first order of the dispersion theory. An interesting situation emerges in this case: the complex envelope of the signal is not distorted in the sense that it remains the same analytic function (with an additional complex shift), but the time dependence of the signal intensity may change considerably because the complex conjugation operation and, hence, the operation of determining the modulus of the complex function are not analytic.

Note that using the concept of complex group velocity (1), we can write formula (9) in the form

$$\tau(z) = \frac{z}{\upsilon_{gr}}, \quad \frac{1}{\upsilon_{gr}} \equiv \frac{\partial k(\omega)}{\partial \omega} = \frac{1}{\upsilon_{gr}^{Re}} + \frac{i}{\upsilon_{gr}^{Im}},$$

$$\frac{1}{\upsilon_{gr}^{Re}} \equiv \mathrm{Re}\,\frac{1}{\upsilon_{gr}} = \frac{1}{\upsilon_{ph}} + \alpha_0 \frac{\partial \mathrm{Im}\, g(\Omega_0)}{\partial \Omega_0}, \tag{10}$$

$$\frac{1}{\upsilon_{gr}^{Im}} \equiv \mathrm{Im}\frac{1}{\upsilon_{gr}} = -\alpha_0 \frac{\partial \mathrm{Re}\, g(\Omega_0)}{\partial \Omega_0}.$$

In this work, we shall call the quantity υ_{gr} the complex group velocity. Although this velocity is not the group velocity in the conventional sense [1–5], it (i) coincides with the ordinary group velocity in a conservative medium , and (ii) is the propagation velocity of the complex envelope of the time dependence of the signal in any medium. In this connection, it is natural to regard the complex group velocity as a generalisation of the concept of group velocity [1–5] to the case of absorbing or amplifying media. It is natural to refer to quantities υ_{gr}^{Re} and υ_{gr}^{Im} as the real and imaginary group velocities of the signal[3]. The imaginary group velocity (as well as the imaginary component of the signal delay time, with which it is connected directly) characterises not the displacement of the signal in the conventional sense of the word, but the change in the shape of its complex envelope in the first order of the classical theory of dispersion. The real group velocity (and the real component of the signal delay time, with which it is connected directly) characterises the velocity of the signal in space. The 'centre of symmetry' of the signal (if it exists) propagates in space precisely with this velocity.

In this work, we are interested not in the distortion of the temporal form of the signal in the first order of dispersion theory, but in the velocity of its propagation. For this reason, we shall confine our subsequent analysis to the specific case of a Gaussian wave packet[4]

$$A^{(0)}(t) = \exp\left(-\frac{t^2}{T^2}\right)$$

with the duration T and the Lorentzian profile of the spectral line [12]

$$g(\Omega) = \left(1 - i\frac{2\Omega}{\Delta\Omega_{1/2}}\right)^{-1} \tag{11}$$

with the width $\Delta\Omega_{1/2}$ and the coherence time $\tau_c \equiv 2/\Delta\Omega_{1/2}$. It should be emphasised that the permittivity of the medium satisfies the Kramers–Kronig relation and, hence the supraluminal velocity of information transfer (as well as the advance response of the medium to the action) are impossible in principle. Nevertheless, the propagation velocity of the wave packet envelope becomes higher than the velocity of light (see below).

For the time dependence of the field intensity $I(z,t) \equiv |A(z,t)|^2$ for various longitudinal coordinates z, we obtain the following relations from Eqn (8):

$$I(z,t) = I_0(z) I_G(z) \exp\left[-\frac{2(t-\Delta t)^2}{T^2}\right],$$

$$I_0(z) \equiv \exp\left(\frac{2\xi}{1+x_0^2}\right),$$

$$I_G(z) \equiv \exp\left[8\left(\frac{\tau_c}{T}\right)^2 \xi^2 x_0^2 \left(1+x_0^2\right)^{-4}\right], \tag{12}$$

