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Intracavity quasi-phase-matched self-frequency conversion in a periodically poled Nd:Mg:LiNbO$_3$ crystal

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***Abstract*.** **The theory of intracavity quasi-phase-matched self-frequency conversion in an active nonlinear periodically poled structure is developed. The processes of quasi-phase-matched self-frequency doubling, self-halving and mixing using the pump wave in a periodically poled Nd:Mg:LiNbO$_3$ crystal are studied. The dependences of the efficiency of nonlinear optical conversion in these processes on the reflection coefficient of the output mirror and on linear losses in the medium are investigated.**

Keywords: *self-frequency doubling and halving, quasi-phase-matching, active nonlinear periodically poled crystals.*

1. Introduction

Investigations of active nonlinear crystals in which the active (laser) properties of the crystal are ensured by the ions of rare-earth elements (Nd, Er, Yb) and the matrix plays the role of a nonlinear medium [1 – 4] are stimulated by wide applications of compact and reliable lasers emitting in the blue – green, red and IR spectral regions. Recent advances in the field of self-frequency conversion are associated with the appearance of new active nonlinear crystals possessing a high nonlinearity and a high concentration of rare-earth elements, as well as a high optical damage threshold and good mechanical characteristics [5 – 7].

It is interesting to use as an active nonlinear medium a periodically poled crystal in which quasi-phase-matched (QPM) wave interactions can be realised. The advantage of such a crystal is the possibility of thermally noncritical phase matching of interacting waves even when the conditions of conventional phase matching cannot be fulfilled [8, 9].

The use of selective pumping together with the QPM wave interaction technique allows the realisation of various three-frequency wave interactions in an active nonlinear crystal involving laser radiation and the pump wave. This provides new schematic solutions for lasers with self-frequency conversion, which can be used in optical memory systems, laser displays, high-resolution digital printing, and telecommunications.

In this work, we propose a theory for the intracavity three-frequency QPM wave interactions in an active nonlinear periodically poled Nd:Mg:LiNbO$_3$ crystal and analyse in detail the self-frequency doubling, halving and mixing using the pump wave.

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2. Theoretical approach and the system of equations

We will analyse the intracavity three-frequency QPM wave interactions in an active nonlinear periodically poled Nd:Mg:LiNbO$_3$ crystal as follows. At first, we consider a system of truncated equations describing nonlinear three-frequency wave interactions in a periodically poled medium and then supplement one of the equations in this system with a term accounting for the amplification of the wave under study by the active medium and add to the obtained system of equations another equation describing the inverse population dynamics in the active medium. The resulting system of equations takes into account simultaneously the nonlinear and active properties of the crystal and is used to analyse three particular cases of self-frequency conversion in an active nonlinear medium (self-frequency doubling, halving and mixing using the pump wave). We assume that the active nonlinear periodically poled crystal is placed in a cavity formed by plane mirrors deposited on the end faces of the crystal at the positions $z = 0$ and $z = L$ (Fig. 1).

In the first approximation of the dispersion theory, the system of truncated equations for three-frequency ($\omega_1 + \omega_2 = \omega_3$) nonlinear interaction of plane waves [10] for squares of moduli of the amplitudes and phases of the waves in the case of a periodically poled medium has the form

$$\pm\frac{\partial S_{1,2}^{\pm}}{\partial z} + \frac{1}{u_{1,2}}\frac{\partial S_{1,2}^{\pm}}{\partial t} + \alpha_{1,2}S_{1,2}^{\pm}$$

$$= -g(z)\beta_{1,2}(S_1^{\pm}S_2^{\pm}S_3^{\pm})^{1/2}\sin\left(\Theta^{\pm} \mp \Delta kz\right), \tag{1}$$

$$\pm\frac{\partial S_3^{\pm}}{\partial z} + \frac{1}{u_3}\frac{\partial S_3^{\pm}}{\partial t} + \alpha_3 S_3^{\pm}$$

$$= g(z)\beta_3(S_1^{\pm}S_2^{\pm}S_3^{\pm})^{1/2}\sin\left(\Theta^{\pm} \mp \Delta kz\right), \tag{2}$$

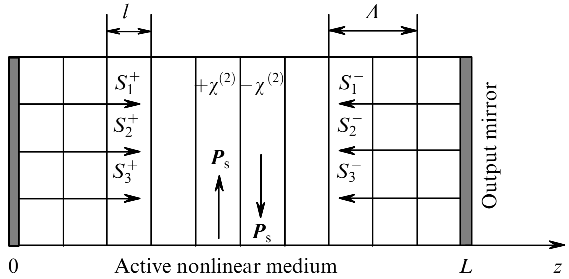

Figure 1. Schematic representation of an intracavity active nonlinear periodically poled medium (Λ is the pole period, l is the layer thickness, $\boldsymbol{P}_s$ is the spontaneous polarisation vector of the medium).

