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Optimal control of optical soliton parameters: Part 2. Concept of nonlinear Bloch waves in the problem of soliton management

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***Abstract*.** **It is shown that optical solitons in nonlinear fibreoptic communication systems and soliton lasers can be represented as nonlinear Bloch waves in periodic structures. The Bloch theorem is proved for solitons of the nonlinear Schrödinger equation in systems with the dispersion, the nonlinearity, and the gain (absorption coefficient) periodically changing over the length. The dynamics of formation and interaction, as well as stability of the coupled states of nonlinear Bloch waves are investigated. It is shown that soliton Bloch waves exist only under certain self-matching conditions for the basic parameters of the system and reveal a structural instability with respect to the mismatch between the periods of spatial modulation of the dispersion, nonlinearity or gain.**

Keywords: *optical solitons, nonlinear waves, soliton management.*

1. Introduction

The use of optical solitons as ideal carriers of a data bit in fibreoptic communication systems and the development of soliton wavelength-division-multiplexed (WDM) communication systems with a bit rate of 40 Gbit s^{-1} is one of the most significant recent achievements of quantum electronics. Recent advances in this rapidly developing field of science and technology are most comprehensively analysed in a collection of papers [1] and review [2].

A group of Nakasawa at NTT Network Innovation Laboratories (Japan) has performed spectacular experiments on soliton data transmission in commercial fibreoptic Tokyo Metropolitan Network. In this network, a commercial fibreoptic cable provides a bit rate of 2.4 Gbit s^{-1} in the usual regime. In the WDM soliton dispersion management regime, a bit rate as high as 40 Gbit s^{-1} was obtained in the network of length 1000 – 2500 km.

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One of the central problems of all-optic soliton fibreoptic communication systems (in which laser amplifiers are used as retransmitters rather than electronic amplifiers) is the problem of the optimal control of the parameters of optical solitons, which is also called the problem of soliton management. The use of various combinations of optical fibres with periodically varying dispersion signs (fibre dispersion management) allows the development of the soliton communication system providing a bit rate as high as several terabits per second (see, for example, Refs [1, 2]). The concept of the soliton management proved to be very useful for the building of new types of femtosecond soliton lasers as well [3, 4].

The mathematical problem of soliton management is a problem of the optimal control of the parameters of optical solitons in the nonlinear Schrödinger equation (NSE) model for a closed or an open system with the system parameters that periodically vary over the system length. These parameters are the group velocity dispersion, the nonlinearity of the refractive index, coefficients of linear losses of emission and of periodic amplification of solitons in communication systems.

This work is devoted to the solution of this problem and to the proof of the fact that Schrödinger optical solitons in periodic structures represent a nonlinear analogue of well-known electron Bloch waves in crystals.

2. Nonlinear Bloch theorem for Schrödinger solitons

The concept of nonlinear Bloch waves was proposed by Haus and Chen [3] to describe the dynamics of optical solitons in the problem of soliton management. The authors of paper [3] pointed out a profound analogy between electron Bloch waves in crystals and nonlinear wave packets in fibreoptic communication systems with spatially varying parameters. In such nonlinear periodic dispersion systems, a soliton pulse produces a nonlinear scattering self-consistent potential for itself due to the self-action during its propagation. Haus and Chen considered the dynamics of a system with a periodically varying dispersion sign in the case when the equivalent averaged dispersion proves to be zero. The theoretical approach to the problem of nonlinear Bloch waves used in papers [3, 4] was based on the variation approximation of Anderson [5] and on the assumption that Gaussian – Hermitian polynomials of a linear eigenvalue problem can be used as test functions. In such an approach, the nonlinear Bloch waves represent the so-called dispersion-managed solitons, which are known to interact with each other inelastically [1 – 8].

We will use an approach that differs from that applied in papers [3, 4], which is based on the concept of quasi-particle wave packets, which are formed in a medium with varying parameters (potentials). By using a direct wave–particle analogy for an electron in a crystal and a soliton in a periodic structure, we will show that the dynamics of Schrödinger solitons in a periodic nonlinear system obeys the nonlinear Bloch theorem, the formulation of the problem having profound quantum-mechanical analogues.

