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# Soliton propagation of a femtosecond laser pulse in a medium with anomalous dispersion

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***Abstract*. The stationary shape of a femtosecond pulse propagating through a nonlinear medium with an anomalous dispersion is obtained. It is shown that the femtosecond-pulse intensity $E_0^2$ at which a soliton propagation takes place is inversely proportional to the pulse duration to the 2/3 power ($E_0^2 \sim 1/\tau_0^{2/3}$). The analytic amplitude dependence of the reconstruction period of the soliton intensity time profile is obtained.**

***Keywords***: *soliton propagation, femtosecond pulse, anomalous dispersion*

## 1. Introduction

It is known that the dispersion – nonlinearity balance during the passage of pico- and femtosecond laser pulses through an optically nonlinear medium with an anomalous dispersion results in the formation of stable pulses, the so-called optical solitons, which retain an almost invariable shape during their propagation through distances exceeding the pulse length $l = c\tau_0$ by factors $10^6 - 10^7$ [1, 2].

In Ref. [3], a numerical solution of the wave equation was obtained, which is different from the nonlinear Schrödinger equation (NSE), describing the dispersive nonlinear propagation of a femtosecond laser pulse through a medium with a normal dispersion and a cubic nonlinearity.

In this paper, the stationary shape of a femtosecond pulse propagating through a dispersive medium with an anomalous dispersion and a cubic nonlinearity was obtained by solving the nonlinear wave equation different from the NSE.

As shown in Ref. [3], for $\omega_0\tau_0 \geqslant 200/\pi$ (where $\omega_0$ is the carrier pulse frequency and $\tau_0$ the pulse duration), the equation describing the dispersive nonlinear propagation of a femtosecond laser pulse through a cubically nonlinear medium has, with the inclusion of the second-order linear dispersion, the following form:

$$\Phi'_\xi - A|\Phi|^2\Phi'_\eta + B\Phi'''_\eta = 0, \qquad (1)$$

where $A = \pi\chi_0^{(3)}E_0^2/n_0^2$; $\chi_0^{(3)}$ is the low-frequency limit of the Fourier transform of the third-order nonlinear susceptibility of the medium $\chi_0^{(3)}(t_1, t_2, t_3)$; $E_0$ is the maximum real amplitude of the electric field vector; $n_0$ is the linear part of the refractive index; $B = \pi\alpha_2/(n_0^2\tau_0^2)$;

$$\alpha_2 = -\int_0^\infty \tau^2\alpha(\tau)\mathrm{d}\tau; \qquad (2)$$

$\alpha(t)$ is the nonlinear medium susceptibility; $\Phi = E/E_0$ is the normalised real modulus of the electric field vector in the medium; $\xi = zn_0/(c\tau_0)$; $\eta = zn_0/(c\tau_0) - t/\tau_0$; $c$ is the velocity of light in vacuum.

It is known that the competition between dispersion and nonlinearity in the spectral range corresponding to the anomalous dispersion of the group velocity ($\alpha_2 < 0$) results in the conservation of the propagating pulse shape at a certain input power [1]. One can see from Eqn (1) that the dimensionless parameter

$$\gamma = \frac{|B|}{A^3} = \frac{|\alpha_2|}{\pi^2(\chi_0^{(3)})^3\tau_0^2E_0^6} \qquad (3)$$

corresponds to the ratio between the characteristic dispersion ($\tau_0^2/|\alpha_2|$) and nonlinear ($[\pi^2(\chi_0^{(3)}E_0^2)^3]^{-1}$) lengths and permits the estimate of their relative contribution to the signal-shape distortion. For $\gamma = 1$, the dispersion spreading of the pulse is exactly compensated for by the nonlinear compression.

Note that, while the intensity $E_0^2$ at which the soliton propagation of picosecond pulses occurs is inversely proportional to the square of duration ($E_0^2 \sim 1/\tau_0^2$) [2], this intensity for femtosecond pulses is, according to expression (3), inversely proportional to the pulse duration to the 2/3 power ($E_0^2 \sim 1/t_0^{2/3}$).

The stationary pulse shape can be obtained by assuming that

$$\Phi(\xi, \eta) = a(\eta)\cos(K\xi - \Omega\eta), \qquad (4)$$

in Eqn (1), where $K, \Omega, a(\eta)$ are the wave number, the frequency, and the amplitude of the stationary pulse, respectively.