$$\Delta t = \mathrm{Re}\,\tau(z) = \tau_0 + \tau_r = \frac{z}{\upsilon_{gr}^{Re}} = \frac{z}{\upsilon_{ph}} + \xi\tau_c\left(1-x_0^2\right)\left(1+x_0^2\right)^{-2},$$

$$x_0 \equiv \tau_c \Omega_0,$$

where $I_0(z)$ is the ordinary factor of the exponential increase in the intensity of a monochromatic wave with frequency $\omega_1 = \omega_0 + \Omega_0$ in an amplifying medium; $I_G(z)$ is an additional (relative to the monochromatic wave) factor describing an increase in the intensity of the Gaussian packet (this factor emerges as a result of inclusion of the imaginary component of the delay time of the Gaussian signal); Δt is the real delay time of the packet; x_0 is the normalised detuning of the carrier frequency of the packet from the line centre.

One can see that for $x_0^2 > 1$ (i.e., on the wings of the spectral line for $|\Omega_0| > \Delta\Omega_{1/2}$), the wave packet propagates in an amplifying medium at a supraluminal velocity (i.e., the signal delay time is shorter than the time of its propagation over distance z with the phase velocity of light[5], $\Delta t < z/\upsilon_{ph}$ and the real group velocity is higher than the phase velocity, i.e., $\upsilon_{gr}^{Re} > \upsilon_{ph}$.

To verify the obtained analytic results, we carried out numerical calculations for the propagation of a wave packet with the initial duration $T = 10\tau_c$ and with a normalised shift of the carrier frequency relative to the spectral line centre $x_0 = 5$. The results of these calculations for $\xi = 0$, 300, and 600 are presented in Fig. 1. One can easily see that the conclusion concerning the supraluminal velocity of propagation of the packet is confirmed. One can also see that the inclusion of the imaginary component of the delay time considerably improves the accuracy of analytic results. Naturally, the accuracy of the first order dispersion theory deteriorates as the distance propagated by the wave packet increases.

We must find out whether the obtained results are applicable to real signals (confined in time). For this purpose, we analysed numerically the propagation of a Gaussian wave packet with a 'truncated' leading front for the same values of parameters T and ξ:

$$A^{(0)} = \exp\left(-\frac{t^2}{T^2}\right)\theta(t - T_1), \tag{13}$$

where T_1 is the time of the signal onset at point $z = 0$; $\theta(t)$ is the Heaviside function.

[3]It should be emphasised that $\upsilon_{gr}^{Re} \neq \mathrm{Re}\,\upsilon_{gr}$ and $\upsilon_{gr}^{Im} \neq \mathrm{Im}\,\upsilon_{gr}$.
[4]It will be shown below that the inclusion of the imaginary component of the delay time for a Gaussian packet does not lead to a distortion of the time dependence of the intensity during its propagation. This is a distinguishing feature of the Gaussian packet (Gaussian beam in the corresponding 3D problem [8]).

[5]In this case, we assume that the 'background' refractive index n_0 is also preserved for an infinitely large frequency (strictly speaking, this is not correct; see Ref. [1]). In this connection, the role of the velocity of light in vacuum in our model is played by the phase velocity of light in a medium with zero dispersion and with the refractive index n_0 $\upsilon_{ph} = c/n_0$. For a rarefied medium (gas), we have $\upsilon_{ph} \simeq c$.