According to the Bloch theorem [9–11], the wave function of an electron in a periodic crystal lattice

$$\Psi(x) = U_k(x)\exp(\mathrm{i}kx) \quad (1)$$

is the wave function $\exp(\mathrm{i}kx)$ of a free electron modulated by the so-called Bloch function, which has a period of the crystal lattice:

$$U_k(x) = U_k(x + nL). \quad (2)$$

Let us show that the Schrödinger soliton in a periodic structure, like an electron in an external periodic potential, can be described by the corresponding soliton Bloch function.

We will describe the self-action of a nonlinear pulse in a system with periodically varying parameters in the NSE approximation, which has, in standard soliton variables, the form [12–16]

$$\mathrm{i}\,\frac{\partial\Psi^{\pm}}{\partial Z} \pm \frac{1}{2}\,D(Z)\,\frac{\partial^2\Psi^{\pm}}{\partial X^2} + N(Z)\left|\Psi^{\pm}\right|^2\Psi^{\pm}$$

$$= -\mathrm{i}\gamma_0\Psi^{\pm} + \mathrm{i}\Gamma(Z)\Psi^{\pm}, \quad (3)$$

where Z is the dimensionless length of the pulse propagation normalised to the dispersion length; $\Psi^{\pm}(Z, X)$ is the field expressed in the units of the amplitude of the so-called fundamental NSE soliton with constant coefficients [12–16]. The coefficients introduced into (3) describe the periodic variation in the parameter of the group velocity dispersion $D(Z) = D(Z + nL)$, the nonlinearity parameter $N(Z) = N(Z + nL)$, and the gain $\Gamma(Z) =$ over the length of interaction of radiation with the spatially inhomogeneous system. Linear losses are described by the parameter γ_0.

Due to the well-known space–time analogy, equation (3) describes both spatial solitons ('slot' beams) and usual temporal solitons (stationary pulses), depending on whether the coordinate X represents time or the spatial coordinate. The sign plus in equation (3) corresponds to the problem for a bright soliton (anomalous dispersion) and the sign minus corresponds to a dark soliton (normal dispersion).

By using the substitution

$$\tilde{\Psi}^{\pm}(Z, X) = \frac{\Psi^{\pm}(Z, X)}{G(Z)}, \quad (4)$$

$$R(Z) = N(Z)G^2(Z), \quad (5)$$

where the function $G(Z)$ satisfies the equation

$$\frac{\partial G(Z)}{\partial Z} = -\gamma_0 G(Z) + \Gamma(Z)G(Z), \quad (6)$$

equation (3) can be reduced to the form

$$\mathrm{i}\,\frac{\partial\tilde{\Psi}^{\pm}}{\partial Z} \pm \frac{1}{2}\,D(Z)\,\frac{\partial^2\tilde{\Psi}^{\pm}}{\partial X^2} + R(Z)\left|\tilde{\Psi}^{\pm}\right|^2\tilde{\Psi}^{\pm} = 0. \quad (7)$$

The nonlinear Bloch theorem for solitons described by the NSE model (7) can be represented, similarly to the Bloch theorem for electrons (1), as the law for the transformation of temporal (for soliton pulses) or spatial (for soliton beams) envelopes of the bright soliton $\tilde{\Psi}^{+}(Z, X)$ and dark soliton $\tilde{\Psi}^{-}(Z, X)$ modulated by the corresponding nonlinear Bloch functions with the periodic parameters of the spatially inhomogeneous structure $D(Z) = D(Z + nL)$ and $R(Z) = R(Z + nL)$:

$$\tilde{\Psi}^{\pm}(Z) = P^{1/2}(Z)\left\{\begin{array}{c}\sqrt{C}\eta\,\mathrm{sech}[\eta P(Z)X] \\ \sqrt{C}\eta\tanh[\eta P(Z)X]\end{array}\right\}$$

$$\times\exp\left[\pm\mathrm{i}\,\frac{P(Z)}{2}\,X^2\right] - \mathrm{i}\alpha\eta^2[P(Z) - P(0)], \quad (8)$$

where η is the form factor of the soliton pulse; $\alpha = 0.5$ for bright solitons and $\alpha = 1$ for dark solitons. To prove the nonlinear Bloch theorem (8), it is necessary first to show that the real function $P(Z)$ describing the modulation in the space of the canonical form of the bright and dark solitons has the same periodicity as the parametric functions $D(Z)$ and $R(Z)$. Second, it is necessary to find the explicit form of the function $R(Z) = R(Z + nL)$.

