

CONTROL OF LASER RADIATION PARAMETERS

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Stable two-frequency generation in a Nd^{3+} : YAG laser with a phase-anisotropic cavity upon intracavity SHG in the laser frequency control mode

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***Abstract*.** **The operating of a two-frequency Nd^{3+} : YAG laser with a phase-anisotropic cavity upon intracavity SHG (ICSHG) is considered theoretically and experimentally. It is shown that for an appropriate choice of the cavity configuration, an intensity-stabilised two-frequency lasing mode may be obtained at the first (1.064 μm) and second (0.532 μm) harmonics, with the mode interval changing continuously over a broad range.**

Keywords: *two-frequency laser, stable two-frequency lasing, mode interval.*

1. Introduction

Intensity-stabilised two-frequency lasers with controlled lasing frequencies are being used at present for carrying out a number of fundamental physical experiments and for solving some applied problems [1 – 3]. Semiconductor-pumped Nd^{3+} : YAG lasers with intracavity SHG (ICSHG) can occupy a special place among such lasers. In particular, such lasers can be used for developing two-wave laser diagnostic systems, laser communication systems, laser interferometers, and optical frequency standards.

In the general case, when a nonlinear crystal (frequency doubler), which is made of a birefringent material and is therefore a phase-anisotropic element, is inserted in a cavity, the spectrum of the initially phase-isotropic cavity is split into two spectra of modes with mutually orthogonal polarisations. For an appropriate choice of the optical length of the cavity, when the gain dominates over the loss, the first-harmonic generation at $\lambda_1 = 1.064$ μm can be obtained at two adjacent modes with frequencies ω_1 and ω_2 with mutually orthogonal polarisations. In this case, the second harmonic generation at $\lambda_2 = 0.532$ μm will occur at three components with frequencies $2\omega_1$, $2\omega_2$, and $\omega_1 + \omega_2$ with parallel polarisations.

According to the results of previous investigations [4 – 8], the presence of a component with the sum frequency $\omega_1 + \omega_2$ gives rise to irregular pulsations in the output laser beam at the first and second harmonics. This is due to uncorrelated intensity fluctuations of individual modes. In order to suppress fluctuations, some authors (see, for example, [5 – 7]) proposed to place a quarter-wave plate in the cavity whose optical axes are oriented at 45° relative to the optical axes of the frequency-doubler crystal. In this case, however, intensity-stabilised lasing at the two harmonics can be obtained for an arbitrary phase anisotropy of the frequency-doubler crystal only for fixed mode intervals: $c/4L_0$ at the first harmonic and $c/2L_0$ at the second harmonic (c is the velocity of light in vacuum and L_0 is the optical length of the cavity).

This work aims at the development and creation of a semiconductor-pumped intensity-stabilised two-wave Nd^{3+} : YAG laser with ICSHG and the frequency difference between the first and second harmonics varied over a broad range (almost, from zero to the maximum possible mode intervals $c/2L_0$ and c/L_0). The stability of such a lasing mode upon the variation of the cavity parameters is also estimated.

2. Basic concepts

To solve the above-formulated problem, we propose a configuration for the cavity whose optical scheme is shown in Fig. 1. Two mirrors (*1*) and (*5*) highly reflecting the first-harmonic radiation, form the cavity. An isotropic active element (*2*), a frequency-doubler crystal (*3*) with the phase incursion difference φ_2 between the ordinary and extraordinary rays, and a phase-anisotropic element (*4*) with the phase incursion difference φ_2 between the ordinary and extraordinary rays are placed between the mirrors along the optical axis. The construction of the cavity envisages the possibility of changing the phase incursion difference φ_2 and the mutual orientation of the optical axes of elements (*3*) and (*4*), which is specified by angle θ.

To find the beam intensities at the first and second harmonics and mode interval, we should consider the Jones polarisation matrix for a round trip over the cavity and find the eigenvectors and eigenvalues of this matrix [10].

