

LASER APPLICATIONS AND OTHER TOPICS IN QUANTUM ELECTRONICS

PACS numbers: 43.35.Sx; 42.79.Gn
DOI: 10.1070/QE2001v031n12ABEH002109

Interaction of whispering-gallery electromagnetic waves with acoustic waves in tapered quartz rods

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Abstract. **The propagation of whispering-gallery waves in a dynamic cavity formed by a tapered quartz rod and the plane interface between two regions of the rod with different refractive indices moving along the axis of the rod is analysed. It is shown that the limiting frequency shift of light in such a cavity is determined by its Q factor and the attainable refractive index discontinuity. The possibility of using acoustic waves for obtaining a dynamic cavity is considered.**

Keywords: *acousto-optics, whispering modes, frequency conversion.*

1. Introduction

In practice, it is often necessary to increase strongly the frequency shift of an optical signal. The simplest way to do this is to use the Raman – Nath acousto-optical diffraction in which the rays diffracted to higher orders are displaced by the frequency kf, k being the diffraction order and f the frequency of sound. However, the efficiency of this method of increasing the frequency shift is quite low. A higher efficiency can be attained in the Bragg diffraction regime. There are several ways of solving this problem [1 – 3].

For example, the fivefold increase in the frequency shift in the Bragg diffraction regime using a single acousto-optical (AO) cell was reported in paper [3]. According to the esti-mates presented in paper [4], the frequency shift in the AO cell proposed in this work may exceed $10^3 f$. A tapered quartz rod along whose axis an acoustic wave and a whispering-gallery optical wave propagate is an example of such a cell.

Because the construction of such a cell requires the understanding of all finer details of the AO interaction, this work is devoted to a detailed description of this process assuming that the frequency shift is caused by the Doppler effect and the total internal reflection of light from a moving discontinuity of the refractive index.

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Received 8 June 2001
Kvantovaya Elektronika **31** (12) 1089 – 1094 (2001)
Translated by Ram Wadhwa

2. Propagation of whispering-gallery waves in a tapered rod

Whispering-gallery waves in a dielectric rod are analogous to the waveguide modes in a dielectric waveguide [5, 6] and are characterised, like the waveguide modes, by the effective refractive index n^* and the specific transverse distribution of the electromagnetic field. The effective refractive index n^* of the mode is expressed in terms of the angle of incidence $\bar{\theta}$ of the light beam from inside on the cylindrical surface of the rod (Fig. 1):

$$n^* = n \sin\left(\frac{\pi}{2} - \frac{\pi}{N}\right) = n \cos\frac{\pi}{N}, \tag{1}$$

where N is the number of reflections experienced by the light beam during a round trip in the rod, and n is the refractive index of the material of the rod. Fig. 1b shows the dependence of n^* on the number N. For a large number of reflections, this dependence can be approximated quite well by the expression

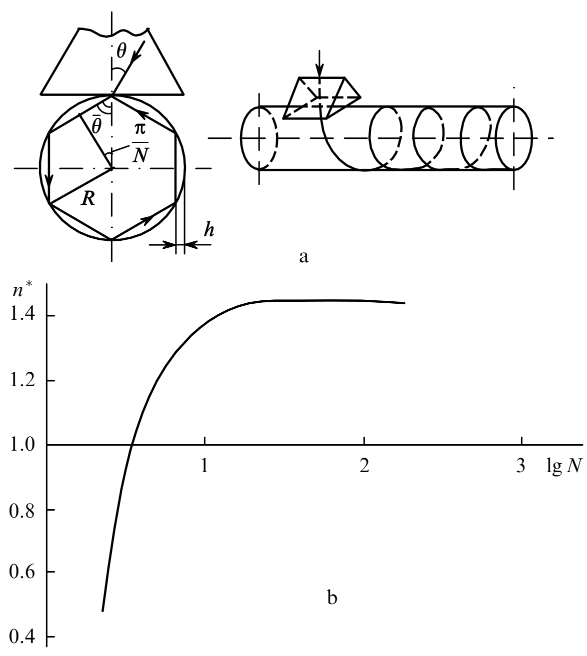

Figure 1. Trajectory of a light beam in the form of a whispering-gallery wave in a cylindrical quartz rod (a); and the dependence of n^* for a whispering-gallery wave on the number of light beam reflections from the surface of the rod (b).

$$n^* = n\left(1 - \frac{\pi^2}{2N^2}\right). \quad (2)$$

The parameter h is connected with N through the relation

$$h = R\left(1 - \cos\frac{\pi}{N}\right) \simeq \frac{R\pi^2}{2N^2}, \quad (3)$$

where R is the radius of the rod at the point where the light beam enters it.