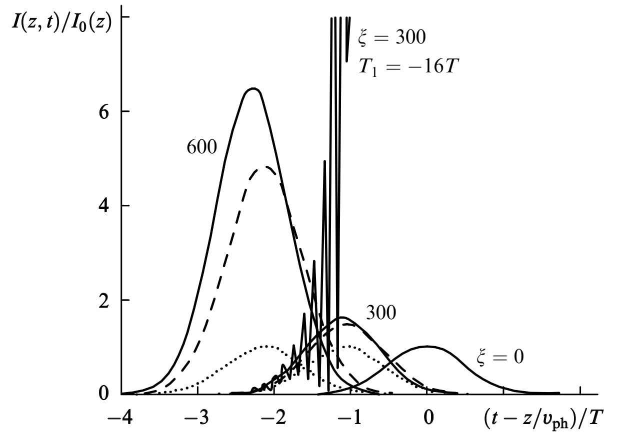

Figure 1. Time dependences of the intensity of the signal with parameters $T = 10\tau_c$ and $x_0 = 5$ for different optical thicknesses ξ of the layer calculated numerically (solid curves) and by formula (12) with (dashed curves) and without (dotted curves) taking into account the factor $I_G(z)$ for T_1 varying from $-\infty$ to $-25T$ (all the curves except the rapidly oscillating one) as well as for $\xi = 300$ and $T_1 = -16T$ (rapidly oscillating curve).

It was found that the truncation of the leading front of the signal for $T_1 = -25T$ does not change the curves in Fig. 1 (within the error of graph plotting). However, the truncation of the leading front of the signal for a (modulo) smaller value of parameter T_1 leads to a distortion of the signal. The same figure shows, by way of an example, the results of calculation of the time dependence of the signal intensity for $\xi = 300$ and $T_1 = -16T$ (rapidly oscillating curve). One can see that the shape of the signal considerably changed compared to the cases with $T_1 = -25T$ or $T_1 = -\infty$ ('untrancated' signal). The reason behind the oscillations becomes clear if we take into account the fact that we are dealing with the interference of two components of the signal with approximately equal energies, but with different frequencies (with the beat period $\sim 2\pi/|\omega_0 - \omega_1|$).

Indeed, the signal gain depends on frequency; therefore, for a large parameter ξ (or sufficiently small truncation parameter $|T_1|$), the spectrum of the signal transmitted through the layer of the medium is concentrated not near the carrier frequency ω_1, but at the central frequency ω_0 of the spectral amplification line. As a result, the first order of dispersion theory is inapplicable. Otherwise, the spectrum of the signal would be concentrated near its carrier frequency and the application of the first order of dispersion theory makes it possible to obtain quantitative results. The transition from the quantitative applicability to complete inapplicability of the first order dispersion theory occurs very quickly: in the case when $\xi = 300$ for $T_1 = -17T$, the results of calculations coincides within the graphical accuracy with the results for an untruncated packet ($T_1 = -\infty$), while for $T_1 = -15T$, it is impossible to depict the results of calculations for a truncated and untruncated packets on the same scale in the same figure.

One can also see from Fig. 1 that the excess of the propagation velocity of the signal envelope over the velocity of light is not necessarily small; for example, for $\xi = 600$, the advance of the signal relative to light is approximately twice the characteristic duration of the signal. The agreement of the numerical data with the results of application of the first order dispersion theory is quite satisfactory. This is quite natural if we take into account the sufficiently narrow angular spectrum of the signal in this case.

The results of similar calculations for a shorter signal with the initial duration $T = 5\tau_c$ and a normalised shift of the carrier frequency relative to the spectrum of the central line $x_0 = 2$ for $\xi = 0$, 10, and 20 are presented in Figs 2–4. Fig. 2 shows the results of calculations for the propagation of a untruncated wave packet. A comparison of Figs 1 and 2 shows that as the signal duration decreases, the range of application of the first order of the classical dispersion theory (with and without taking into account the imaginary component of the delay time) becomes smaller (which is quite natural). At the same time, the optical thickness ξ of the layer after passage through which the advance of the signal relative to light becomes comparable with its duration also decreases considerably.

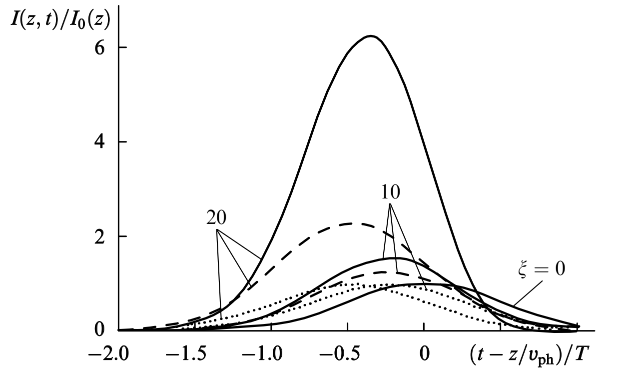

Figure 2. Time dependences of the intensity of a signal with parameters $T = 5\tau_c$ and $x_0 = 2$ for various optical thicknesses ξ of the layer, obtained under the same conditions as in Fig. 1.

