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Effect of inhomogeneous optical properties of a nonlinear medium on the propagation of powerful light beams

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***Abstract*. The propagation of powerful light beams in nonlinear media with the Kerr nonlinearity is considered. The effect of local inhomogeneities on self-focusing of the beams is studied.**

Keywords: *light beams, nonlinear medium, optical inhomogeneities.*

The properties of powerful light beams propagating in nonlinear media have been studied in detail in the literature [1 – 6]. A special attention in these papers has been paid to self-focusing and self-trapping of the beams. The parameters characterising linear and nonlinear properties of the medium have been assumed, as a rule, constant. In this paper, we perform a numerical analysis of the effect of local inhomogeneities of the nonlinearity and absorption coefficients of a medium on self-focusing of light beams near focus.

We will describe the propagation of optical beams in nonlinear media along the z axis ($z \geqslant 0$) by the approximate Schrödinger equation

$$\frac{\partial E}{\partial z} + \frac{\mathrm{i}}{2k}\Delta_{\perp}E + \mathrm{i}F(I)E = 0. \quad (1)$$

Here, E is the complex amplitude; I is the intensity; $k = 2\pi/\lambda$ is the wave number; λ is the wavelength; $\Delta_{\perp} = \partial^2_{xx} + \partial^2_{yy}$ is the Laplace operator; and $F(I)$ is the function describing the nonlinear properties of the medium.

We considered the solution of equation (1) satisfying the conditions

$$E|_{z=0} = E_0 \exp\left(-\frac{x^2 + y^2}{2r_0^2}\right)$$

and

$$E|_{x^2+y^2\to\infty} = 0,$$

where E_0 is the beam amplitude at the medium surface and r_0 is its characteristic width.

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By solving equation (1), it is convenient to normalise the functions and independent variables as

$$x' = x/r_0, \quad y' = y/r_0, \quad z' = z/2kr_0^2,$$

$$E' = E/E_0, \quad F'(I') = 2kr_0^2 F(I_0 I').$$

After the normalisation, equation (1) takes the form (the prime is omitted for brevity)

$$\frac{\partial E}{\partial z} + \mathrm{i}\Delta_{\perp}E + \mathrm{i}F(I)E = 0. \quad (2)$$

We will write the initial condition in the form

$$E|_{z=0} = \exp\left[-\frac{(x^2 + y^2)}{2}\right]. \quad (3)$$

Within the framework of this approach, the influence of the medium on the propagation of optical beams with different initial amplitude distributions is reduced to the specifying of nonlinearity of one or other type in the equation. We represent the function $F(I)$ in equation (2) in the form

$$F(I) = \alpha I - \mathrm{i}\beta I, \quad (4)$$

where α is the Kerr nonlinearity coefficient and β is the nonlinear two-photon absorption coefficient. Recall that $\alpha/4 = P_0/P_{\mathrm{cr}}$ is the ratio of the input and critical powers.

The coefficients α and β are usually assumed constant. In our case, they have the form

$$\alpha = \alpha_0\left\{1 + \varepsilon\sum_j \exp\left[-\frac{(x - x_j)^2 + (y - y_j)^2}{\eta_j^2}\right]\right.$$

$$\left.\times \exp\left[-\frac{(z - z_j)^2}{\zeta_j^2}\right]\right\},$$

$$\beta = \delta\sum_i \exp\left[-\frac{(x - x_i)^2 + (y - y_i)^2}{\sigma_i^2}\right]\exp\left[-\frac{(z - z_i)^2}{\xi_i^2}\right],$$

where α_0 is the unperturbed Kerr nonlinearity coefficient; $x_j, y_j, z_j, x_i, y_i, z_i, \eta_j, \zeta_j, \sigma_i, \xi_i$ are the parameters determining the location and size of the regions where coefficients α and β are inhomogeneous; and ε and δ are the maximum inhomogeneities.

We solved the problem (2) – (5) numerically by the method of variable directions (the longitudinal – transverse scheme) of the second-order accuracy described in Ref. [7].

Consider first the case when $\varepsilon = 0$, $\delta = 0$. The numerical calculations were performed to the distance $z = 0.046$, which was close to the focal distance z_f. The parameter α_0 was assumed equal to 90. Fig. 1 shows the spatial profiles of a light beam for $z = 0$, 0.042, 0.043, 0.0446. One can see that the beam retains a Gaussian shape with increasing intensity.