Whispering-gallery waves can be excited in a rod by using a prism whose refractive index is higher than the refractive index n of the material of the rod. When a light wave is introduced into a cylindrical quartz rod at a certain angle φ to the generatrix of the cylinder, a whispering-gallery wave propagates in the rod along a helical trajectory with a constant pitch of the spiral. Light also propagates along a helical trajectory in a tapered rod, with the pitch of the spiral decreasing with each turn until the light beam reaches the point of inversion (turning point) and reverses the direction of its propagation (Fig. 2). The turning point is separated from the point of entry of light into the rod by a distance

$$H = R\,\frac{1 - \sin\varphi}{\tan\gamma}, \quad (4)$$

where 2γ is the apex angle of the cone. Expression (4) was obtained for whispering-gallery waves with n^* close to the limiting value, i.e., for $N > 10^2$.

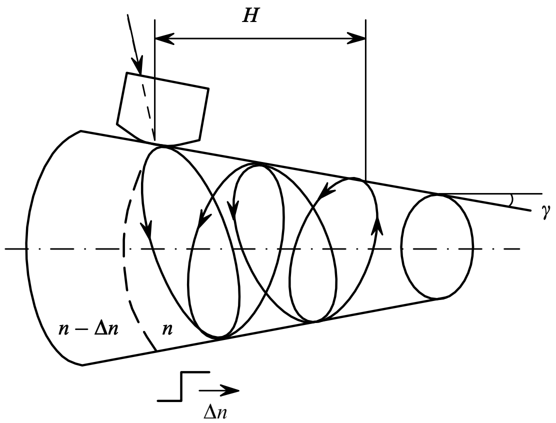

Figure 2. Excitation of the 'multi-turn' mode in a conical cavity.

Note here that we observed the helical trajectories of a whispering-gallery wave in experiments with cylindrical and tapered quartz rods of diameter between 10 and 1.5 mm, and the cone apex angle 2γ varying between 0 and 2°. The whispering-gallery waves were excited by a He–Ne laser ($\lambda = 0.63$ μm). For an appropriate choice of the angle φ, the backward propagation of the beam was also reliably observed in tapered rods.

Our experiments confirm the validity of the geometrical optics approach (approximation) for describing the process under study. At the turning point, the whispering-gallery wave moves along the normal to the generatrix of the cone. The backward wave intersects the plane perpendicular to the cone axis and passing through the point at which the light beam enters the rod. If this plane is the boundary between two regions of the rod with different refractive indices and the beam is in the region with the higher refractive index, it becomes localised under the condition that the angle φ is larger than the critical angle associated with the refractive index discontinuity Δn at the interface. In fact, a cavity is formed for the whispering-gallery wave.

The type of the cavity mode is determined by the number W of turns of the spiral, which a light beam must cross during a single round trip in the cavity, as well as by the number m of reflections experienced by the light beam from the plane interface (discontinuity Δn). For example, Fig. 2 shows the trajectory of a light beam representing a mode with four turns [$W = (\pi - 2\varphi)/(2\pi\sin\gamma)$], while Fig. 3 shows a mode with a single turn, but with six reflections from the plane interface between media with different refractive indices. The minimum possible angle of incidence φ of a light beam for the cavity mode is defined by the discontinuity Δn:

$$\sin\varphi = 1 - \frac{\Delta n}{n}. \quad (5)$$

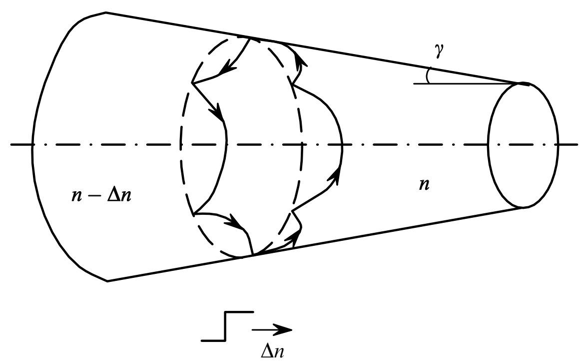

Figure 3. Trajectory of a beam representing a 'single turn' mode in a conical cavity.

