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Talbot effect in Gaussian optical systems

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***Abstract*. It is shown that the diffraction reproduction of a periodically modulated wave field takes place when light propagates through Gaussian optical systems. Generally, such a reproduction is accompanied by image scaling. Equations are derived that relate the reproducton distance and scaling factor to the *ABCD* matrix elements of the optical system. The Talbot effect in a convergent (divergent) wave is considered.**

Keywords: *Talbot effect, diffraction self-reproduction, Talbot cavity, phase locking, laser gratings.*

The diffraction self-reproduction of a periodically modulated wave field (Talbot effect) has been known for nearly two hundred years now [1, 2]. The interest in this classical effect is steadily growing, along with the number of its practical applications [3]. Schemes of phase locking of laser gratings in Talbot cavities have been demonstrated experimentally, the Talbot filters are used in systems of optical image processing, etc. The essence of the effect is that a monochromatic wave field with a periodical space modulation in the plane normal to its propagation direction is self-reproduced due to diffraction at the distances $z_{\mathrm{t}}^{(m)} = z_{\mathrm{t}} m$ ($m = 1, 2, ...$) that are multiple of the Talbot distance z_{t}. If the modulation is one-dimensional with the period a, the Talbot distance is

$$z_{\mathrm{t}} = 2a^2/\lambda, \tag{1}$$

where λ is the wavelength. In fact, the Talbot effect is a Fresnel diffraction. The spatial phasing of harmonics of the angular spectrum, on which the effect is based, is analogous to mode locking upon formation of ultrashort laser pulses. Clear demonstration of such a phasing of harmonics of a discrete equidistant spectrum is a periodical coincidence of a deviation angle in the Chebotaev pendulum ensemble [4].

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In this paper, we generalise the Talbot effect to the wave fields propagating in Gaussian optical systems. Consider Gaussian optical systems [5], where the light-beam field transformation is described in the Fresnel diffraction approximation. We restrict ourselves to a one-dimensional case. The complex amplitude $u(x, z)$ of the field propagated through such an optical system is related to the initial distribution $u(x, z = 0)$ by the integral expression [5]

$$u(x, z) = \left(\frac{\mathrm{i}k}{2\pi B}\right)^{1/2} \int u(\xi, z = 0) \times \exp\left[\frac{\mathrm{i}k}{2B}\left(A\xi^2 + Dx^2 - 2x\xi\right)\right] \mathrm{d}\xi, \tag{2}$$

where A, B, D are the elements of the $ABCD$ matrix of the optical system and $k = 2\pi/\lambda$. The element C is not independent because the determinant of the $ABCD$ matrix should be equal to unity. We will use a periodically modulated field with the period a

$$u(x \pm a, 0) = u(x, 0). \tag{3}$$

as the initial field. Let us represent it as a superposition of spatial Fourier harmonics with spatial frequencies q_n:

$$u(x, z = 0) = \sum_{n=-\infty}^{\infty} c_n \exp(iq_n x), \quad q_n = \frac{2\pi}{a} n. \tag{4}$$

By substituting (4) into (2), we find the field at the output of the optical system:

$$u(x, z) = A^{-1/2} \exp\left[\frac{\mathrm{i}kx^2}{2AB}(AD - 1)\right] \times \sum_n c_n \exp\left(\frac{\mathrm{i}q_n x}{A} - \frac{\mathrm{i}q_n^2 B}{2kA}\right). \tag{5}$$

By requiring that the phase incursion of all the harmonics would be a multiple of 2π, we obtain the equation for determining the self-reproduction distance

$$\frac{B}{A} = \frac{2a^2}{\lambda}. \tag{6}$$

The image is scaled as the field is reproduced. One can see from (4) and (5) that the period a' of the image reproduced is determined by the element A of the $ABCD$ matrix:

$$a' = Aa. \tag{7}$$

The radius of curvature of the wave front in the reproduction plane is

$$R' = \frac{AB}{AD - 1}. \tag{8}$$

The optical system for the classical Talbot effect is a free space, and the $ABCD$ matrix has the form

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}. \tag{9}$$

Substituting the elements of (9) into (6)–(8), we obtain the classical formula (1); the image period coincides with the initial period, and the wave front is plane.