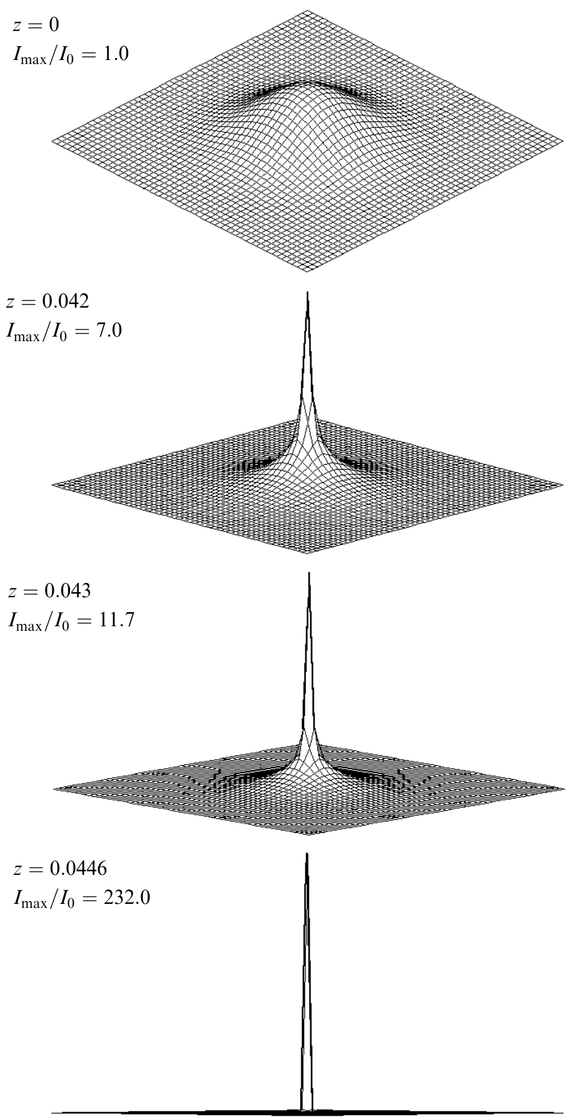

Figure 1. Spatial profile of the light-beam intensity for different z $z\,(\varepsilon = 0,\ \delta = 0)$.

The inhomogeneity of the nonlinearity coefficient was considered for several groups of parameters.

The first group was $x_1 = 0.28$, $x_2 = 0.2$, $x_3 = -0.27$, $x_4 = -0.27$; $y_1 = 0.28$, $y_2 = -0.28$, $y_3 = -0.2$, $y_4 = 0.17$; $z_j = 0.0428, \eta_j = 0.1, \zeta_j = 0.316 \times 10^{-3}, j = 1-4$. These coordinates correspond to the characteristic width of the beam in the plane $z = 0.0428$ for $\varepsilon = 0$ and $\delta = 0$. In this case, the beam intensity at its axis is approximately 10.4. Fig. 2 shows the surface demonstrating the inhomogeneity of the nonlinearity coefficient of the medium.

The second group was $x_1 = 0.083, x_2 = 0.04, x_3 = -0.06, x_4 = -0.066; y_1 = 0.083, y_2 = -0.083, y_3 = -0.041, y_4 = 0.17$; $z_j = z_j = 0.0428, \eta_j = 0.1, \zeta_j = 0.316 \times 10^{-3}, j = 1-4$. These coordinates determine the points that are located closer to the beam axis than in the first case.

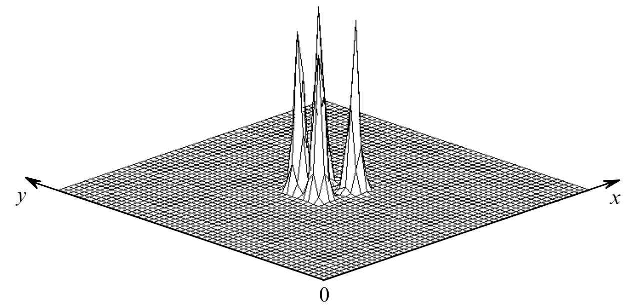

Figure 2. Surface demonstrating the inhomogeneity of the nonlinearity coefficient of the medium for $z = z_j$.