For the cavity under study, the round-trip matrix in the region between the left mirror and the frequency-doubler crystal has the form

$$A = R\Phi_1 S(\theta)\Phi_2 R\Phi_2 S(\theta)\Phi_1, \quad (1)$$

where R is the matrix of reflection from the mirror; $S(\theta)$ is the matrix of rotation through angle θ; and Φ_1 and Φ_2 are the matrices of the frequency-doubler crystal and the phase-

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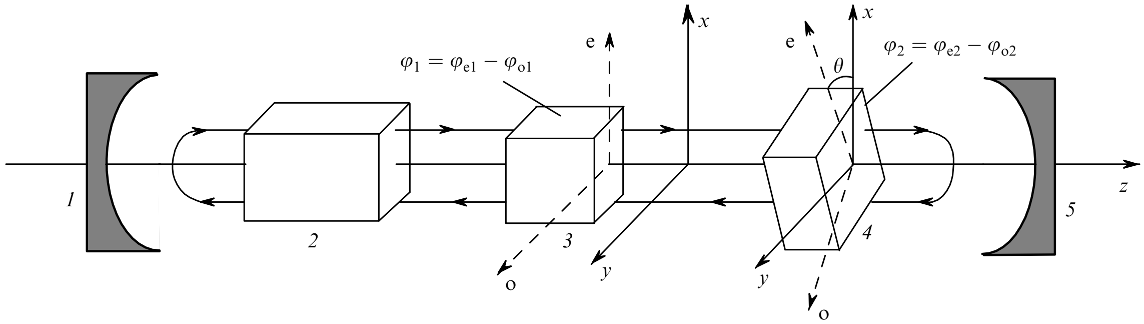

Figure 1. Optical diagram of a phase-anisotropic cavity: (*1*) highly reflecting mirror, (*2*) active element, (*3*) frequency doubler, (*4*) phase-anisotropic element, (*5*) output mirror.

anisotropic element with the phase difference φ_1 and φ_2, respectively. The specific form of the matrices appearing in Eqn (1) can be found, for example, in paper [9].

Having solved the problem of the eigenvectors and eigenvalues of the matrix A, we can find the vectors of electromagnetic fields of two orthogonally polarised modes $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$, the frequency difference between adjacent modes, and the second-harmonic radiation intensity.

In the general case, the mode spectrum for a phase-anisotropic cavity consists of equidistant pairs of modes with different polarisations with the mode interval ω_{12} and period $c/2L_0$. In this paper, we are interested only in the two-frequency lasing and, hence, we will consider the lasing parameters for two adjacent modes in the following.

According to [6, 10], the expressions for $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ can be written in the form

$$\boldsymbol{E}_1(\omega_1) = \frac{|E_1|}{N_1}\exp[\mathrm{i}(\omega_1 t + \alpha_1)]\begin{pmatrix} B_1 \\ C_1 \end{pmatrix},$$
$$\boldsymbol{E}_2(\omega_2) = \frac{|E_2|}{N_2}\exp[\mathrm{i}(\omega_2 t + \alpha_2)]\begin{pmatrix} B_2 \\ C_2 \end{pmatrix}, \quad (2)$$

where α_1 and α_2 are the initial phase shifts;

$$B_1 = \cos^2\theta\sin(\varphi_1+\varphi_2) + \sin^2\theta\sin(\varphi_1-\varphi_2)$$
$$+ \Big\{\big[\cos^2\theta\sin(\varphi_1+\varphi_2) + \sin^2\theta\sin(\varphi_1-\varphi_2)\big]^2$$
$$+ \sin^2 2\theta\sin^2\varphi_2\Big\}^{1/2}; \;\; C_1 = \sin 2\theta\sin\varphi_2;$$

$$B_2 = \frac{1}{\sin 2\theta\sin\varphi_2}\Big\{\cos^2\theta\sin(\varphi_1+\varphi_2) \quad (3)$$
$$+ \sin^2\theta\sin(\varphi_1-\varphi_2) - \Big\{\big[\cos^2\theta\sin(\varphi_1+\varphi_2)$$
$$+ \sin^2\theta\sin(\varphi_1-\varphi_2)\big]^2 + \sin^2 2\theta\sin^2\varphi_2\Big\}^{1/2}\Big\};$$

$$C_2 = 1; \;\; N_1 = \left(B_1^2 + C_1^2\right)^{1/2}; \;\; N_2 = \left(B_2^2 + C_2^2\right)^{1/2}.$$

According to [6], the total intensity of lasing at the second harmonic is

$$\Im = \frac{d_{\mathrm{eff}}^2}{4}\left[g\left(I_1^2 + I_2^2\right) + 4(1-g)I_1 I_2\right], \quad (4)$$

where I_1^2 and I_2^2 are the lasing intensities at the second harmonic at doubled frequencies $2\omega_1$ and $2\omega_2$; $I_1 I_2$ corresponds to the SHG intensity at the sum frequency $\omega_1+\omega_2$; d_{eff} is the effective coefficient of conversion into the second harmonic; and

$$g = 4\left(\frac{B_1 C_1}{N_1^2}\right)^2 \quad (5)$$

is the doubling coefficient.