This relation allows us to present H in terms of Δn:

$$H = \frac{R\Delta n}{n\tan\gamma}. \quad (6)$$

This expression can be used to estimate H from the parameters of the AO cell. For example, for $\Delta n = 10^{-4}$, $\gamma = 10^{-2}$, $n = 1.5$, and $R = 10^3$ μm, we have $H = 6.6$ μm. If the parameters of the AO cell are such that the relation $\gamma \leqslant \pi^{-1}(2\Delta n/n)^{1/2}$ is satisfied, the cavity mode is a 'single-turn' mode. Otherwise, a multiple turn trajectory of the beam corresponds to the mode. In the case of a 'single-turn' mode, the number m of reflections of the beam from the plane boundary of the cavity can be estimated from the equation

$$m = \frac{\pi\gamma}{(2\Delta n/n)^{1/2}}. \quad (7)$$

An important factor in the above analysis of conical cavity modes is the ratio of the refractive index discontinuity required for confining the 'multiple turn' mode in the cavity, to the discontinuity required for confining the 'single turn' mode:

$$\frac{\Delta n_w}{\Delta n_m} = W^2 m^2, \tag{8}$$

from which it follows that Δn_w is much larger than Δn_m. For example, the ratio $\Delta n_w/\Delta n_m = 576$ for the modes presented in Figs 2 and 3 for identical γ and n.

3. Dynamic conical cavity

Let us now assume that the interface between the regions of a tapered rod with different n moves with a velocity υ into the cone, i.e., towards the turning point of the light beam. Consider the behaviour of a whispering-gallery wave excited in such a cavity. First of all, the frequency ν of light will increase according to the Doppler effect when light is reflected from the moving interface:

$$\nu = \nu_0\left(1 + \frac{2\upsilon}{c}\cos\varphi\right), \tag{9}$$

where c is the velocity of light in the quartz rod.

Let us estimate the frequency shift of the whispering-gallery wave during its lifetime in a dynamic cavity. Let us assume that this time is $t = 1$ μs [7], while the velocity $\upsilon = 6 \times 10^5$ cm s^{-1}.

We will first estimate the frequency shift for a ‘single turn’ mode during its motion into the cone due to reflection of light from the moving plane interface between the media. The angle of incidence of light on this interface is $\varphi = \pi/2 - \pi\gamma/m$, where m is the number of reflections from the plane boundary during a round trip in the cavity. After each reflection, the relative frequency shift is

$$\frac{\Delta\nu}{\nu} = \frac{2\upsilon}{c}\cos\varphi = \frac{2\upsilon}{c}\frac{\pi\gamma}{m}, \tag{10}$$

while for one circumvention (one turn) of light, we obtain $\Delta\nu/\nu = 2\upsilon\pi\gamma/c$. The number of such turns in time t is $\bar{W} = tc/(2\pi R)$.

Therefore, the total shift is

$$\Delta\nu = \nu\frac{\upsilon t\gamma}{R}, \tag{11}$$

where $2R = R_0 + R_t$, R_0 and R_t being respectively the radii of the rod at the points where the beam enters and leaves it. For the case mentioned above ($R = 0.1$ cm, $\gamma = 10^{-2}$, $t = 10^{-6}$ s, and $\upsilon = 6 \times 10^5$ cm s^{-1}), this estimate gives $\Delta\nu/\nu = 6 \times 10^{-2}$.