Fig. 3 shows the results of calculations for the same wave packet with a truncated leading front for various values of the truncation parameter $x \equiv T_1/T$ and a fixed optical thickness of the layer $\xi = 10$ and 20. One can see that the truncation of the leading front of the signal (even at considerable distances from the main peak) may cause considerable distortion in the time dependence of the signal intensity. This distortion is manifested primarily in the emergence of 'beats' with the characteristic period $2\pi/|\omega_0 - \omega_1|$ and then (see Fig. 3b) in a considerable increase in the amplitude of the detected signal whose time dependence in determined by the parameters of the spectral line and the initial jump in the signal amplitude at the moment of its actuation rather than by the initial form of the signal.

Thus, the supraluminal group velocity of a signal can be realised only for signals with a sufficiently 'prolonged' leading front: the signal peak can indeed propagate at a velocity higher than the velocity of light in vacuum without a significant distortion of the time dependence of the signal intensity, but only until it starts 'lean' against the actual onset of the signal, i.e., against the initial jump which propagates at the velocity of light in vacuum (velocity v_{ph} in our model). When the signal attempts to 'pass its actual onset' (actually, long before the passing), it is strongly distorted. A similar situation was noted in Refs [6, 7] for a high-power signal.

The situation with the truncated trailing edge of a signal seems to be more interesting. In this case, it is the limiting

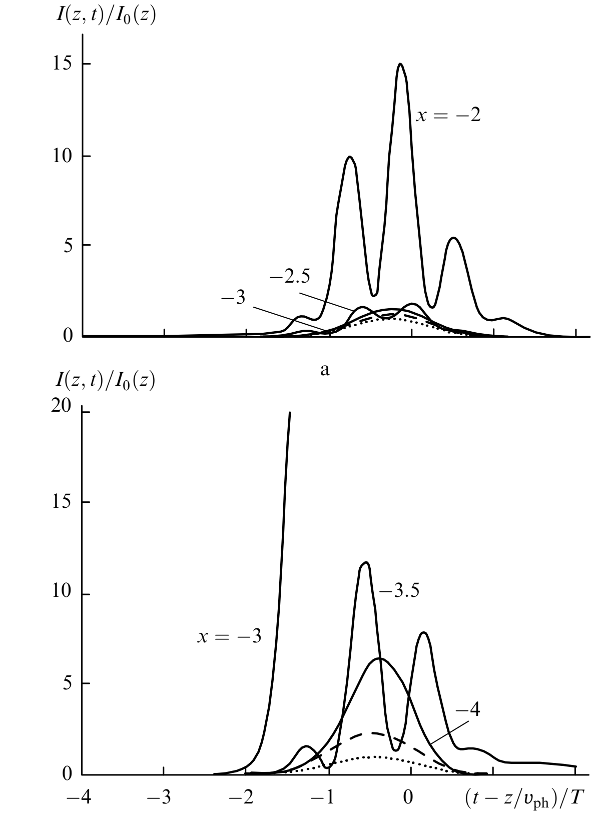

Figure 3. Distortion of the time dependence of the signal intensity due to the truncation of the leading front of the signal for the optical thickness of the layer $\xi = 10$ (a) and 20 (b).