Consider an optical scheme in which a 'classical' reproduction of the field also occurs, i.e., $a' = a, R' = \infty$. The periodically modulated field is focused at the centre of a cylindrical mirror of radius r (Fig. 1a) by a cylindrical convergent lens (with the focal distance R). The field is reconstructed in the initial plane after reflection from the mirror, if the relation

$$R = \frac{r}{2} + \frac{r}{2}\left(1 + \frac{2}{r}\frac{2a^2}{\lambda}\right)^{1/2} \tag{10}$$

is valid. This can be verified by solving Eqns (5)–(7) for the $ABCD$ matrix elements:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 2R^2/r - 2R \\ 0 & 1 \end{pmatrix}. \tag{11}$$

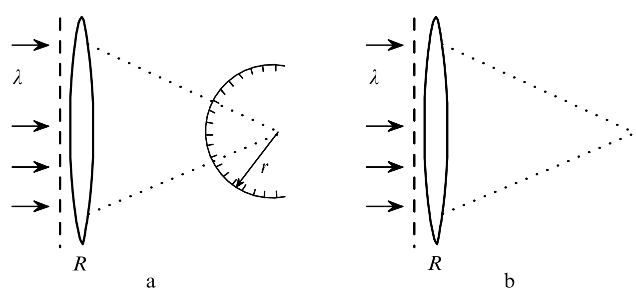

Figure 1. Talbot effect in Gaussian optical systems: lens–free space–spherical mirror telescopic system (a) and diffraction reproduction of a periodic field in a converging wave (b).

The scheme described above is used for telescope imaging upon phase locking in laser gratings based on the Talbot effect [6]. In paper [7], a scheme of the cylindrical Talbot cavity is proposed, whose parameters can be calculated using Eqn (10).

The developed approach permits one to generalise the diffraction self-reproduction of a periodic wave field to the case of convergent and divergent waves. Let the field of a plane wave be focused by a convergent lens with the focal distance R just after its propagation through a periodical grating (Fig. 1b). The $ABCD$ matrix of the lens–free space system is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - z/R & z \\ -1/R & 1 \end{pmatrix}. \tag{12}$$

Using the matrix elements (12), we find from Eqns (6)–(8) that the image of the initial periodic structure is reproduced at the distance

$$z^* = \frac{2a^2}{\lambda}\left(1 + \frac{1}{R}\frac{2a^2}{\lambda}\right)^{-1}. \tag{13}$$

In this case, the structure period decreases:

$$a^* = a\left(1 + \frac{1}{R}\frac{2a^2}{\lambda}\right)^{-1}. \tag{14}$$

The wave front in the $z = z^*$ plane has the radius of curvature

$$R^* = R\left(1 + \frac{1}{R}\frac{2a^2}{\lambda}\right)^{-1}. \tag{15}$$

The Talbot effect in a convergent wave is the consequence of the invariance of the parabolic diffraction equation $2iku_z = u_{xx}$ with respect to the 'lens' transformation [8]:

$$x' = \frac{x}{1 - z/R}, \quad z' = \frac{z}{1 - z/R},$$

$$u' = \frac{u}{(1 - z/R)^{1/2}} \exp\left(-\frac{ik}{2R}\,\frac{x^2}{1 - z/R}\right). \tag{16}$$

That means that the Talbot effect of higher orders and the so-called fractional effects [9] take place in the convergent wave with the accuracy to scaling factor for image period and the radius of curvature of the wave front. The mth-order effect can be observed in the convergent wave at the distance

$$z_m^* = \frac{2a^2 m}{\lambda}\left(1 + \frac{2a^2}{\lambda R}m\right)^{-1}. \tag{17}$$

The image period a_m^* and the radius R_m^* of wave-front curvature are decreased by a factor of $\left[(1 + 2a^2 m/(\lambda R)\right]$. Of course, in a real experiment, only a finite number of higher-order effects can be observed because the periodic structure is finite.

Expression (17) also describes the analogues of the 'fractional' Talbot effects [8]. For example, the structure is doubled (taking into account the scaling) in the plane $z = z_{1/4}^*$, and the image is shifted by half period $a_{1/2}^*$ at $z = z_{1/2}^*$. Note that Eqns (12)–(17) describe the reproduction of the periodic field in the divergent wave when R is negative. In this case, the planes of the reconstructed image lie at larger distances than in the case of a plane wave, while the image period increases.

Therefore, the Talbot effect is observed in Gaussian optical systems described by $ABCD$ matrices. The distance of a diffraction reproduction of a periodically modulated wave field is determined by the matrix elements A and B of the optical system. The element A is also a scaling factor, which describes an increase or a decrease in the image period. The Talbot effect exists both in the convergent and divergent waves. The 'fractional' Talbot effects and higher-order effects are also observed taking into account the scaling.

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