Note that an explicit dependence of the frequency shift on the angle γ can be obtained in this case. It follows from this dependence that for a fixed lifetime t of photons in the cavity, an increase in the frequency shift of whispering-gallery waves can be attained by increasing the cone apex angle 2γ of the rod. One should remember, however, that such a variation in the angle γ leads to a corresponding increase ($\sim \gamma^2$) in the value of Δn.

During the analysis of the frequency shift of the whispering-gallery waves in a dynamic cavity, the possibility of the steady-state propagation of light beyond the turning point for the light beam was not considered. How is such a process possible? In the first place, it is possible because light is reflected from a moving mirror in a different manner than from a stationary mirror. For a stationary mirror, the angle of reflection of light is equal to the angle of its incidence on the mirror. For a moving mirror, these angles are not equal: the angle of reflection of light from the mirror moving towards the beam is smaller than the angle of incidence of light on this mirror [8]. Fig. 4a shows the pattern of reflection and the angles corresponding to this process. The angles $\bar{\varphi}_{i,r}$ are defined in the reference frame of the mirror and are connected with each other through the equality $\bar{\varphi}_i = \bar{\varphi}_r$. The angles $\varphi_{i,r}$ are defined in the reference frame of the observer and are connected with $\bar{\varphi}_{i,r}$ through the following relations:

$$\sin\bar{\varphi}_i = \frac{\alpha\sin\varphi_i}{1 + (\upsilon/c)\cos\varphi_i}, \qquad \sin\bar{\varphi}_r = \frac{\alpha\sin\varphi_r}{1 - (\upsilon/c)\cos\varphi_r},$$
$$\alpha = \left(1 - \frac{\upsilon^2}{c^2}\right)^{1/2}. \tag{12}$$

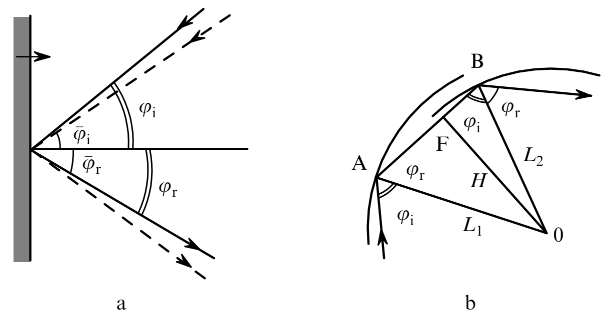

Figure 4. Diagram of reflection of light from (a) a moving mirror and (b) a moving discontinuity Δn.

Consider how these relations can be used to describe the propagation of a light beam beyond the turning point.

Fig. 4b shows the diagram of two successive reflections of a light beam in a dynamic conical cavity. The arcs in Fig. 4b indicate the position of the discontinuity Δn at two different instants of time t_1 and t_2 ($t_1 - t_2 = \tau$). The broken line shows the beam trajectory. The path of the beam from one reflection to the next is given by

$$\mathrm{AF} + \mathrm{FB} = \frac{H}{\tan\varphi_r} + \frac{H}{\tan\varphi_i} = c\tau\,. \tag{13}$$

The path length of the discontinuity Δn (of the moving mirror of the cavity) is

$$L_1 - L_2 = \frac{H}{\sin\varphi_r} - \frac{H}{\sin\varphi_i} = \upsilon\tau. \tag{14}$$

Expressions (13) and (14) give the relation between the angles φ_i and φ_r:

$$\frac{\upsilon}{c} = \frac{\sin\varphi_i - \sin\varphi_r}{\sin(\varphi_i + \varphi_r)}. \tag{15}$$

Obviously, the relation obtained here must conform to the laws of reflection of light. Indeed, it can be easily verified using expression (12) that relation (15) describing the process of penetration of a light beam into a cone (beyond the turning point) becomes an identity. This means that the behaviour of the light beam beyond the turning point can be described as follows: for a uniform motion of the discontinuity Δn, the light beam propagates in a dynamic conical cavity along a helical trajectory into the cone, the pitch of the spiral being constant and determined by the velocity υ of motion of the discontinuity Δn. An illustrative representation of the process described here is the motion of a surfer over the sea waves – slow towards the beach and fast along the crest of the wave.