$$\pm\frac{\partial\Theta^{\pm}}{\partial z}+\left(\frac{1}{u_3}-\frac{1}{u_2}-\frac{1}{u_1}\right)\frac{\partial\Theta^{\pm}}{\partial t}$$

$$=\frac{g(z)}{2}(S_1^{\pm}S_2^{\pm}S_3^{\pm})^{1/2}\left(\frac{\beta_3}{S_3^{\pm}}-\frac{\beta_2}{S_2^{\pm}}-\frac{\beta_1}{S_1^{\pm}}\right)$$

$$\times\cos\left(\Theta^{\pm}\mp\Delta kz\right), \qquad (3)$$

where $\Theta^{\pm}=\varphi_3^{\pm}-\varphi_2^{\pm}-\varphi_1^{\pm}$ is the phase difference; $S_j^{\pm}$ and $\varphi_j^{\pm}$ are the slowly varying square of the modulus of the amplitude and phase of a wave with frequency ω_j propagating along the z axis ('+') and in the opposite direction ('−') (Fig. 1); $\beta_j=4\pi\omega_j(\boldsymbol{e}_j d^{(2)}\boldsymbol{e}_i\boldsymbol{e}_k)/cn_j$ is the nonlinear coupling factor; $d^{(2)}$ is a second-order tensor of nonlinear susceptibility of the medium; c is the velocity of light in vacuum; $\Delta k=k_1+k_2-k_3$ is the phase mismatch; $\boldsymbol{e}_j$, n_j, u_j and $\boldsymbol{k}_j$ are the unit polarisation vector, the refractive index, the group velocity and the wave vector of a wave with frequency ω_j, respectively; $g(z)=(-1)^{M(z)}$ is a sign-alternating function characterising the modulation of the nonlinear susceptibility with a period $\Lambda=2l$; $M(z)=[z/l+1]$ is the layer number in the medium; l is the thickness of an individual layer; $L=N_0\Lambda=2N_0 l$ is the cavity length; $2N_0$ is the number of domains; the coefficient α_j characterises the linear losses in an active nonlinear medium; and $j=1,2,3$. If the QPM conditions are fulfilled, we can write $\Delta k=2\pi m/\Lambda$, where m is an odd number (quasi-phase-matched order).

Let us average both sides of Eqns (1)–(3) over the length of the cavity taking into account the small variation in $S_j^{\pm}$ and $\varphi_j^{\pm}$ on this length. Such an approach is justified under the condition that the length of the active nonlinear periodically poled crystal is much smaller than the characteristic length of nonlinear interaction. This condition is fulfilled in the range of the parameters of the crystal and the cavity studied by us (their values are close to the real parameters of the experiment). Integrals of the type

$$\frac{1}{L}\int_0^L g(z)\sin\left(\Theta^{\pm}\mp\Delta kz\right)\mathrm{d}z \text{ and } \frac{1}{L}\int_0^L g(z)\cos\left(\Theta^{\pm}\mp\Delta kz\right)\mathrm{d}z$$

appearing during averaging are taken using the layer-by-layer calculation. For example,

$$\frac{1}{L}\int_0^L g(z)\sin\left(\Theta^{\pm}\mp\Delta kz\right)\mathrm{d}z=$$

$$=\frac{1}{L}\,\mathrm{Im}\left[\exp\left(\mathrm{i}\Theta^{\pm}\right)\int_0^L g(z)\exp(\mp\mathrm{i}\Delta kz)\mathrm{d}z\right]$$

$$=\frac{1}{L}\,\mathrm{Im}\left[\exp\left(\mathrm{i}\Theta^{\pm}\right)\sum_{N=0}^{2N_0-1}(-1)^N\int_{Nl}^{(N+1)l}\exp(\mp\mathrm{i}\Delta kz)\mathrm{d}z\right]$$

$$=\pm\tan(\Delta k\Lambda/4)\mathrm{sinc}(\Delta kL/2)\cos\left(\Theta^{\pm}\mp\Delta kL/2\right).$$