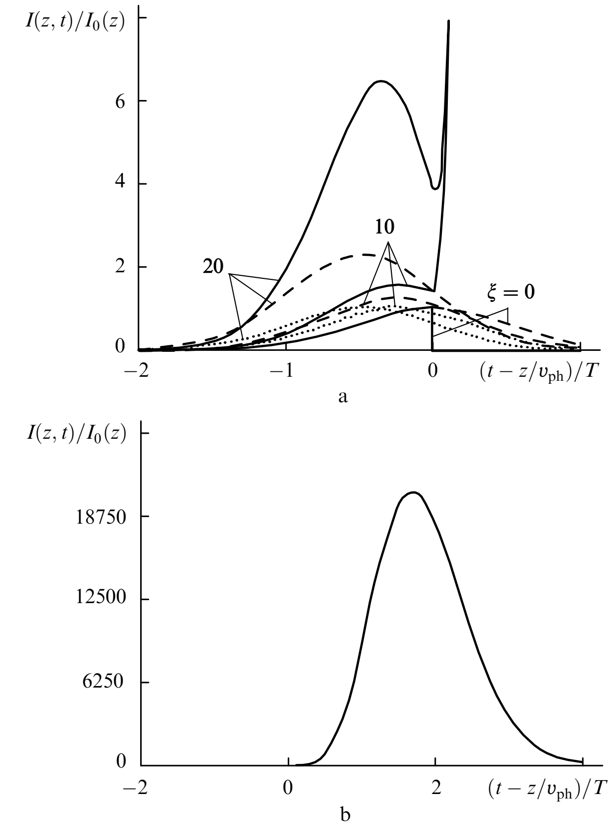

Figure 4. Time dependences of the intensity of a signal with parameters $T = 5\tau_c$ and $x_0 = 2$, whose transmission is terminated abruptly at the instant $t = 0$ for various optical thicknesses ξ of the layer (a) and for $\xi = 10$ (b).

nature of the velocity of light that leads to the 'anticipation' of the signal. Indeed, in the case of a supraluminal velocity of propagation of the signal envelope, the information on its abrupt termination propagates less rapidly than the signal (at the velocity of light); consequently, it is possible to receive the rear part of the signal at the point of reception even if it has not been transmitted at the transmission point.

Fig. 4a presents the results of the corresponding calculations for the propagation of a signal of duration $T = 5\tau_c$ with a normalised shift $x_0 = 2$ of the carrier frequency for $\xi = 0$, 10, and 20. The calculations were made for the signal whose transmission was terminated abruptly at the instant $t = 0$ (i.e., only the first half of the Gaussian signal was transmitted, after which the amplitude of the signal being transmitted abruptly dropped to zero). A comparison of Figs. 4a and Fig. 2 (showing the results of calculations for a untruncated signal with the same values of parameters) readily shows that a part of the second half of the Gaussian curve is successfully received away from the transmission point even if it was absent at the point of transmission.

The reception of the nonexisting signal continues until the information on the signal termination reaches (with velocity v_{ph}) the point of reception[6]. Then the signal amplitude sharply increases since the signal spectrum expands considerably (as compared to the 'expected' spectrum) as a result of truncation of the trailing edge of the signal, and a considerable part of this signal falls at the centre of the spectral amplification line. This can be seen clearly in Fig. 4b depicting the time dependence of the intensity of a signal truncated at $t = 0$ for the layer thickness $\xi = 10$ and differing from Fig. 4a only in the scale. It can easily be noted that in the case of an abrupt termination of the signal transmission, the expansion of the signal spectrum leads to the propagation of the main part of the signal with a group velocity smaller than the velocity of light in vacuum and has approximately Gaussian shape. Nevertheless, the less intense 'leading' part of the signal is not lost against its background just because it propagates at a supraluminal velocity and is separated in time from the main part of the signal.

Note also that for a sufficiently high gain α_0 of the signal, not only the propagation of the signal envelope faster than light (i.e., the propagation of the signal in the medium at a velocity higher than the velocity of light, $\Delta t < z/v_{ph}$, $v_{gr}^{Re} > v_{ph}$) is possible, but also the emergence of a negative signal delay, when it becomes negative even when the phase delay of the signal is taken into account ($\Delta t < 0$, $v_{gr}^{Re} < 0$). In this case, away from the transmission point, the peak of the signal appears earlier than at the transmission point; i.e., the signal is received not 'earlier than expected', but 'earlier than transmitted'.