The eigenvalues of matrix A can also be used to find the frequency difference between the modes:

$$\omega_{12} = \frac{c}{2\pi L_0}\arccos\left[\cos^2\theta\cos 2\varphi + \sin^2\theta\cos(2\delta\varphi)\right], \quad (6)$$

where $\varphi = (\varphi_1+\varphi_2)/2$; $\delta\varphi = (\varphi_1-\varphi_2)/2$.

3. SHG at doubled frequencies $2\omega_1$ and $2\omega_2$ only

It was shown in papers [4–8] that in the general case, the second harmonic emission spectrum of a laser operating in the ICSHG mode in a nonlinear crystal in which type II wave matching is realised with two orthogonally polarised modes generated at the fundamental harmonic, consists of three components. The intensities of the components at the doubled frequencies $2\omega_1$ and $2\omega_2$ and at the sum frequency $\omega_1+\omega_2$ are given by expression (4).

The investigations carried out in papers [5–8] showed that the sum-frequency component is responsible for the emergence of considerable intensity instabilities in an ICSHG laser. The authors of these papers explain this by the fact that, for $g \neq 1$, modes with different polarisations are coupled through the generation of the sum frequency in the nonlinear crystal; the changes in their intensity are anticorrelated, i.e., occur in antiphase, and the intensity of the sum-frequency component is $\sim (1-g)I_1 I_2$, which explains the dominating contribution of this component to large instabilities of the total second-harmonic intensity under this condition.

On the other hand, if the condition $g = 1$ is satisfied, the modes with different polarisations are not coupled with each other through the sum-frequency generation in the nonlinear crystal because the intensity of the sum-frequency component is zero and, hence, the main cause of significant instabilities in an ICSHG laser operating in the double-frequency mode vanishes.

A similar conclusion can also be drawn concerning the behaviour of the first-harmonic radiation intensity upon ICSHG. It was noted in [7] that the loss in each mode associated with the coupling of modes with different polarisations amounts to $\sim(1-g)I_1 I_2$, while the loss associated with the coupling of modes with identical polarisations is $\sim g I_{1,2}^2$. This means that as in the case of SHG, the main source of large intensity instabilities at the first harmonic for $g \neq 1$ is the existence of a strong coupling between modes with different polarisations though the sum-frequency generation.

In accordance with Eqn (5), the condition $g = 1$ corresponds to the relation $B_1 = \pm C_1$. An analysis of this relation taking into account expressions (3) for the coefficients B_1 and C_1 makes it possible to find the relation between the parameters θ, φ_1, and φ_2 for $g = 1$:

$$\varphi_2 = \operatorname{arccot}(-\cos 2\theta \cot \varphi_1), \tag{7}$$

where $\theta \neq 0,\ \pi/2$.

Thus, for a fixed phase incursion difference φ_1 in the frequency-doubler crystal, the condition $g = 1$ can be satisfied by varying the phase incursion φ_2 of the phase-anisotropic element and by choosing an appropriate angle θ in accordance with expression (7). It was noted above that, in this case, the second harmonic generation can be realised only at components with frequencies $2\omega_1$ and $2\omega_2$ for intensity-stabilised output radiation both at first and second harmonics.