Neglecting the difference in the group and phase velocities ($u_j\approx c/n_j$) and introducing the dimensionless intensities in the form $I_j^{\pm}=cn_jS_j^{\pm}/8\pi I_s$, where I_s is the saturation intensity of the active medium, we arrive at the following system of equations for dimensionless intensities $I_j^{\pm}$ and phase differences $\Psi^{\pm}=\varphi_3^{\pm}-\varphi_2^{\pm}-\varphi_1^{\pm}\mp\Delta kL/2$ averaged over the cavity length:

$$\pm\frac{2}{n_{1,2}T_c}\left[I_{1,2}^{\pm}(L)-I_{1,2}^{\pm}(0)+I_{1,2}^{\pm}\alpha_{1,2}L\right]+\frac{\mathrm{d}I_{1,2}^{\pm}}{\mathrm{d}t}$$

$$=\mp\frac{1}{n_{1,2}T_c}\left(\mu_{1,2}I_1^{\pm}I_2^{\pm}I_3^{\pm}\right)^{1/2}\cos\Psi^{\pm}, \qquad (4)$$

$$\pm\frac{2}{n_3T_c}\left[I_3^{\pm}(L)-I_3^{\pm}(0)+I_3^{\pm}\alpha_3L\right]+\frac{\mathrm{d}I_3^{\pm}}{\mathrm{d}t}$$

$$=\pm\frac{1}{n_3T_c}\left(\mu_3I_1^{\pm}I_2^{\pm}I_3^{\pm}\right)^{1/2}\cos\Psi^{\pm}, \qquad (5)$$

$$\pm\frac{2}{T_c}\left[\Psi^{\pm}(L)-\Psi^{\pm}(0)\right]+(n_1+n_2-n_3)\frac{\mathrm{d}\Psi^{\pm}}{\mathrm{d}t}$$

$$=\pm\frac{(I_1^{\pm}I_2^{\pm}I_3^{\pm})^{1/2}}{2T_c}\left(\frac{\sqrt{\mu_3}}{I_3^{\pm}}-\frac{\sqrt{\mu_2}}{I_2^{\pm}}-\frac{\sqrt{\mu_1}}{I_1^{\pm}}\right)\sin\Psi^{\pm}, \qquad (6)$$

where $T_c=2L/c$ is the round-trip transit time for light in the cavity; and $\mu_j=2048\pi^5L^2I_s(\boldsymbol{e}_jd^{(2)}\boldsymbol{e}_i\boldsymbol{e}_k)^2\tan^2(\Delta k\Lambda/4)\times\mathrm{sinc}^2(\Delta kL/2)/cn_1n_2n_3\lambda_j^2$. The factor $\tan^2(\Delta k\Lambda/4)$ is related to a periodic modulation of the nonlinear susceptibility of the medium. In the case of an exact QPM, we have $\tan^2(\Delta k\Lambda/4)\mathrm{sinc}^2(\Delta kL/2)=4/\pi^2m^2$.

We assume that the left mirror in Fig. 1 reflects radiation with frequency ω_j totally, while the right (output) mirror has the intensity reflection coefficient R_j for a wave with frequency ω_j. In this case, the system of equations (4)–(6) is supplemented by the following boundary conditions:

$$I_j^-(L)=R_jI_j^+(L),\quad I_j^-(0)=I_j^+(0),$$

$$\Psi^+(L)=\Psi^-(L)-\pi-\Delta kL-\delta\Psi_L, \qquad (7)$$

$$\Psi^+(0)=\Psi^-(0)-\pi-\Delta kL+\delta\Psi_0,$$

where $\delta\Psi_{0,L}$ are the additional phase shifts introduced by the left and right cavity mirrors, respectively; $\delta\Psi=\delta\Psi_L+\delta\Psi_0$ is the total phase shift. Because variations in the intensities and phases of the waves over the cavity length are small, we have

$$I_j^-=R_jI_j^+=R_jI_j,\quad \Psi^+=\Psi^--\pi-\Delta kL-\delta\Psi=\Psi \qquad (8)$$

and the system (4)–(6), taking (7) into account, can be written as:

$$\frac{dI_{1,2}}{dt} = \frac{1}{n_{1,2}T_c}\left[-v_{1,2}I_{1,2} - (\varepsilon_{1,2}I_1I_2I_3)^{1/2}\sin(\varphi+\theta)\right], \quad (9)$$

$$\frac{dI_3}{dt} = \frac{1}{n_3T_c}\left[-v_3I_3 + (\varepsilon_3I_1I_2I_3)^{1/2}\sin(\varphi+\theta)\right], \quad (10)$$

$$\frac{d\varphi}{dt} = \frac{(I_1I_2I_3)^{1/2}}{2(n_1+n_2-n_3)T_c}\left[\frac{\sqrt{\xi_2}}{I_2}\cos(\varphi+\psi_2)\right.$$

$$\left.+\frac{\sqrt{\xi_1}}{I_1}\cos(\varphi+\psi_1) - \frac{\sqrt{\xi_3}}{I_3}\cos(\varphi+\psi_3) + \frac{2\delta\Psi}{(I_1I_2I_3)^{1/2}}\right], \quad (11)$$

where $v_j = 2(1-R_j)/(1+R_j) + 2\alpha_jL$ are dimensionless linear losses for a wave with frequency ω_j inside the cavity;

