

NONLINEAR OPTICAL PHENOMENA

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Double resonance at inhomogeneously broadened transitions in common-upper-level scheme during the formation of self-induced transparency pulses in the pump channel

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***Abstract*.** **The transient double resonance excited in a common-upper-level scheme is studied theoretically in a medium whose spectral inhomogeneity is caused by the Doppler effect. The formation of the 0π, 2π- and 4π-pulses in the pump channel is considered. Studies of the pulse formation dynamics have revealed, in particular, that the exponential gain of a signal pulse is possible only if the pump channel contains at least one 2π-pulse. If there are more than one 2π-pulses in the pump channel, each pump pulse generates its own signal pulse whose characteristics are determined by the parameters of the pump pulse.**

Keywords: *double resonance, self-induced transparency, inhomogeneous broadening*

1. Introduction

Double resonance has been the subject of theoretical investigation and practical application for a long time [1]. Transient double resonance emerges for pulse durations that are comparable with the irreversible relaxation times for quantum transitions or are much shorter than these. The theory of the linear regime of the transient double resonance (when the effect of a weak field on a strong field is negligible) shows that a strong pulse can trap a weak pulse and exponentially amplify it [2 – 5]. Investigations of this phenomenon [3, 6, 7] provided a rigorous justification for the possibility of formation of simultons and the complete transfer of energy from a strong pulse to a weak pulse. The theory of transient double resonance under the conditions when both interacting pulses satisfy the adiabaticity criterion was proposed in paper [8]. The transient double resonance was considered in paper [9] using a scheme with a common upper level during the formation of a lower frequency pulse from spontaneous noise radiation.

The results of the analytical theory of the nonlinear stage of transient double resonance obtained so far pertain to a spectrally homogeneous medium. Real media like rarefied gases or impurity centres in crystals have inhomogeneously broadened spectral transitions.

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In this paper, we simulate numerically transient double resonance (including its linear and nonlinear stages) taking into account the spectral inhomogeneity of the medium. The latter is required if the spectral widths of the interacting pulses are not much broader than inhomogeneous widths of resonance transitions. According to the theory of self-induced transparency (SIT) [10], the distance over which an input pulse splits into solitons is directly proportional to the inhomogeneous width of the resonance transition. Therefore, the consideration of inhomogeneous broadening is significant in the case of two-frequency interaction also for determining the distances at which the input pulses are transformed. Note that the numerical simulation taking into account the spectral inhomogeneity of the medium was performed in paper [11] for the case of double resonance in a scheme with a common middle level.

2. Mathematical model

We will simulate a gaseous medium by an ensemble of three-level objects, hereafter called atoms, with simple levels labelled as 1, 2, 3 in increasing order of energy. The symbols p_{13} and p_{23} denote the z components of the electric dipole moments of the transitions $1-3$ and $2-3$, respectively, while the frequencies of these transitions for atoms at rest are denoted by ω_{p} and ω_{s}, respectively. The $1-2$ transition is assumed to be forbidden in the electric-dipole approximation. We will describe [4] the spectral inhomogeneity of all quantum transitions involved in the process caused by the thermal motion of atoms by the only parameter – the time T_{p} of polarisation decay at the $1-3$ transition caused by the Doppler effect: $T_{\mathrm{p}} = c(M/2kT)^{1/2}/\omega_{\mathrm{p}}$, where M is the mass of an atom and T is the absolute temperature.