Let us represent the nonlinear Bloch function for the problem (7) in the general form as

$$\tilde{\Psi}^{\pm}(Z) = P^{1/2}(Z)Q^{\pm}(S)$$

$$\times\exp\left[\pm\mathrm{i}\,\frac{P(Z)}{2}\,X^2 + \mathrm{i}\int_0^Z K^{\pm}(\zeta)\mathrm{d}\zeta\right], \quad (9)$$

where $Q^{\pm}$ and $K^{\pm}$ are the amplitude and phase of a soliton wave. By substituting (9) into (7) and separating the real and imaginary parts in the equation obtained, we obtain the system

$$\pm\frac{1}{2}\,\frac{\partial^2 Q^{\pm}}{\partial S^2} + \frac{R}{DP}\,(Q^{\pm})^3 - Q^{\pm}\,\frac{K^{\pm}}{DP^2}$$

$$\mp\,\frac{S^2 Q^{\pm}}{2DP^4}\left(DP^2 + \frac{\partial P}{\partial Z}\right) = 0\,, \quad (10)$$

$$\left(DP^2 + \frac{\partial P}{\partial Z}\right)\left(\frac{1}{2}\,Q^{\pm} + S\,\frac{\partial Q^{\pm}}{\partial S}\right) = 0\,, \quad (11)$$

where $S(Z, X) = P(Z)X$; $\partial S/\partial Z = X\partial P/\partial Z$; $\partial S/\partial X = P(Z)$.

Equation (10) is a quantum-mechanical Schrödinger wave equation for a harmonic oscillator in the self-consistent nonlinear potential

$$U(S,Z) = \mp\frac{R(Z)}{D(Z)P(Z)}[Q^{\pm}(S)]^2 \pm \frac{S^2}{2D(Z)P^4(Z)}\left(DP^2 + \frac{\partial P}{\partial Z}\right). \quad (12)$$

A particular solution of equation (10) was obtained numerically for the first time in paper [17] assuming that the basic parameters of the problem satisfy the condition

$$\frac{\partial P}{\partial Z} + DP^2 = \text{const} = 1. \quad (13)$$

The numerical solution, which was called the NSE quasi-soliton with a harmonic potential, has the intermediate form between a Gaussian and hyperbolic secant describing the form of a canonical NSE soliton with constant coefficients.

Unlike papers [17 – 19], we will consider another particular solution of the problem at which the functions $Q^{\pm}$ describe the bright and dark NSE quasi-solitons modulated by the nonlinear Bloch function. We should find the compatible solution of the nonlinear system (10), (11) for the Bloch function (9) under study. As follows from (1), the condition

$$\frac{1}{2}Q^{\pm} + S\frac{\partial Q^{\pm}}{\partial S} = 0 \quad (14)$$

leads to a singular solution of the form

$$Q(S) = \frac{\text{const}}{\sqrt{S}}. \quad (15)$$

To find a nonsingular solution of the system (10), (11), we require that the relation

$$\frac{\partial P}{\partial Z} + DP^2 = 0. \quad (16)$$

would be satisfied. One can easily see that condition (16) for the existence of a nonsingular solution allows the transformation of equation (10) into a nonlinear Schrödinger equation with variable coefficients. As was shown in the first part of this paper [20], this equation has the Lax representation and, therefore, it is completely integrable then and only then when the so-called condition of mutual matching between the coefficients at nonlinear and dispersion terms is satisfied:

$$R(Z) = CD(Z)P(Z), \quad (17)$$

where C is an arbitrary constant.

Therefore, if conditions (16) and (17) are satisfied, then the solution of the system (10), (11) is represented in the form of the bright or dark soliton modulated by the function $P(Z)$:

$$Q^{\pm}(S) = \begin{cases} \eta\,\text{sech}[\eta P(Z)X], \\ \eta\tanh[\eta P(Z)X]. \end{cases} \quad (18)$$

It follows from (5) and (17) that the soliton amplitude and duration will be modulated $[P(Z) = P(Z + nL)]$ if the conditions

$$\frac{R(Z+nL)}{R(Z)} = \frac{D(Z+nL)}{D(Z)} = \frac{N(Z+nL)G^2(Z+nL)}{N(Z)G^2(Z)} = 1. \quad (19)$$

are satisfied.