$$\theta = \arcsin\left\{\frac{[1+(R_1R_2R_3)^{1/2}]\cos(\Delta kL/2+\delta\Psi/2)}{[1+R_1R_2R_3+2(R_1R_2R_3)^{1/2}\cos(\Delta kL+\delta\Psi)]^{1/2}}\right\};$$

$$\varepsilon_j = \frac{\mu_j[1+R_1R_2R_3+2(R_1R_2R_3)^{1/2}\cos(\Delta kL+\delta\Psi)]}{(1+R_j)^2};$$

$$\psi_j =$$

$$\arcsin\left\{\frac{[1+(R_1R_2R_3/R_j^2)^{1/2}]\cos(\Delta kL/2+\delta\Psi/2)}{[1+R_1R_2R_3/R_j^2+2\cos(\Delta kL+\delta\Psi)(R_1R_2R_3/R_j^2)^{1/2}]^{1/2}}\right\};$$

$$\varphi = \Psi^+ + \Delta kL/2 + \delta\Psi/2;$$

$$\xi_j = \frac{\mu_j[1+R_1R_2R_3/R_j^2+2\cos(\Delta kL+\delta\Psi)(R_1R_2R_3/R_j^2)^{1/2}]}{4}.$$

The system of equations (9)–(11) describes the nonlinear interaction of waves by neglecting the active properties of the medium. As mentioned above, this system of equations should be modified slightly by taking into account the active properties of the medium. These properties are described by the system of Statz–de Mars equations [11] which can be written in the dimensionless form as

$$\frac{d(I_q^+ + I_q^-)}{dt} = \frac{v_q(I_q^+ + I_q^-)}{n_qT_c}(N-1), \quad (12)$$

$$\frac{dN}{dt} = \frac{1}{T_1}\left[1+\eta - N(I_q^+ + I_q^- + 1)\right], \quad (13)$$

where N is the ratio of the inverse population density to the threshold density; $1+\eta = P_{\text{pump}}/P_{\text{th}}$ is the ratio of the pumping power to the threshold power; T_1 is the inverse-population relaxation time; $I_q^{\pm} = cn_qS_q^{\pm}/8\pi I_s$ is the dimensionless intensity of a wave with frequency ω_q, which is amplified in the active medium. In our case, $q = 1$ or 2 or 3 (one of the three interacting waves is amplified). Taking (8) into account, we can represent the system (12), (13) in the form

$$\frac{dI_q}{dt} = \frac{v_qI_q}{n_qT_c}(N-1), \quad (14)$$

$$\frac{dN}{dt} = \frac{1}{T_1}\left[1+\eta - N(I_q + I_qR_q + 1)\right]. \quad (15)$$

The system of equations (9)–(11), (14), (15) describes the three-frequency QPM nonlinear wave interaction in an active nonlinear periodically poled medium and is a generalisation of the equations for self-frequency doubling [12] to the case of arbitrary R_j and QPM wave interactions. In this case, the right-hand side of Eqn (14) should be substituted into one of Eqns (9) or (10) for the term v_jI_j/n_jT_c depending on the frequency ($\omega_{1,2}$ or ω_3) of the wave that is amplified by the active medium.

Below, we will consider three particular cases: (1) second harmonic generation $\omega+\omega\to2\omega$ in an active nonlinear medium (self-frequency doubling); (2) subharmonic generation $\omega\to\omega/2+\omega/2$ in an active nonlinear medium (self-frequency halving); (3) frequency mixing $\omega+\omega_{\text{pump}}\to\omega_{\text{sum}}$ involving a pump wave. In the cases under study, a wave with frequency ω is amplified in the active medium.

3. QPM self-frequency doubling

Consider the QPM self-frequency doubling $\omega+\omega=2\omega$. In this case, $\omega_1=\omega_2=\omega$, $\omega_3=2\omega$, $\lambda=2\pi c/\omega$, and taking into account that a wave with frequency ω_1 is amplified in the active medium Eqns (9)–(11), (14), (15) assume the form:

$$\frac{dI_1}{dt} = \frac{I_1}{n_1T_c}\left[v_1(N-1) - (\varepsilon_1I_3)^{1/2}\sin(\varphi+\theta)\right], \quad (16)$$

$$\frac{dI_3}{dt} = \frac{1}{n_3T_c}\left[-v_3I_3 + (\varepsilon_3I_3)^{1/2}I_1\sin(\varphi+\theta)\right], \quad (17)$$

$$\frac{d\varphi}{dt} = \frac{I_1\sqrt{I_3}}{2(2n_1-n_3)T_c}\left[\frac{2\sqrt{\xi_1}}{I_1}\cos(\varphi+\psi_1)\right.$$

$$\left.-\frac{\sqrt{\xi_3}}{I_3}\cos(\varphi+\psi_3)\right] + \frac{\delta\Psi}{(2n_1-n_3)T_c}, \quad (18)$$

$$\frac{dN}{dt} = \frac{1}{T_1}[1+\eta - N(I_1+I_1R_1+1)], \quad (19)$$

where $I_{1,3}$ is the dimensionless intensity of the first and second harmonics respectively.