The third group contained the points that were located in the vicinity of the beam axis and the groups of points consisting of a superposition of the first and second groups.

The calculations were performed for $\varepsilon = 0.5, 1, 2, 4$ and for $\varepsilon = -0.5, -1, -2, -4$. For $\varepsilon = 0.5, 1, 2, 4$ and $\delta = 0$ for the first group of inhomogeneity points, a qualitative picture is similar to that shown in Fig. 1. The amplitude shape remains invariable. The only difference is that the intensity of the beam propagating along the z axis increases stronger and the focus shifts to the lower values of z.

In the case of the inhomogeneities of the same type, whose location is characterised by the points of the second and third groups, self-focusing occurs behind $z \approx z_j$ more efficiently, and the focus is formed in the vicinity of z_j.

The case of negative ε ($\varepsilon = -0.5, -1, -2, -4$), when the local defocusing of the beam occurs, substantially differs from the above cases. In this case, the beam intensity decreases in the vicinity of inhomogeneities and the beam is decomposed into several components. The beam decomposition continues over the distance $|z - z_j| \approx \zeta_j$. Then, the beam again converges and is focused. The focal distance increases with increasing $|\varepsilon|$.

These effects occurred for all the three groups of the inhomogeneity parameters considered above. Fig. 3 shows the spatial profiles of the beam intensity for $\delta = 0$ and $\varepsilon = -2$ for the third group of parameters.

The inhomogeneity of the nonlinear two-photon absorption coefficient was also specified by three groups of parameters. The calculations were performed for $\varepsilon = 0$ and $\delta = 0.01, 0.1, 0.5, 1, 2$. For small values of δ ($\delta = 0.01, 0.1, 0.5$), the inhomogeneity weakly affects the beam focusing compared to the case $\varepsilon = 0$, $\delta = 0$. The only difference is that the beam intensity becomes lower in the region of inhomogeneity influence ($|z - z_j| \approx \zeta_j$) and then it is focused. For large δ ($\delta = 1, 2$) in the region of inhomogeneity influence, the beam intensity exhibits a hole at the beam axis. As z increases, the intensity peak is formed against this hole, resulting in the beam focusing at somewhat larger values of z. These features of the effect of the medium inhomogeneity are typical for all the three groups of parameters. Fig. 4 shows the development of the process for the third group of parameters for $\varepsilon = 0$, $\delta = 2$.

The simultaneous effect of the inhomogeneities of the nonlinearity and absorption coefficients of the medium can be interpreted as a superposition of the situations considered above. In this case, the beam self-focusing occurs qualitatively as in the above cases.