It should be emphasised that we obtained analytic solutions for soliton Bloch waves (8), (16), (17) in quadratures in the most general form. The phase of a soliton wave is determined by the relation

$$K^{+}(Z) = \frac{1}{2}\eta^2 D(Z)P^2(Z) \quad (20)$$

for the bright soliton and by the relation

$$K^{-}(Z) = \eta^2 D(Z)P^2(Z) \quad (21)$$

for the dark soliton. It has the same translational symmetry as the periodic perturbations of the parameters of a nonlinear medium. Note that the moduli of phases of the dark and bright solitons differ by a factor of two.

3. Nonlinear dynamics of soliton Bloch waves

The predicted soliton Bloch waves can be found under different specific experimental conditions. When the dispersion parameter $D(Z)$ is assumed known upon planning a particular experiment, we can approximate it by an analytic periodical function $D(Z) = D(Z + nL)$ and find the main parameters describing a soliton Bloch wave from the equation

$$P(Z) = P_0\left[1 + P_0\int_0^Z D(Z')\mathrm{d}Z'\right]^{-1}, \qquad R(Z) = \frac{P(Z)D(Z)}{C}. \quad (22)$$

If, on the contrary, an analytic approximation $R(Z) = R(Z + nL)$ of the nonlinearity parameter $R(Z)$ is known in a particular experiment, then the dispersion characteristic and the parameters of a Bloch wave in the system are determined by the expressions

$$P(Z) = P_0\exp\left[-C\int_0^Z R(Z')\mathrm{d}Z'\right], \qquad D(Z) = \frac{CR(Z)}{P(Z)}. \quad (23)$$

The nonlinear Bloch theorem for Schrödinger solitons (8), (16), (17) also allows one to solve a more general problem of the optimal control of the soliton parameters in communication systems and soliton lasers. A remarkable feature of the solution (8) is the fact that the soliton amplitude, duration, and phase in an inhomogeneous medium are determined only by one function $P(Z) = P(Z + nL)$. We will treat this function as a function of the optimal control of the soliton parameters. By choosing the function $P(Z)$ in accordance with the requirements of a particular problem as an analytic function of the variable Z, we obtain the following conditions for the existence of the soliton Bloch wave:

$$D(Z) = -\frac{1}{P^2(Z)}\frac{\partial P}{\partial Z}, \quad R(Z) = -\frac{1}{CP(Z)}\frac{\partial P}{\partial Z}. \tag{24}$$

As a rule, it is much more convenient to use in the experiment a certain periodic gain $\Gamma(Z) = \Gamma(Z + nL)$ of solitons. If this function can be approximated by a specified analytic function of the variable Z, then the so-called map of the main parameters – dispersion and nonlinearity – can be found from the system of equations

$$\frac{D(Z)P(Z)}{N(Z)} = \frac{D_0 P_0}{N_0}\exp\left[-2\gamma_0 Z + 2\int_0^Z \Gamma(\zeta)\mathrm{d}\zeta\right], \tag{25}$$

$$P(Z) = P(0)\exp\left\{-\int_0^Z N(\xi)CG(0) \times \exp\left[-2\gamma_0\xi + 2\int_0^\xi \Gamma(\zeta)\mathrm{d}\zeta\right]\mathrm{d}\xi\right\}. \tag{26}$$

Consider a number of particular examples. Let us assume that a periodic fibreoptic structure containing laser amplifiers has a length that is multiple of the period L of variation of the main parameters of the structure and is closed to form a ring. Such a scheme of the optical soliton memory has been proposed in paper [21]. The fundamental solutions for this soliton memory model can be written in terms of trigonometric functions. By approximating a periodic dispersion function, for example, by the expression

$$D(Z) = 1 + \delta\sin^m(\kappa Z) \tag{27}$$

and considering the simplest situation when the nonlinearity of the refractive index does not change over the pulse propagation length ($N(Z) = 1$), we obtain the following solutions for a soliton periodically circulating in the ring:

$$P^{-1}(Z) = -\left(C - Z - \frac{\delta}{\kappa}\int_0^{\kappa Z}\sin^m x\,\mathrm{d}x\right), \tag{28}$$

$$2\Gamma(Z) = -D(Z)P(Z) + \frac{\delta\kappa m\sin(2\kappa Z)\sin^{2m-2}(\kappa Z)}{D(Z)}, \tag{29}$$

where an arbitrary integration constant C may be both positive and negative.