Assuming a high-Q cavity for a wave with frequency ω ($R_1=R_2=1$), we obtain

$$\varepsilon_1 = \mu_1[1+R_3+2\sqrt{R_3}\cos(\Delta kL)]/4,$$

$$\varepsilon_3 = \mu_3[1+R_3+2\sqrt{R_3}\cos(\Delta kL)]/(1+R_3)^2,$$

$$\xi_1 = \mu_1[1+R_3+2\sqrt{R_3}\cos(\Delta kL+\delta\Psi)]/4,$$

$$\xi_3 = \mu_3[1+1/R_3+2\cos(\Delta kL+\delta\Psi)/\sqrt{R_3}]/4,$$

$$\theta = \psi_{1,3} = \arcsin\left\{\frac{(1+\sqrt{R_3})\cos(\Delta kL/2+\delta\Psi/2)}{[(1+R_3+2\sqrt{R_3}\cos(\Delta kL+\delta\Psi)]^{1/2}}\right\}.$$

For $\delta\Psi=0$, the system of equations (16)–(19) has two steady-state solutions:

$$I_3 = \frac{\varepsilon_3}{4v_3^2}\left\{-v_1v_3(\varepsilon_1\varepsilon_3)^{-1/2} - 1/2\right.$$

$$\left.+\left[\left(v_1v_3(\varepsilon_1\varepsilon_3)^{-1/2}+1/2\right)^2 + 2\eta v_1v_3(\varepsilon_1\varepsilon_3)^{-1/2}\right]^{1/2}\right\}^2, \quad (20)$$

$$I_3 = \frac{4v_1\eta\sqrt{R_3} - v_3(1+R_3)}{4\sqrt{R_3}[4v_1\sqrt{R_3} + v_3(1+R_3)]}. \tag{21}$$

Depending on the values of the parameters $v_{1,3}$, $\varepsilon_{1,3}$, R_3 and η, one of the two steady-state solutions is realised. The solution (20) exists for all values of the parameters $v_{1,3}$, $\varepsilon_{1,3}$, R_3, η, while the solution (21) exists only for $4v_1\eta\sqrt{R_3} \geqslant v_3(1+R_3)$. In the case $R_1 = R_2 = 1$ and $\delta\Psi \neq 0$, the system (16)–(19) does not have a simple analytic solution for the second harmonic intensity.

Figs 2 and 3 show the characteristic dependences corresponding to the stable branches of the solutions (20) and (21). One can see from these dependences that there exists an optimal reflection coefficient $R_{2\omega}$ of the output mirror for the second-harmonic wave for which the second harmonic has the maximum output power. Calculations were made for a Nd:Mg:$LiNbO_3$ crystal of length $L = 0.5$ cm and the pole period $L = 7$ mm placed in a double cavity, one of whose mirrors has reflection coefficients $R_\omega = R_{2\omega} = 100\,\%$, while the other has $R_\omega = 100\,\%$ and $R_{2\omega} < 100\,\%$. It is assumed that the ee–e interaction takes place in the crystal, involving the highest nonlinear coefficient d_{33} for lithium niobate for $\lambda = 2\pi c/\omega = 1.084$ μm, $I_s = 10$ kW cm^{-2} (calculated according to the data presented in Ref. [4]), $P_{th} = 0.1$ W, and for a beam radius $r_0 = 10^{-4}$ m in the cavity.

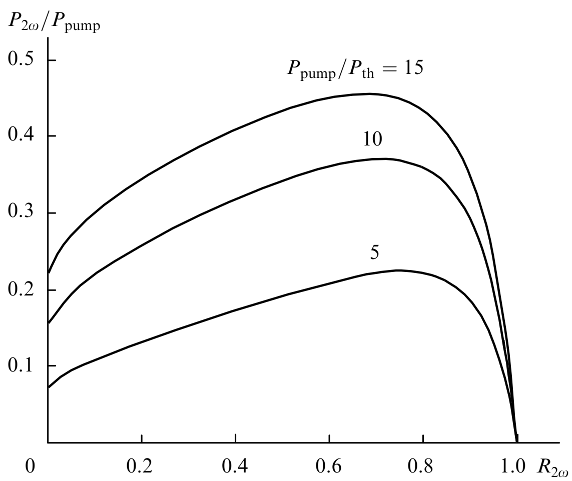

Figure 2. Dependences of the normalised second-harmonic output power on the reflection coefficient of the output mirror for the second harmonic for different pump powers ($v_1 = 0.08$, $\alpha_{2\omega} = 0.1$ cm^{-1}, $m = 1$).