Consider now the behaviour of the mode interval at the fundamental frequency upon a variation of the cavity parameters under the condition that relation $g = 1$ holds. Expression (6) for the mode interval at the fundamental frequency under condition (7) is a function of only one independent variable whose role can be played either by φ_2 or by θ. The corresponding expressions for ω_{12} have the form

$$\omega_{12}(\varphi_2) = \frac{c}{2\pi L_0} \arccos\left(\frac{\cos\varphi_2}{\cos\varphi_1}\right), \quad \text{for} \quad \varphi_1 \neq \frac{\pi}{2},$$

$$\omega_{12}(\theta) = \begin{cases} \dfrac{c}{2\pi L_0} \arccos\left\{\dfrac{\cos[\operatorname{arccot}(-\cos 2\theta \cot\varphi_1)]}{\cos\varphi_1}\right\}, & \text{for } \varphi_1 \neq \dfrac{\pi}{2}, \\ \dfrac{c}{2\pi L_0} \arccos(-\cos 2\theta), & \text{for } \varphi_1 = \varphi_2 = \dfrac{\pi}{2}. \end{cases} \tag{8}$$

For a given difference φ_1 in phase incursions in the frequency-doubler crystal, one of expressions (8) [either the dependence $\omega_{12}(\theta)$ or the dependence $\omega_{12}(\varphi_2)$] can be used upon stable lasing, when φ_2 and θ are interrelated.

When lasers with a controlled mode interval are used, it is important to have information on the behaviour of mode interval and the range of its variation upon a change in the cavity parameters. In this section, these questions are considered under the condition that the relation $g = 1$ holds. Fig. 2a shows the curves $\varphi_2(\theta)$ plotted by using expression (7), while Fig. 2b shows the normalised curves $\omega_{12}(\theta)$ plotted for three phase incursion differences φ_1 in the frequency-doubler crystal: $\pi/18$, $\pi/4$, $\pi/2$. One can see from Fig. 2 that the dependences $\varphi_2(\theta)$ and $\omega_{12}(\theta)$ are nonlinear in the general case.

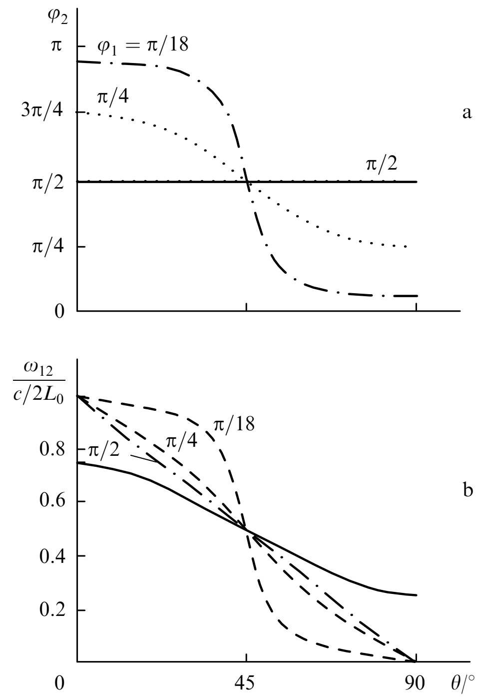

Figure 2. Dependences of (a) the phase incursion φ_2 and (b) mode interval ω_{12} on the angle θ upon stable lasing for different values of φ_1 and for the cavity configuration described in [5–8] (solid curve).

For small φ_1 ($\varphi_1 = \pi/18$ in Fig. 2b), the dependence $\omega_{12}(\theta)$ is essentially nonlinear and has a plateau for angles θ close to 0 and $\pi/2$. As φ_1 increases to $\varphi_1 = \pi/2$, the dependences $\omega_{12}(\theta)$ approach a linear dependence. In the vicinity of point $\theta \approx \pi/4$, the curvature of the curves $\omega_{12}(\theta)$ changes, and these dependences become close to linear for any value of φ_1. At point $\theta = \pi/4$, the mode interval is independent of the phase incursion difference φ_1:

$$\omega_{12}\left(\frac{\pi}{4}\right) = \frac{c}{2\pi L_0} \arccos\left\{\frac{\cos[\operatorname{arccot}(0)]}{\cos\varphi_1}\right\} = \frac{c}{4L_0}.$$

The situation when $\varphi_1 = \varphi_2 = \pi/2$ must be considered separately. In this case, according to Eqns (8), the dependence $\omega_{12}(\theta)$ takes the form

$$\omega_{12}(\theta) = \frac{c}{2\pi L_0} \arccos(-\cos 2\theta) = \frac{c}{2\pi L_0}(\pi - 2\theta), \quad \text{for} \quad 0 < \theta < \frac{\pi}{2},$$

$$\omega_{12}(\theta) = \frac{c}{2\pi L_0}(2\theta - \pi), \quad \text{for} \quad \frac{\pi}{2} < \theta < \pi,$$

i.e., for given parameters of the frequency-doubler crystal and the phase-anisotropic element, the dependence $\omega_{12}(\theta)$ is linear in the entire range of angles θ.