The space-time structure of bright and dark solitons (27)–(29), whose amplitude and duration change periodically (soliton Bloch waves), is shown in Fig. 1. Fig. 2 shows the dynamics of interaction of periodic solutions (27)–(29) compared to the elastic interaction of canonical NSE solitons with constant coefficients. The results presented in Figs 2a–c (in-phase interaction) and Fig. 2d (out-of-phase interaction) were obtained by direct integration of equation (3) for different values of the parameter κ of the structure periodicity (27).

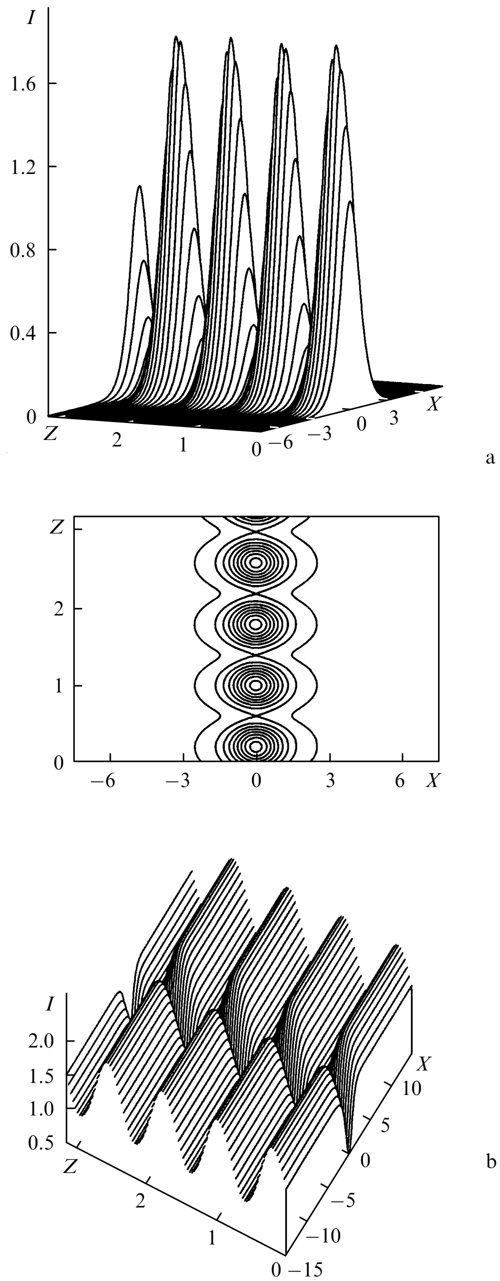

Figure 1. Spatial interaction dynamics for bright (a) and dark (b) soliton Bloch waves in a periodic nonlinear system (27)–(29) for $\delta = 0.75$, $m = 1$, and $C = 10^5$

Let us represent the control function $P(Z)$ in the form

$$P(Z) = \frac{(1-\beta^2)^{1/2}}{1-\beta\cos(\kappa Z)}. \tag{30}$$

When the parameter $\beta \to 1$, this function represents a periodic grating of delta functions. For $\beta \to 0$, function (30) tends to the constant value $P(Z) = 1$ undergoing periodic oscillations. Note that function (30) modulates a periodic jump-like amplification of solitons over the propagation length of a signal in soliton communication systems. The results presented in Figs 3, 4 illustrate the dynamics of a coupled state of initially immobile solitons (in the soliton

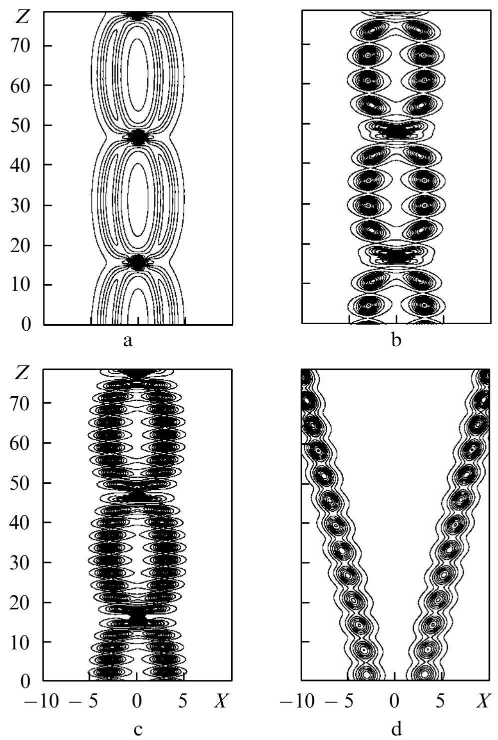

Figure 2. Nonlinear interaction dynamics for in-phase (a–c) and out-of-phase (d) soliton Bloch waves compared to the interaction dynamics for canonical NSE solitons with constant coefficients. The periodic dispersion $D(Z)$ and gain $\Gamma(Z)$ are specified by relations (27)−(29) with parameters $m = 2$, $\kappa = 2$ (b, c) and $\kappa = 4$ (d). The distance between the soliton centres is $\Delta x = 6$ for $Z = 0$.