4. QPM self-frequency halving

Consider now the QPM self-frequency halving $\omega = \omega/2 + \omega/2$. In this case, taking into account that a wave with frequency ω_3 is amplified in the active medium, $\omega_1 = \omega_2 = \omega/2$, $\omega_3 = \omega$, $\lambda = 2\pi c/\omega$ and Eqns (9)–(11), (14), (15) assume the form:

$$\frac{dI_1}{dt} = \frac{I_1}{n_1 T_c}\left[-v_1 - (\varepsilon_1 I_3)^{1/2}\sin(\varphi + \theta)\right], \tag{22}$$

$$\frac{dI_3}{dt} = \frac{1}{n_3 T_c}\left[v_3 I_3(N-1) + (\varepsilon_3 I_3)^{1/2} I_1 \sin(\varphi + \theta)\right], \tag{23}$$

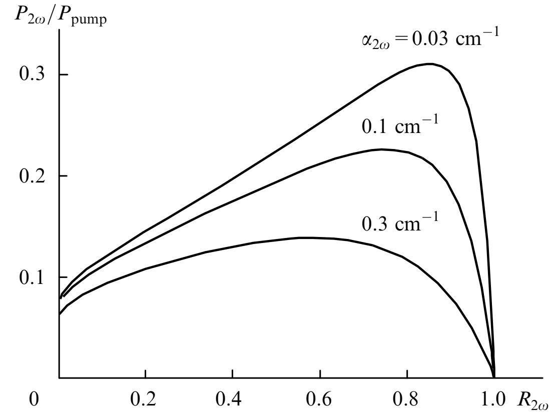

Figure 3. Dependences of the normalised second-harmonic output power on the reflection coefficient of the output mirror for the second harmonic for different linear losses $\alpha_{2\omega}$ ($v_1 = 0.08$, $P_{pump}/P_{th} = 5$, $m = 1$).

$$\frac{d\varphi}{dt} = \frac{I_1\sqrt{I_3}}{2(2n_1 - n_3)T_c}\left[\frac{2\sqrt{\xi_1}}{I_1}\cos(\varphi + \psi_1) - \frac{\sqrt{\xi_3}}{I_3}\cos(\varphi + \psi_3)\right] + \frac{\delta\Psi}{(2n_1 - n_3)T_c}, \tag{24}$$

$$\frac{dN}{dt} = \frac{1}{T_1}[1 + \eta - N(I_3 + I_3 R_3 + 1)], \tag{25}$$

where $I_{1,3}$ are dimensionless intensities of the waves with frequencies $\omega/2$ and ω, respectively. For the parametric generation of a low-frequency wave, the cavity must have a high Q for waves with frequencies $\omega/2$ and ω simultaneously ($R_1 = R_2 \approx 1, R_3 = 1$). For such a cavity, we can write

$$\varepsilon_1 = \mu_1[1 + R_1^2 + 2R_1\cos(\Delta kL + \delta\Psi)]/(1 + R_1)^2,$$

$$\varepsilon_3 = \xi_3 = \mu_3[1 + R_1^2 + 2R_1\cos(\Delta kL + \delta\Psi)]/4,$$

$$\xi_1 = \mu_1[1 + \cos(\Delta kL + \delta\Psi)]/2,$$

$$\theta = \psi_3 \approx \psi_1 \approx (\pi - \Delta kL - \delta\Psi)/2.$$

For $\delta\Psi = 0$, the system of equations (22)–(25) has two steady-state solutions for the intensity I_1:

$$I_1 = \frac{v_1 v_3(\eta\varepsilon_1 - 2v_1^2)}{(\varepsilon_1\varepsilon_3)^{1/2}(\varepsilon_1 + 2v_1^2)}, \tag{26}$$

$$I_1 = \frac{\eta v_3 R_1^{1/2} - v_1(1 + R_1)}{v_3 R_1 + v_1(1 + R_1)R_1^{1/2}}. \tag{27}$$

As in the case of self-frequency doubling, one of the two steady-state solutions is obtained, depending on the values of parameters $v_{1,3}$, $\varepsilon_{1,3}$, R_1 and η. The solution (26) exists for $\eta\varepsilon_1 \geqslant 2v_1^2$, and (27) for $\eta v_3 R_1^{1/2} \geqslant v_1(1 + R_1)$. The characteristic dependences corresponding to stable branches of solutions (26) and (27), presented in Figs 4 and 5, show that being a parametric process, the self-frequency halving has a threshold. Also, the optimal reflection coefficient $R_{\omega/2}$ exists for which the output power of the subharmonic is maximum. Calculations were made for the ee – e interaction in a $Nd:Mg:LiNbO_3$ crystal with $L = 0.5$ cm and $\Lambda = 22$ μm placed in a double cavity, one of whose mirrors has the reflection coefficient $R_{\omega/2} = R_{\omega} = 100\,\%$, and the other $R_{\omega} = 100\,\%$ and $R_{\omega/2} < 100\,\%$.