Consider now the question associated with the range of variation in the mode interval $D = \omega_{12}^{\max} - \omega_{12}^{\min}$ (where $\omega_{12}^{\max}$ and $\omega_{12}^{\min}$ are the maximum and minimum mode intervals) under stable lasing conditions. To derive the dependence $D(\theta, \varphi_2)$, we should consider the limits of functions (8) for $\theta \to 0$ and $\theta \to \pi/2$ at a fixed phase incursion difference φ_1 in the frequency-doubler crystal. An analysis of relations (8) proved that

$$\lim_{\theta \to 0} \omega_{12}(\theta) = \frac{c}{2L_0}, \quad \lim_{\theta \to \pi/2} \omega_{12}(\theta) = 0,$$

irrespective of φ_1.

Thus, for any phase incursion difference φ_1 in the frequency-doubler crystal upon stable lasing, the maximum possible ranges $D_1^{\max} = c/2L_0$ and $D_2^{\max} = c/2L_0$ of variation in the mode interval ω_{12} can be realised at the first and second harmonics, respectively.

For comparison of the results obtained for different configurations of a phase-anisotropic cavity, Fig. 2b shows the solid curve plotted for the cavity configuration proposed in papers [5–8] and $\varphi_1 = \pi/4$. One can see that the range of variation in the mode interval in this case is noticeably narrower that in the cavity configuration proposed by us here. In addition, as noted above, the intensity-stabilised lasing can be realised only for the fixed mode interval $c/4L_0$.

4. Estimate of the tuning accuracy for elements of a phase-anisotropic cavity upon stable two-frequency lasing

It was noted above that under the conditions of stable two-frequency lasing ($g = 1$), the parameters φ_1, φ_2 of a phase-anisotropic cavity and the angle θ characterising the mutual orientation of phase-anisotropic elements are connected through relation (6). The deviation of even one of these parameters from the values satisfying relation (6) destabilises the lasing.

In the general case, the coefficient g is a complex function of parameters φ_1, φ_2, θ; for this reason, several typical values of angles θ were taken for an analysis of the tuning accuracy for the elements of a phase-anisotropic cavity upon stable double-frequency lasing in order to find the required accuracy of setting φ_2.

Fig. 3 shows the dependences $g(\varphi_2)$ for different values of θ from the interval $0 < \theta < 90°$. The phase incursion differences φ_1 are chosen in the interval $0 < \varphi_1 \leqslant \pi/2$. The maxima of these two dependences correspond to the stable lasing case considered above. Let us compare the behaviour of the functions $g(\varphi_2)$ for different angles θ, but for identical values of φ_1. One can see that for small angles θ, the dependences $g(\varphi_2)$ are sharper near the maxima, which indicates that it is difficult to tune a phase anisotropic cavity and to maintain stable lasing. As the angle θ between the optical axes of phase-anisotropic elements increases, the conditions for exact tuning of the phase-anisotropic cavity over the parameter φ_2 become less stringent. The least stringent requirements for maintaining a stable two-frequency lasing over the parameter φ_2 are imposed for an angle $\theta \approx 45°$. As the angle θ increases further (from 45 to 90°), the behaviour of parameter g becomes mirror symmetric (relative to the straight line $\varphi_2 = \pi/2$) to that considered above on the half-open interval $0 < \theta \leqslant 45°$.

5. Experimental investigation of a two-frequency ICSHG Nd^{3+} : YAG laser

We developed and constructed a combined Nd^{3+} : YAG laser emitting simultaneously at wavelengths $\lambda_1 = 1.064$ μm and $\lambda_2 = 0.532$ μm. A Nd^{3+} : YAG crystal of diameter 5 mm and length 5 mm was used as the active element. Longitudinal pumping was carried out by two 500-mW cw semiconductor lasers with mutually orthogonal polarisations. Radiation beams emitted by the pump lasers were aligned and focused by a telescopic system at the centre of the active element. A 20-mm-long laser cavity was formed by two interference mirrors; the input mirror was mounted on a piezoelectric corrector.