In conclusion of this section, note that a mode in a conical cavity occupies a space of volume

$$V = 2\pi \frac{R^2 \bar{h}}{\tan\gamma} \frac{\Delta n}{n}, \tag{16}$$

where $\bar{h}$ is the transverse size of the mode (along the radius). If t is the time of AO interaction, the change in the mode volume during this time is

$$V_0 - V_t = \Delta V = 2V_0 \frac{\upsilon t \tan\gamma}{R} \simeq 2V_0 \frac{\upsilon t \gamma}{R}, \tag{17}$$

which gives

$$\Delta \nu = \nu \frac{\Delta V}{2V_0} = \nu \frac{\Delta S}{2S_0}. \tag{18}$$

Here $S = V/\bar{h}$ is the area of the conical surface confining the cavity mode. The form of expression (18) suggests that it also remains valid for the 'multiple turn' mode of the cavity.

4. Realisation of a dynamic cavity

The possibility of generating a powerful shock wave inside cylindrical rods made of various metals was demonstrated earlier in paper [9]. The shock wave is generated upon irradiation of the end of the rod by 10 – 20-ns laser pulses at a wavelength which is absorbed strongly in the material of the rod. A rapid surface heating of the rod generates a compression – dilatation wave propagating along the rod at the sound velocity. The motion of the wave front is accompanied by a refractive index discontinuity determined by the laser pulse energy and, in the limiting case, by the strength of the material of the rod.

Such a refractive index discontinuity can be created easily in a tapered quartz rod, and it is quite possible to realise a dynamic cavity in it. Note here that in actual practice the duration of the discontinuity Δn is not infinitely small, hence, it is very important to find out its effect on the total internal reflection of light at this discontinuity. We assume that the refractive index in the cone varies linearly along its axis. This dependence can be described approximately by two equal refractive index discontinuities. Let d be the thickness of the layer with the refractive index $n - \Delta n/2$. Consider now Fig. 5 showing the development of the surface of a cone into a plane. The trajectory of a whispering-gallery wave is shown in this development with a broken line with kinks at the boundaries of the layer of thickness d. One can easily see that the angle ψ of reflection of the wave from the outer boundary of this layer depends on its thickness d. For a sufficiently large thickness d, the angle ψ may become smaller than the total internal reflection angle $\psi_{\text{cr}} = \arcsin[(n - \Delta n)/(n - \Delta n/2)]$. Our estimates show that the limiting thickness of the intermediate layer is given by

$$d_{\text{lim}} \simeq \frac{4R\Delta n}{n \tan\gamma} = 4H. \tag{19}$$

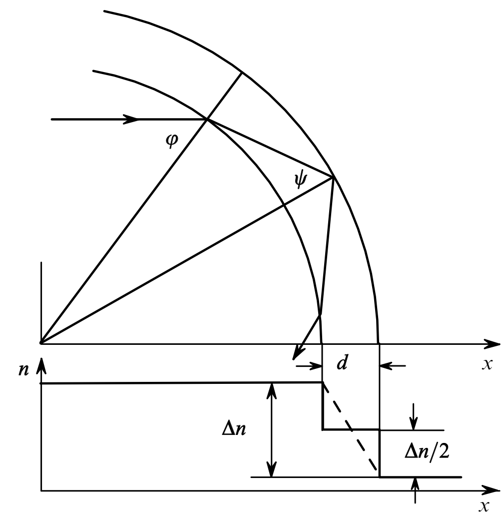

Figure 5. Total internal reflection of light in a tapered rod with an intermediate insulating layer.

This means that the length of the discontinuity Δn should not differ significantly from the mode size in a conical cavity with an instantaneous discontinuity of the refractive index. For $\Delta n = 10^{-4}$, $\gamma = 10^{-2}$, $n = 1.5$, and $R = 10^3$ μm, the thickness d is found to be only 27 μm. The duration of this discontinuity is $\Delta t = d/\upsilon$, i.e., has the value 4.5 ns in the example considered.