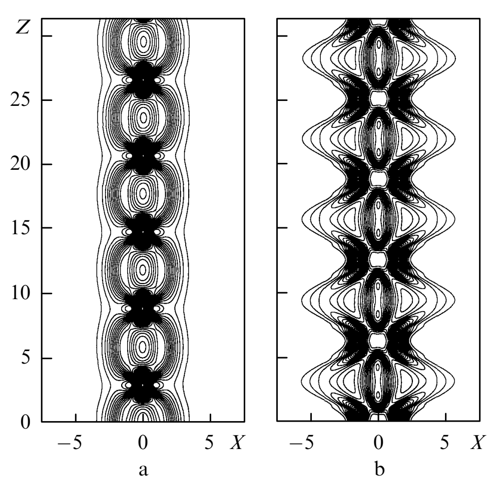

Figure 3. Interaction dynamics for strongly overlapped soliton Bloch waves (b) for the control function (30) compared to the interaction dynamics for canonical NSE solitons with constant coefficients (a) calculated for $\beta = 0.75$, $\kappa = 1$, $x = 3$ and $Z = 0$.

coordinate system), which was calculated using the control function (30). A comparison of the interaction dynamics of solitons (8), (30) (Fig. 3) with the dynamics of canonical NSE solitons with constant parameters shows that, despite a substantial difference in details, the nature of the elastic interaction of solitons (8), (30) does not change substantially: soliton Bloch wave interact elastically. Fig. 4 illustrates

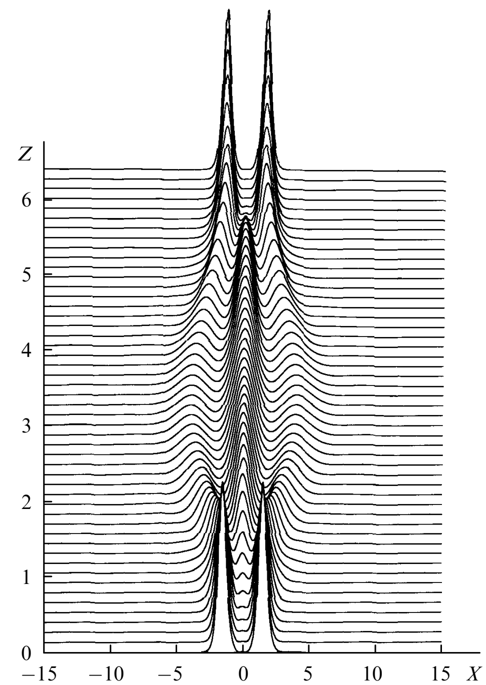

Figure 4. Detailed structure of the field distribution corresponding to one period of the interaction of strongly overlapped dark soliton Bloch waves.

the details of formation of a complicated structure upon the interaction of strongly overlapped soliton Bloch waves.

The fact that the problem of soliton Bloch waves can be reduced to the completely integrable model [see (10), (11) and (16), (17)] means that the coupled states of nonlinear Bloch waves can be formed both in soliton data transmission and storage systems and in soliton fibre lasers.

To generate nonlinear Bloch waves of higher orders, the initial conditions should be specified in the form

$$\tilde{\Psi}^{+}(Z = 0) = NP_0^{1/2}\eta\,\mathrm{sech}(\eta P_0 X)\exp\left(\mathrm{i}\,\frac{P_0}{2}\,X^2\right). \quad (31)$$

The principal difference from a standard generation of coupled soliton states in optical fibres with constant parameters over the fibre length [12−16] is that now in order to form a coupled state, it is necessary not only to increase the soliton amplitude by several times but also to specify properly the initial phase modulation of the soliton at the entrance to the medium. For spatial solitons, this means the specification of the initial parabolic wave front of the beam.