After substitution of expression (4) in (1), we obtain the system of equations:

$$-Ka - Aa^3\Omega + 3B\Omega a''_\eta - B\Omega^3 a = 0,$$
$$-Aa^2a'_\eta + Ba'''_\eta - 3B\Omega^2 a'_\eta = 0. \qquad (5)$$

After integration of the second equation of the system (5),

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taking into account that $a_\eta(\eta), a'_\eta(\eta), a''_\eta(\eta) \to 0$ for $|\eta| \to \infty$ (this corresponds to the soliton propagation through the unperturbed medium), we obtain

$$\left(\frac{\partial a}{\partial \eta}\right)^2 = \frac{A}{6B}a^4 + 3\Omega^2 a^2. \tag{6}$$

By multiplying the last equation of the system (5) by $a'_\eta(\eta)$ and integrating, we obtain

$$\left(\frac{\partial a}{\partial \eta}\right)^2 = \frac{A}{6B}a^4 + \left(\frac{\Omega^2}{3} + \frac{K}{3B\Omega}\right)a^2. \tag{7}$$

A comparison of Eqns (6) and (7) gives

$$K = 8B\Omega^3. \tag{8}$$

For $B < 0$, Eqn (6) has the solution

$$a(\eta) = a_0 \operatorname{sech}\left(\frac{\eta}{\tau_s}\right) \tag{9}$$

provided that the soliton duration $\tau_s$ and its amplitude $a_0$ satisfy the relation

$$a_0^2 = \frac{6|B|}{A\tau_s^2} = \frac{9|B|}{A}\Omega^2. \tag{10}$$

Therefore, taking into account (8), the expression for the stationary shape of a femtosecond pulse has the form:

$$\Phi(\eta, \xi) = a_0 \operatorname{sech}\left(\frac{\eta}{\tau_s}\right)\cos(8B\Omega^3\xi - \Omega\eta). \tag{11}$$

One can see from expression (11) that the reconstruction period of the soliton intensity profile

$$\Lambda = \frac{\pi}{8B\Omega^3} = \frac{27\pi}{8Ba_0^3}\left(\frac{|B|}{A}\right)^{3/2} \tag{12}$$

is inversely proportional to the cube of the soliton amplitude.

Fig. 1 shows the dynamics of the time envelope and the spectral density of the soliton over a length of one reconstruction period for $a_0 = 1, \gamma = 1$. Fig. 2 shows the reconstruction period of the soliton time profile as a function of the amplitude $a_0$.

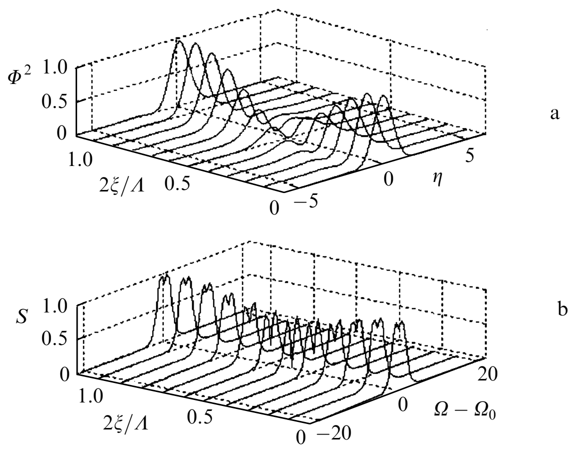


**Figure 1.** Dynamics of the time envelope (a) and the spectral density (b) of the soliton over a length of one reconstruction period for $a_0 = 1$ and $\gamma = 1$.

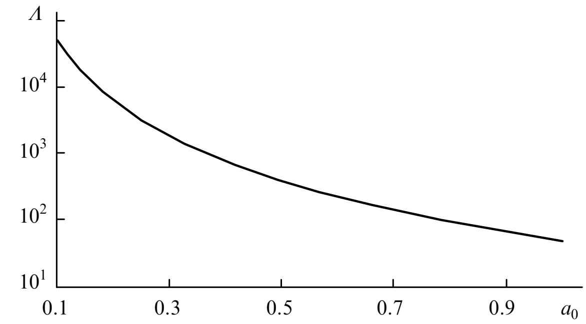


**Figure 2.** Amplitude dependence of the reconstruction period of the soliton time profile.

Therefore, the stationary shape of a femtosecond pulse propagating through a nonlinear medium with an anomalous dispersion was derived from truncated Eqn (1).

## References

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