A frequency doubler and a phase anisotropic element were placed between the active element and the output mirror along the optical axis of the cavity. A KTP crystal of length 7 mm was chosen for frequency doubling. The frequency-doubler crystal was cut in such a way as to ensure type II phase matching during SHG. It was mounted into a special frame permitting a variation of the mutual orientation of the optical axes (angle θ) of the frequency-doubler crystal and the phase-anisotropic element. Preliminary experiments with the laser showed that the phase incursion difference introduced by the frequency-doubler crystal for ordinary and extraordinary rays was $\varphi_1 = 0.42\pi$.

A quartz crystal wedge was used as the phase-anisotropic element. Such an element made it possible to change the phase shift φ_2 smoothly by linearly displacing the phase-anisotropic wedge at right angles to the cavity axis. The position of the axial modes on the gain line profile was varied by applying a controlled dc voltage across the piezoelectric corrector on which the input mirror was mounted. The optical length L_0 of the cavity taking into account the refractive indices of the active element, fre-

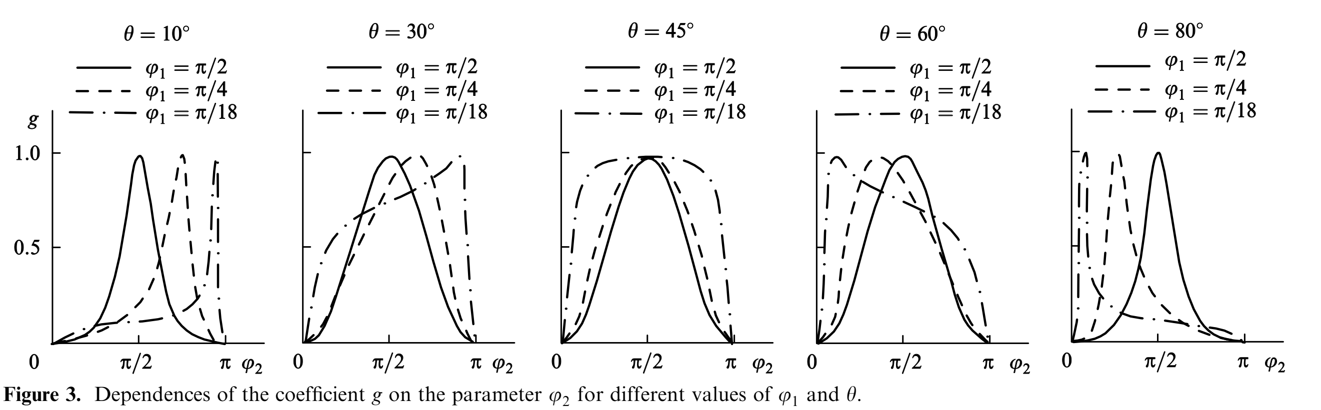

Figure 3. Dependences of the coefficient g on the parameter φ_2 for different values of φ_1 and θ.

quency-doubler crystal, and the phase-anisotropic wedge was ~ 30 mm. This length corresponds to the frequency difference ~ 5 GHz between two adjacent unsplit axial modes.

The predominance of the gain over the loss could be varied smoothly by controlling the current of the semiconductor pump lasers. It was chosen so that the active part of the gain line profile (the frequency range in which the gain exceeds the loss) approximately corresponded to the frequency difference between the unsplit axial modes of the Nd^{3+}:YAG laser.

In the experiments, the mode interval and the emission spectra of the first and second harmonics were controlled continuously. The mode intervals in the range $0.1 \leqslant \omega_{12} \leqslant 1.5$ GHz were recorded by a spectrum analyser, while mode intervals $\omega_{12} > 1.5$ GHz were measured by using the calibrated wedge method [2].

The emission spectra of the first and second harmonics were monitored with the help of scanning interferometers. The interferometer used for studying the spectrum of the first harmonic had a free spectral range (FSR) of 5 GHz and a resolution of ~ 100 MHz. The interferometer used for studying the spectrum of the second harmonic had a FSR of 10 GHz and a resolution of ~ 100 MHz.