Naturally, this circumstance should not be regarded as a violation of the causality principle (in the same way as the supraluminal group velocity of propagation of a packet

[6]For this reason, Fig. 4a shows, by way of analytic results, the results of calculations on the basis of formula (12) (derived for a untruncated signal). Indeed, the jump of a truncated signal propagates at a velocity v_{ph} and, hence, is just 'standing' (on the chosen scale), being behind the signal itself (which makes it possible to 'predict' the signal at the reception point).

cannot be regarded as a violation of the limiting nature of the velocity of light). In this case, we are speaking of the prediction of the missing part of a signal from its received part, which occurs naturally (without human interference). This circumstance is well illustrated by the above data on the propagation of a signal with a truncated trailing edge; i.e., its 'reconstruction' at the point of reception is possible just due to the fact that the information on the rear part of the signal is contained in the leading part which has already been received in contrast to the information concerning the abrupt termination of signal transmission which can affect the signal being received only after a delay time $\tau_0 = z/\upsilon_{\rm ph}$.

In other words, the propagation of a signal at a supraluminal velocity can be regarded as the transfer of information about the signal at the velocity of light $\upsilon_{\rm ph}$ (in this case, the signal delay time is positive and independent of the gain factor) followed by the prediction[7] of the time dependence of the complex amplitude of the signal (the delay time in this case is negative and depends on the signal amplification). The result of such a delay followed by a prediction can be positive (for a weak amplification) or negative (for a strong amplification) and these two versions do not differ in principle.

In conclusion, we consider the possibilities of experimental observation of the above phenomena. Clearly, the situation in which the group velocity of the signal in a medium noticeably exceeds its phase velocity is of main interest. Using relations (10) and (11), we obtain the following expression for $\upsilon_{\rm gr}^{\rm Re}$, the real group velocity[8]

$$\upsilon_{\rm gr}^{\rm Re} = \upsilon_{\rm ph}[1 - \alpha_2 f(x_0)]^{-1}, \quad \upsilon_{\rm ph} \equiv \frac{c}{n_0},$$
$$\alpha_2 \equiv \alpha_0 \upsilon_{\rm ph} \tau_{\rm c}, \quad f(x_0) \equiv (x_0^2 - 1)(x_0^2 + 1)^{-2}. \qquad (14)$$

The function $f(x_0)$ is bounded for $x_0 \sim 1$ and achieves its maximum value equal to 1/8 for $x_0 = \sqrt{3}$. Consequently, the effect is essentially described by the dimensionless parameter α_2: for $\alpha_2 \ll 1$, the group velocity virtually coincides with the phase velocity, while for $\alpha_2 \sim 1$ or $\alpha_2 \gg 1$, the group velocity differs noticeably from the phase velocity (is smaller or larger depending on the detuning x_0)[9]. For $\alpha_2 > 8$, the group velocity may become negative (in a certain interval of detuning x_0)[10].

For most of the widely used laser systems, the parameter α_2 is small, but some exceptions are also observed:

(1) low-pressure CO_2 laser (for $\lambda = 10.6$ μm, $2\alpha_0 = 4$ dB m^{-1}, and $\Delta\nu_{\rm D} = 50$ MHz, we have $\alpha_2 = 0.88$; for $x_0 = \sqrt{3}$, we have $\upsilon_{\rm gr}^{\rm Re}/\upsilon_{\rm ph} = 1.12$)[11];

(2) He–Ne laser generating at a wavelength 3.39 μm (for $\lambda = 3.39$ μm, $2\alpha_0 = 20$ dB m^{-1}, and $\Delta\nu_{\rm D} = 280$ MHz, we have $\alpha_2 = 0.79$; for $x_0 = \sqrt{3}$, we have $\upsilon_{\rm gr}^{\rm Re}/\upsilon_{\rm ph} = 1.11$);

(3) YAG laser (for $\lambda = 1.06$ μm, $2\alpha_0 = 20$ cm^{-1}, $\Delta\nu_{\rm D} = 6$ cm^{-1}, and $n_0 = 1.82$, we have $\alpha_2 = 0.58$; for $x_0 = \sqrt{3}$, we have $\upsilon_{\rm gr}^{\rm Re}/\upsilon_{\rm ph} = 1.08$).