Because of the use of high-power laser radiation for obtaining the discontinuity Δn, a finite-sized region with a higher refractive index moves inside the cone. We have estimated above the admissible thickness d of the intermediate part of this region. However, the existence of the mode in the dynamic cavity is ensured not only by the reflection of light from the intermediate (or discontinuity) part of the region with a higher refractive index, but also by the reflection of light from the conical surface of the rod, i.e., by the passage of light through the turning points.

If the turning points of the whispering-gallery wave lie outside the higher refractive index region, light will be reflected at the second intermediate region, i.e., in the moving region of decrease of the refractive index. In this case, the whispering-gallery waves will not shift. This means that the length of the moving 'steady-state' part of the region with the higher refractive index should exceed the size of the mode that has already been defined [see formula (3)]. The following estimate seems to be quite reasonable for the parameter D, i.e., the length of the 'steady-state' part of the discontinuity region Δn:

$$D \geqslant 2H. \tag{20}$$

If this relation is satisfied, the length of the region of decrease in the refractive index will have no effect whatsoever on the frequency shift of the whispering-gallery wave.

In connection with estimates presented above, the question arises of whether the frequency shift of the whispering-gallery waves can be obtained by using acoustic waves that cause a periodic variation in the density of the medium and, hence, its refractive index. The above analysis of the required spatial and temporal dependences of the discontinuity Δn can be used to estimate the acoustic wavelength required for obtaining the frequency shift of whispering-gallery waves. This estimate is expressed by the relation

$$\Lambda \simeq 10H. \tag{21}$$

It follows from (21) that in our case ($\Delta n = 10^{-4}$, $\gamma = 10^{-2}$, $n = 1.5$, $R = 10^3$ μm), the acoustic frequency is $f = 90$ MHz.

As an acoustic wave propagates along a cylindrical quartz rod, a set of layers with a higher refractive index is formed inside the rod, alternating with an identical set of layers with a lower refractive index. The thickness of these layers is equal to half the acoustic wavelength. The cylindrical layer with a higher refractive index for whispering-gallery waves is a channel waveguide, while the set of such layers constitutes a family of channel waveguides.

If a whispering-gallery wave is excited in one of the waveguide channels, the light propagating along this waveguide will spread along all other channel waveguides. Such a spreading of light is caused by the tunnel coupling of the waveguides with one another. The intensity of light in the initially excited waveguide depends on the coordinate as:

$$I(z) = J_0^2(2Kz)\mathrm{e}^{-\alpha z}, \tag{22}$$

where $J_0(z)$ is a zeroth-order Bessel function; K is the coupling coefficient between two adjacent waveguides; and α is the light attenuation coefficient in the waveguide. Assuming that $\alpha = 0$, we find that the intensity of light in the waveguide decreases by a factor of 20 for $2Kz = 13.2$. This is equivalent to a decrease in the lifetime of light in the cavity, which determines the frequency shift of the whispering-gallery waves (see above).

Let us estimate the quantity K in our case ($\Delta n = 10^{-4}$, $\gamma = 10^{-2}$, $n = 1.5$, $R = 10^3$ μm, $f = 90$ MHz, and $\lambda = 1$ μm). Because the field of a whispering-gallery wave is localised at the surface of the quartz rod (in a layer of thickness $h = \lambda/n$), a good approximation of the system of channel waveguides formed by the acoustic wave is a system of rectangular channel waveguides in a quartz layer of thickness h in contact with air. Let us assume that the width of channel waveguides and the interval between them are equal to $\Lambda/2$. In this case, we obtain according to [10], $K = 0.05$ cm^{-1}.

If the signal at the shifted frequency with the intensity lowered by a factor of 20 can be detected experimentally, the attainable frequency shift $\Delta\nu/\nu$ will be equal to 4×10^{-4}. This frequency shift is more than 1000 times larger than the shift caused by the single Bragg diffraction of light from an acoustic wave. Note here that the coupling coefficient decreases significantly with increasing Δn. For $\Delta n = 10^{-3}$, for example, the coefficient $K = 1.6 \times 10^{-5}$ cm^{-1}, and this corresponds to a threefold decrease in the loss of light in the cavity as compared to the initial losses (when $t = 1$ μs).