6. Experimental results

We studied the time dependences of the first and second harmonic intensities for various mutual orientations (determined by the angle θ) of the frequency-doubler crystal and the phase-anisotropic element and for different phase anisotropies φ_2 of the wedge; the behaviour of the mode interval ω_{12} and the range D of its variation upon stable lasing were also investigated.

The experiments showed that, in the general case, the laser generated two linear orthogonally polarised axial modes at the first harmonic frequencies ω_1 and ω_2. This radiation was partially converted into the second harmonic radiation with spectral components at frequencies $2\omega_1$, $2\omega_2$, and $\omega_1 + \omega_2$; all components of the second harmonic radiation had parallel polarisations. To achieve stable lasing, we specified the angle θ, and then the angle φ_2 was varied by linear displacement of the wedge until the condition $g = 1$ was satisfied.

We found that stable lasing both at the first (see Fig. 4c) and at the second harmonic (Fig. 4f) could be obtained by an appropriate choice of parameters θ and φ_2. The unstable lasing (Figs 4a, d) was first replaced by lasing with regular instabilities (Figs 4b, e) and then became stable upon a further displacement of the wedge. In Fig. 4, the angle $\theta = 35^\circ$, and the range of variation of the mode interval ω_{12} varied from 760 MHz to 1100 MHz upon a transition from the regime shown in Fig. 4a to that in Fig. 4c.

Our experiments also confirmed the possibility of controlling the mode interval at the first and second harmonics upon stable lasing in the ranges from ~ 0 to $\sim c/2L_0$ and from ~ 0 to $\sim c/L_0$, respectively.

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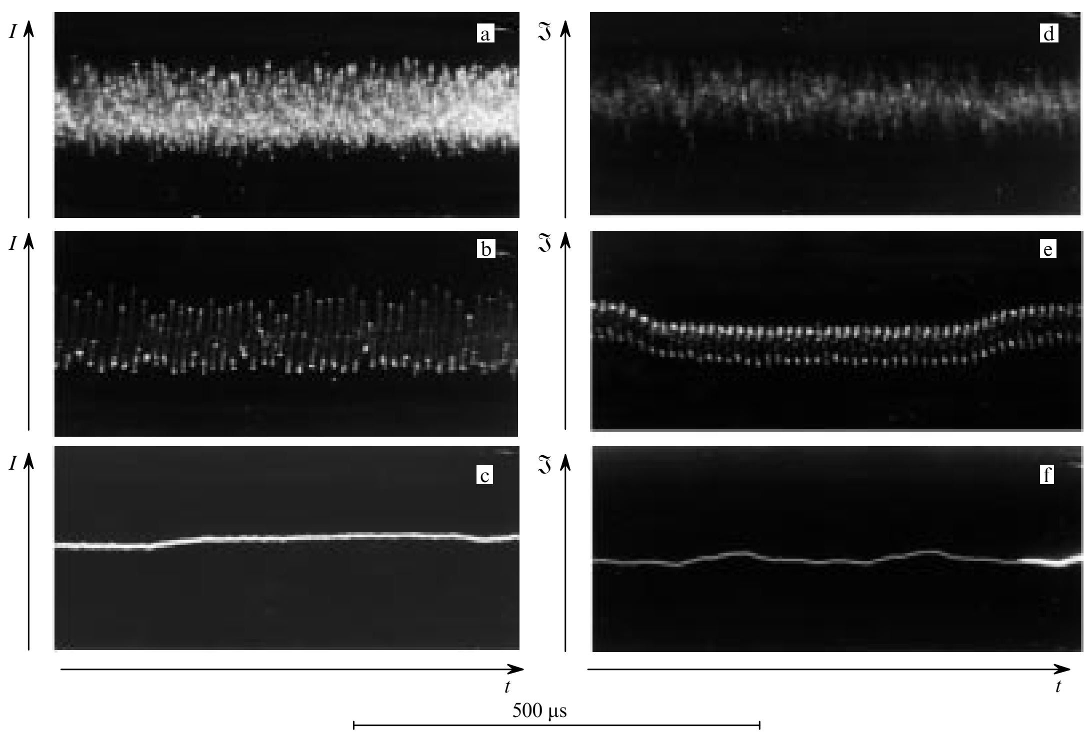

Figure 4. Experimental time dependences of the radiation intensity of the (a–c) first and (d–f) second harmonics.

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