These estimates show that the problem of 'reconstruction' can in principle also be solved using the standard systems, while for solving the 'prediction' problem, the line must be narrowed or the gain factor must be increased approximately by an order of magnitude as compared to the standard systems. In addition, the above effect will be manifested in practice if the advance Δt of the signal over light is at least comparable with the signal duration T. The duration of the signal is limited from below by the condition $T \gg \tau_{\rm c}$ (if this condition is violated, the first order dispersion theory is inapplicable and we cannot speak of the velocity of propagation of the signal as a whole). Comparing this condition with relations (12), we can easily deduce that the signal advance over light can be comparable with its duration only in the case of a large optical thickness of the layer of the substance ($\xi \gg 1$).

This circumstance gives rise to a new difficulty: the gain for a signal for a large optical thickness is exponentially large[12] and the analysis carried out by us here (disregarding the saturation of nonlinearity) is inapplicable. By the way, the gain can be easily reduced to an admissible value by introducing a (concentrated or distributed) absorption which is not selective (or weakly selective) relative to frequency.

Another difficulty lies in the fact that the supraluminal group velocity in an amplifying medium is realised at the periphery of the spectral gain line, where the gain is much smaller than at the line centre. This leads to an exponential decrease (upon an increase in the optical thickness of the layer) of the signal-to-noise ratio (if the signal means a 'regular' signal with a carrier frequency at the periphery of the spectral line and the noise means the 'noise' signal with the frequency close to the centre of the spectral line).

In our opinion, this difficulty is apparent to a considerable extent. Indeed, the regular and noise (in the sense indicated above) signals propagate with different group velocities, the regular signal leading the noise. As a result, the regular and noise signals are separated in time and the noise distortion of the regular signal is unlikely. In actual practice, the noise signal in a single-pass laser–amplifier simply removes the population inversion in the active medium remaining after the passage of a regular signal[13].

Moreover, the above peculiarity in the behaviour of a regular signal with an supraluminal group velocity makes it possible to overcome the previous difficulty also to a considerable extent. Indeed, for the applicability of the linear theory to the propagation of a regular signal at a supraluminal velocity, it is sufficient to prevent the saturation of amplification only for the regular signal whose gain is noticeably smaller than the gain at the centre of the spectral line. The saturation of amplification relative to the

[7]We are speaking just of the prediction which may also contain errors like the reception of the signal which has not been transmitted.

[8]In order to avoid confusion, we emphasise once again that we apply the term 'group velocity' to the velocity of motion of the temporal envelope of a signal. This velocity (in a nonconservative medium) is not the velocity of energy transfer and not even the propagation velocity of the spatial distribution of the field intensity (which differs from the time dependence of the signal intensity due to the exponential variation of the field in space). In a conservative system, however, all the three types of group velocity coincide (see, for example, Ref. [1]).

[9]This is sufficient, for example, for solving the problem of 'reconstruction' of the untransmitted part of the signal.

[10]This is sufficient for the 'advance' reception of the signal being transmitted.

[11]In the present work, α_0 is the amplitude gain, while $2\alpha_0$ is the intensity gain.

[12]This also follows from our calculations. For example, for $x_0 = 2$ (Figs. 2–4a), the optical thickness $\xi = 10$ of the layer corresponds to the amplification of the signal intensity of about 17 dB (at the carrier frequency) and the enhancement of noise of about 87 dB (at the line centre).

[13]As a result, there emerges a pattern similar to that depicted in Fig. 4: the intensity of a regular signal may be much smaller than that of noise, but this does not hamper its observation.

noise signal following the regular one only improves the characteristics of the system (since it suppresses the noise).

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