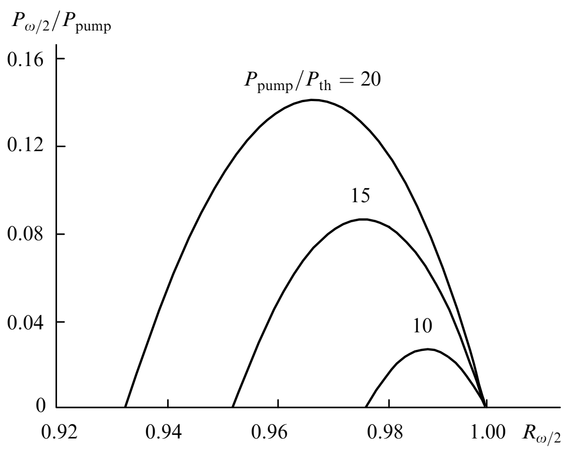

Figure 4. Dependences of the normalised second-harmonic output power on the reflection coefficient of the output mirror for the subharmonic for different pump powers ($v_3 = 0.08, \alpha_{\omega/2} = 0.08$ cm^{-1}, $m = 1$).

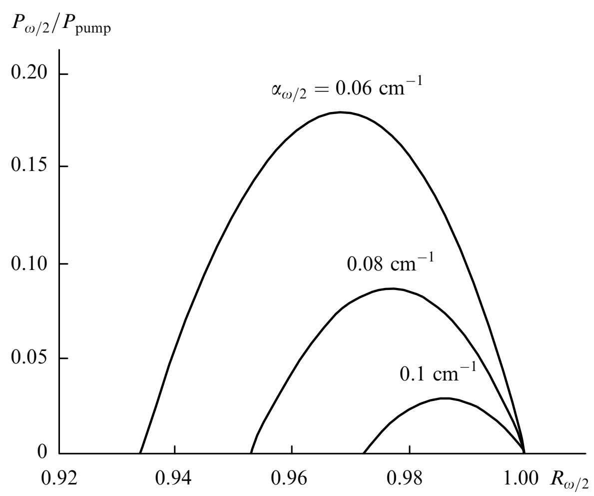

Figure 5. Dependences of the normalised second-harmonic output power on the reflection coefficient of the output mirror for the subharmonic for different linear losses $\alpha_{\omega/2}$ ($v_3 = 0.08, P_{\text{pump}}/P_{\text{th}} = 15, m = 1$).

5. QPM frequency mixing using a pump wave

Semiconductor lasers have been used actively in recent years for pumping active media. A part of the pumping radiation is absorbed by the active medium, while the unabsorbed pumping radiation can participate in nonlinear optical interaction. Consider the intracavity nondegenerate frequency mixing $\omega_1 + \omega_2 = \omega_3$, where ω_1 is the lasing frequency and ω_2 is the pump frequency.

For QPM frequency mixing at a constant pump wave power $I_2 = \text{const}$, $\varphi_2^{\pm} = \text{const}$, and in the absence of a cavity ($R_2 = 0$) at the pump frequency, Eqns (9) – (11), (14) and (15) have the form

$$\frac{\mathrm{d}I_1}{\mathrm{d}t} = \frac{1}{n_1 T_{\mathrm{c}}}\left[v_1 I_1 (N-1) - \frac{(\mu_1 I_1 I_2 I_3)^{1/2}}{1+R_1}\sin\Phi\right], \tag{28}$$

$$\frac{\mathrm{d}I_3}{\mathrm{d}t} = \frac{1}{n_3 T_{\mathrm{c}}}\left[-v_3 I_3 + \frac{(\mu_3 I_1 I_2 I_3)^{1/2}}{1+R_3}\sin\Phi\right], \tag{29}$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{(I_1 I_2 I_3)^{1/2}\cos\Phi}{4(n_1+n_2-n_3)T_{\mathrm{c}}}\left(\frac{\sqrt{\mu_1}}{I_1} - \frac{\sqrt{\mu_3}}{I_3}\right) + \frac{\delta\Psi}{(n_1+n_2-n_3)T_{\mathrm{c}}}, \tag{30}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1}{T_1}[1 + \eta - N(I_1 + I_1 R_1 + 1)], \tag{31}$$

where $I_2 = (1/\delta - 1)P_{\text{pump}}/\pi r_0^2 I_{\mathrm{s}}$; δ is a coefficient characterising absorption of pumping radiation by the active medium $(0 < \delta \leqslant 1)$; $\Phi = \varphi + \arcsin[\cos(\Delta k L/2 + \delta\Psi/2)]$. In the case of exact QPM and for $\delta\Psi = 0$, the system of equations (28) – (31) has two steady-state solutions:

$$I_3 = \frac{\mu_3 I_2\left[\eta v_1 v_3 (1+R_1)(1+R_3) - I_2(\mu_1\mu_3)^{1/2}\right]}{v_3^2(1+R_1)(1+R_3)^2\left[v_1 v_3(1+R_1)(1+R_3) + I_2(\mu_1\mu_3)^{1/2}\right]}, \tag{32}$$

$$I_3 = \frac{(\mu_1/\mu_3)^{1/2}}{(1+R_1)}\,\frac{\eta v_1(1+R_1) - v_3(1+R_3)}{v_1(1+R_1) + v_3(1+R_3)}. \tag{33}$$

Figs 6 and 7 show the characteristic dependences corresponding to stable branches of the solutions (32) and (33). One can see that as in the case of self-frequency doubling, the use of a high-Q cavity for small α_3 makes it possible to considerably increase the efficiency of nonlinear optical conversions. Calculations were made for a $Nd:Mg:LiNbO_3$ crystal with $L = 0.5$ cm and $\Lambda = 4.2$ μm placed in a double cavity, one of whose mirrors has the reflection coefficient $R_1 = R_3 = 100\,\%$, and the other $R_1 = 100\,\%$ and $R_3 < < 100\,\%$. As before, it is assumed that the ee – e interaction is realised in the crystal and $\lambda_1 = 2\pi c/\omega_1 = 1.084$ μm, $\lambda_2 = 2\pi c/\omega_2 = 0.81$ μm, $\lambda_3 = 2\pi c/\omega_3 = 0.464$ μm, $1/\delta - 1 = 0.3$.

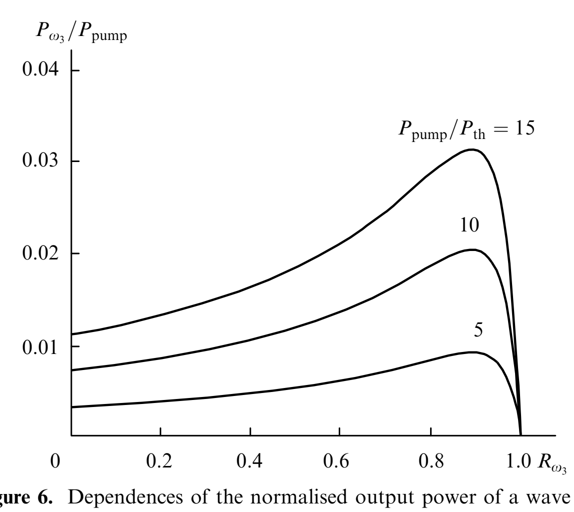

Figure 6. Dependences of the normalised output power of a wave with frequency ω_3 on the reflection coefficient of the output mirror for I_3 for different pump powers ($R_1 = 1$, $v_1 = 0.08$, $\alpha_3 = 0.1$ cm^{-1}, $m = 1$).

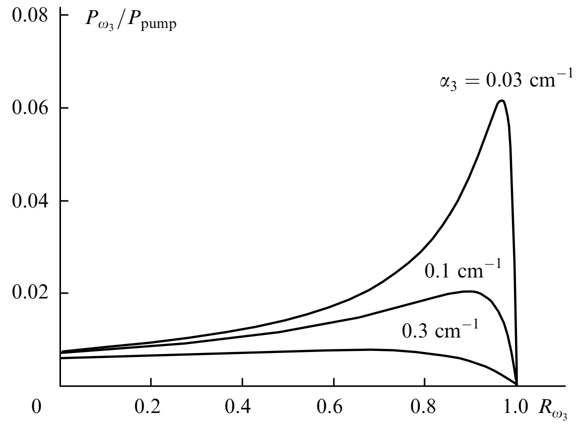

Figure 7. Dependences of the normalised output power of a wave with frequency ω_3 on the reflection coefficient of the output mirror for I_3 for different linear losses α_3 ($R_1 = 1$, $v_1 = 0.08$, $P_{\text{pump}}/P_{\text{th}} = 10, m = 1$).

6. Conclusions

We have presented in this work the theory of intracavity three-frequency QPM interactions of light waves in active nonlinear periodically poled crystals. We analysed in detail QPM processes of self-frequency doubling, halving and mixing using the pump wave in an active nonlinear periodically poled Nd:Mg:$LiNbO_3$ crystal. The existence of optimal reflection coefficients of output cavity mirrors for the efficient generation of the second harmonic, subharmonic, and a wave with the sum frequency is shown.

The results presented in this work confirm that the use of semiconductor pumping and QPM wave interactions opens up new avenue for applying active nonlinear media to realise three-frequency wave interactions.

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