Fig. 5 shows the results of computer simulations of the formation dynamics of coupled soliton states in a periodically nonlinear system (8), (30), which were obtained by varying the period of spatial modulation of parameters $D(Z)$ and $R(Z)$ in the model (7). It follows from Fig. 5 that, as the modulation period increases, the number of nonlinear foci, which characterise the coupled states of Schrödinger solitons [12−16], noticeably decreases. A further increase in the modulation period results in the stabilisation of the emission structure and formation of a quasi-soliton pulse. Computer simulations showed that the coupled states of soliton Bloch waves are unstable, and even a slight mismatch of about 1 %−2 % between the periods of the spatial

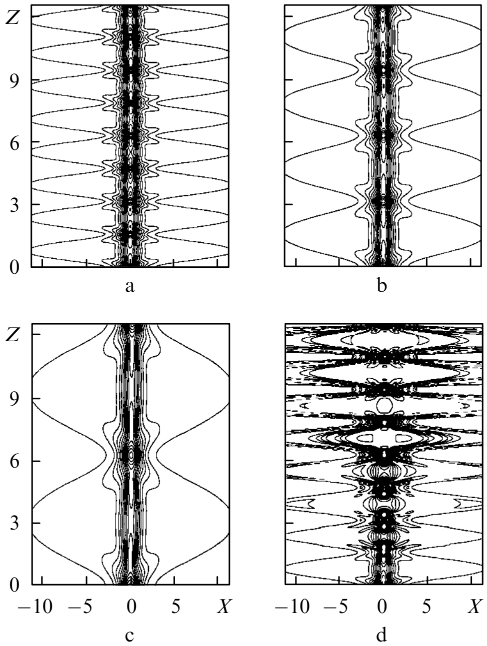

Figure 5. Formation dynamics for coupled states of four-soliton Bloch waves calculated for the control function (30) for variable periods of the spatial modulation of dispersion and nonlinearity and parameters $\beta = 0.75$, $\kappa = 8$ (a), 4 (b), 2 (c) and also for the 2 % mismatch between the periods of spatial modulation of dispersion and nonlinearity ($\kappa = 8$) (d).

modulation of the dispersion and the gain results in a complete stochastic behaviour of the system, as shown in Fig. 5.

4. Conclusions

The results of computer simulations confirm a conclusion about the 'classical' elastic interaction of nonlinear Bloch waves and the possibility of formation of their coupled states. Note that one of the simplest methods for shaping the dispersion over the propagation length of a soliton pulse is the use of the fragments of different fibres with opposite dispersion signs [22]. The importance of using the initial phase modulation (chirp) in soliton transmission systems was recently demonstrated in paper [23].

However, the approach used by the authors of paper [23], which is based on the variational approximation of Anderson [5], did not allow them to obtain an exact analytic solution of the problem. Unlike previous papers, we obtained in this paper the exact analytic solutions for soliton Bloch waves and showed that the nonlinear Bloch theorem gives the transformation law for Schrödinger solitons in systems with periodically changing nonlinearity, dispersion, and amplification. Soliton nonlinear Bloch waves exist only for a certain relation between the main parameters of the system. This means that in real soliton data transmission and storage systems, as well as in soliton lasers, the dispersion and nonlinearity cannot be chosen arbitrarily but should be related by the conditions of the nonlinear Bloch theorem. In this case, along with the phase-modulated nonlinear Bloch waves with periodically varying amplitudes and duration (8), there also exist soliton Bloch waves without phase modulation with a constant duration [24].

The spatial analogue of the nonlinear Bloch waves is well known in the literature over more than 25 years. This is a periodic variation in the temporal and spatial parameters of ultrashort light pulses in a laser cavity upon the intracavity self-focusing of radiation. The possibility of using the intracavity self-focusing of radiation for compression of ultrashort light pulses and for increasing self-mode-locking (the so-called radiation contrast) was predicted in paper [25]. At present, passive mode locking due to intracavity self-focusing of radiation is widely used for the building of Kerr-lens mode locking lasers (see, for example, papers [26–29] and references therein). It is also well known that such an analogue of the space–time nonlinear Bloch waves exists only under certain conditions; otherwise, the opposite effect of stochastic radiation is developed [30, 31]. It is obvious that the further development of femtosecond laser systems of the Ti : Al_2O_3 type will be based on the concept of soliton Bloch waves.

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