In our first experiments on the measurement of the discontinuity Δn produced by an acoustic wave, we used cylindrical quartz rods of diameter 2 mm and sound sources ($f = 40$ MHz) based on $LiNbO_3$ plates operating in the pulse mode. It was shown that for an electric power of 60 W, a change in the refractive index at the level $\Delta n \sim 10^{-4}$ was detected in the rod at a distance of 40 mm from the sound source.

To demonstrate the interaction between the whispering-gallery waves and acoustic waves, we carried out an experiment whose schematic diagram is shown in Fig. 6. A cylindrical quartz rod of diameter 10 mm and length 60 mm was used in the experiment. An annular piezoelectric sound source, which was fixed to one of the ends of this rod, generated an acoustic wave adjoining the side surface of the rod. The opposite end of the rod was skewed for eliminating the reflected acoustic waves. The central acoustic frequency was 40 MHz, while the bandwidth of the piezoelectric transducer was up to 20 MHz. A whispering-gallery wave was excited with the help of a prism ($n = 1.75$) in such a way that the light beam was directed towards the acoustic wave front at the Bragg angle θ_{B}. Light diffracted from the acoustic wave was extracted with the help of the same prism. The frequency dependence of the intensity of the diffracted light was measured and a sharp resonance dependence of the efficiency ξ of the Bragg diffraction was observed. This efficiency at the peak of the resonance curve was estimated as $\xi = 50\,\%$ for an electric power of 1 W at the transducer.

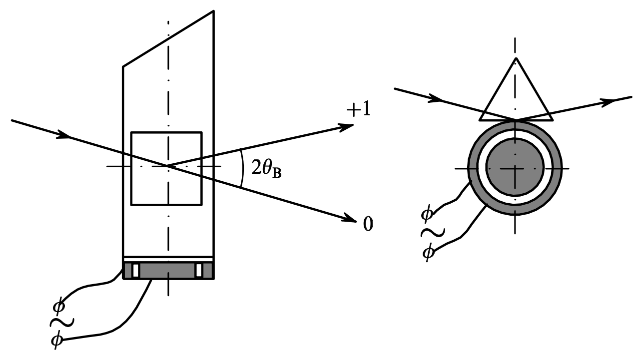

Figure 6. Experimental setup for the Bragg diffraction of a whispering-gallery wave in a cylindrical rod.

Note that a quite high efficiency of the Bragg diffraction of light in our experiment is determined first of all by the concentration of the acoustic energy near the surface of the rod. An extremely high concentration of energy near the surface and, hence, an extremely high efficiency of the AO interaction can be expected by producing the frequency shift of whispering-gallery waves with the help of surface acoustic waves (SAW) whose penetration depth into the substrate is close to the wavelength of sound in it.

For the acoustic frequency $f = 90$ MHz, the wavelength Λ in quartz rods is 67 μm. Because it is rather difficult to create a piezoelectric transducer for SAW on the conical surface of a quartz rod another approach is preferable for increasing the efficiency of the AO interaction. This approach is based on the use of quartz rods of a small diameter ($\sim 2\Lambda$). In this case, the efficiency of interaction using SAW and bulk acoustic waves is virtually identical. The problem of exciting bulk waves of quite high power in a quartz fibre of diameter ~ 300 μm can be solved by using different types of concentrators. For example, focusing of a plane acoustic beam of diameter 10 mm at the end of a tapered optical fibre ($\varnothing\,0.3$ mm) not only increases the efficiency of AO interaction significantly (by a factor of $\sim 10^3$), but also the range of frequency shift due to an increase in the discontinuity Δn.

Thus, the above analysis of the propagation and confinement of light in a dynamic cavity formed by a moving discontinuity of the refractive index in a tapered quartz rod, and obtaining of the Bragg diffraction of light in a quartz rod show that a considerable frequency shift of the whispering-gallery waves is possible and is determined by the lifetime of photons in this cavity and the maximum possible refractive index discontinuity.

Acknowledgements. This work was supported by the International Science and Technology Centre (Grant No. 1043). One of the authors (VAS) thanks I K Krasyuk and B A Usievich for useful discussions and their help in this research.

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