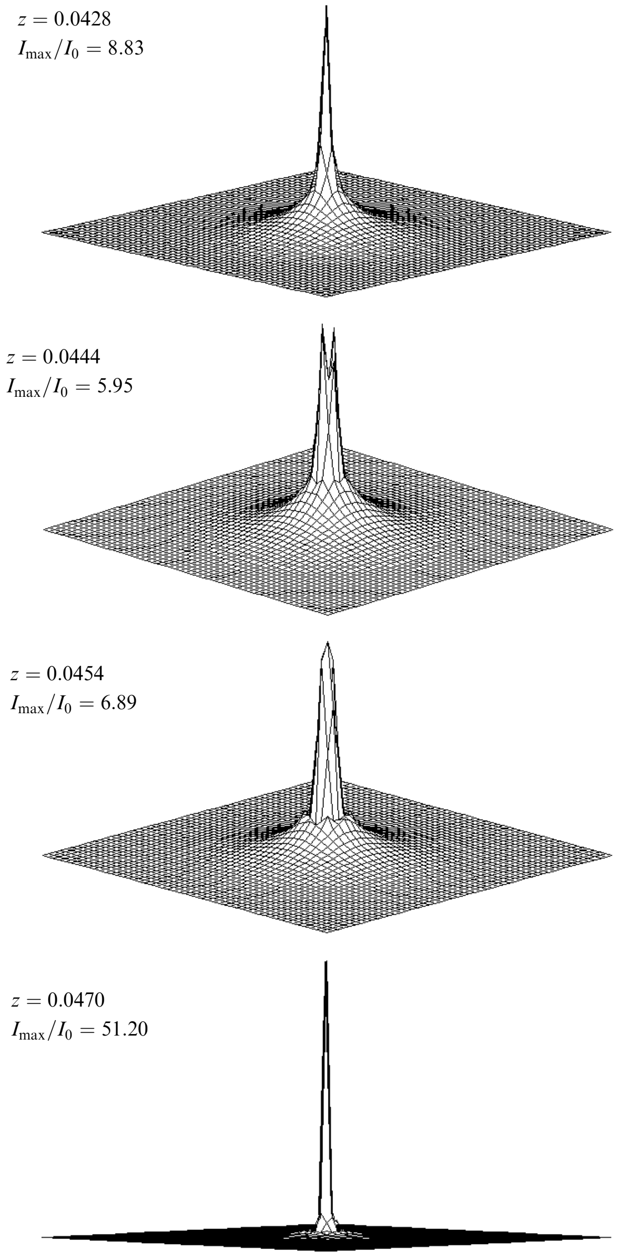

Figure 3. Spatial profile of the light-beam intensity for different $z\,(\varepsilon = -2,\ \delta = 0)$.

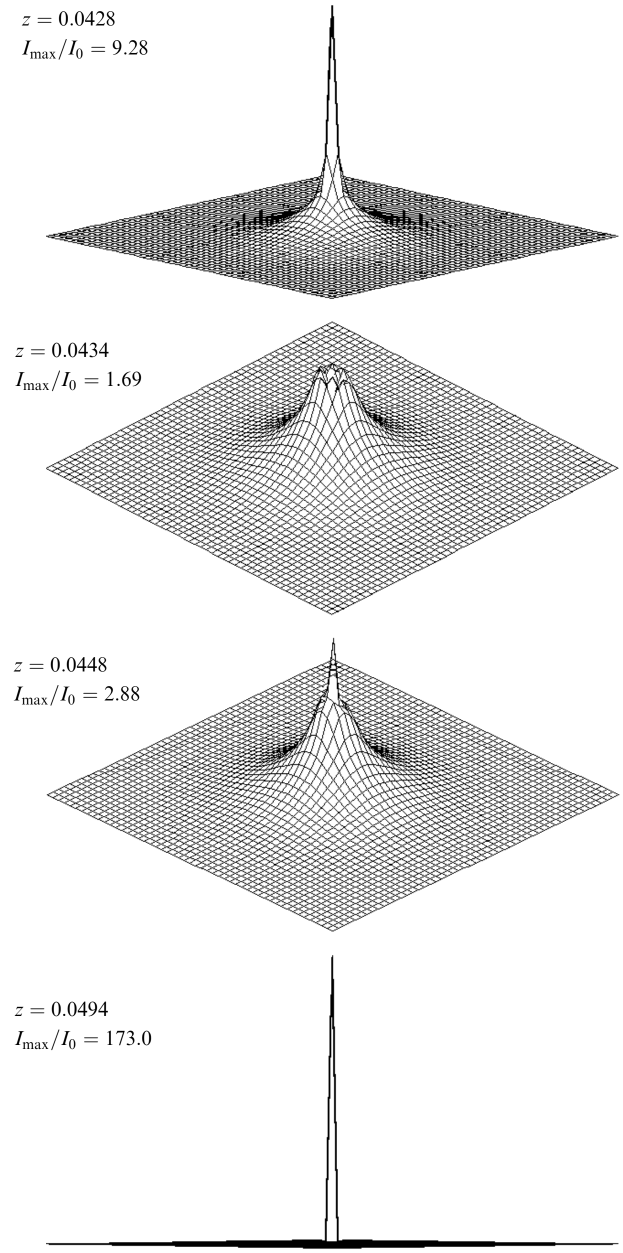

Figure 4. Spatial profile of the light-beam intensity for different $z\,(\varepsilon = 0,\ \delta = 2)$.

The main result of our study is that the focusing of powerful optical beams propagating in a medium with the Kerr nonlinearity and two-photon absorption is stable. This statement means mathematically that a small inhomogeneity of the parameters of the problem causes small perturbations of the solution for any interval $[0, z'_{\mathrm{f}}]$ $(z'_{\mathrm{f}} = z_{\mathrm{f}} - \varepsilon_1,\ \varepsilon_1 > 0)$. Physically, this means that the beam intensity can change qualitatively when the parameters of the medium inhomogeneity are varied in a broad range, but then the beam nevertheless undergoes self-focusing.

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