An electric field polarised along the z axis and propagating along the x axis is represented in the form

$$E(x,t) = \frac{1}{2}\mu_{\mathrm{p}} E_{\mathrm{p}}(x,t)\exp[\mathrm{i}(k_{\mathrm{p}}x - \omega_{\mathrm{p}}t)] + \frac{1}{2}\mu_{\mathrm{s}} E_{\mathrm{s}}(x,t)\exp[\mathrm{i}(k_{\mathrm{s}}x - \omega_{\mathrm{s}}t)] + \mathrm{c.c.}, \quad (1)$$

Here, $\omega_{\mathrm{p,s}}$ and $k_{\mathrm{p,s}} = \omega_{\mathrm{p,s}}/c$ are respectively the carrier frequencies and wave numbers of the quasiharmonics at the entrance ($x = 0$) to the resonance medium; $\mu_{\mathrm{p}} = \hbar/(T_{\mathrm{p}}|p_{13}|)$; and $\mu_{\mathrm{s}} = \hbar/(T_{\mathrm{p}}|p_{23}|)$.

Below, we will call the quasi-harmonics with frequencies ω_{p} and ω_{s} ($\omega_{\mathrm{p}} > \omega_{\mathrm{s}}$) as a pump and a signal, respectively. We described the evolution of pulses using a system of truncated Maxwell equations and of the equations of motion for the density matrix elements of the medium. The irreversible relaxation processes were neglected. The resulting system of equations has the form:

$$\frac{\partial E_p}{\partial s} = \frac{i}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp(-\varepsilon^2)\sigma_{31} d\varepsilon,$$

$$\frac{\partial E_s}{\partial s} = \frac{i}{\sqrt{\pi}} \alpha \int_{-\infty}^{+\infty} \exp(-\varepsilon^2)\sigma_{32} d\varepsilon,$$

$$\frac{\partial \sigma_{31}}{\partial w} = -i\varepsilon\sigma_{31} - iE_p(\sigma_{33} - \sigma_{11}) + iE_s\sigma_{21},$$

$$\frac{\partial \sigma_{21}}{\partial w} = -i(1-\beta)\varepsilon\sigma_{21} + \frac{i}{4}\left(E_s^*\sigma_{31} - E_p\sigma_{32}^*\right), \quad (2)$$

$$\frac{\partial \sigma_{32}}{\partial w} = -i\beta\varepsilon\sigma_{32} - iE_s(\sigma_{33} - \sigma_{22}) + iE_p\sigma_{21}^*,$$

$$\frac{\partial \sigma_{11}}{\partial w} = \frac{1}{2}\mathrm{Im}\left(E_p\sigma_{31}^*\right), \quad \frac{\partial \sigma_{22}}{\partial w} = \frac{1}{2}\mathrm{Im}\left(E_s\sigma_{32}^*\right),$$

$$\frac{\partial \sigma_{33}}{\partial w} = -\frac{1}{2}\mathrm{Im}\left(E_p\sigma_{31}^* + E_s\sigma_{32}^*\right),$$

where

$$w = \frac{1}{T_p}\left(t - \frac{x}{c}\right), \qquad s = \frac{x}{x_p} \quad (3)$$

are independent dimensionless variables; N is the concentration of atoms; and $x_p = c\hbar/(2\pi\omega_p|p_{13}|^2 N T_p)$ is the distance over which the amplitude of the weak steady-state pump radiation decreases by a factor of e due to inhomogeneous broadening [10]. The parameters α and β are defined as follows:

$$\alpha = \frac{\omega_s|p_{23}|^2}{\omega_p|p_{13}|^2}, \qquad \beta = \frac{\omega_s}{\omega_p},$$

the parameter β appearing due to the difference in inhomogeneous broadening of the 1–3 and 2–3 transitions.

The system of equations (2) was supplemented by the boundary conditions ($s = 0$):

$$E_p(w, s=0) = A_p \mathrm{sech}(w - w_0), \quad E_s(w, s=0) = A_s,$$

$$0 \leqslant w < \infty, \quad (4)$$

where A_p and A_s are constants. Expressions (4) describe the input pump pulse of duration $2T_p$ (at the level $\mathrm{sech}\, x = 1$) and correspond to the case when the input signal pulse is much longer than the input pump pulse. It was also assumed that all the atoms are at the lowest energy level at the initial instant ($w = 0$).

The boundary value problem formulated for the system (2) was solved numerically with the help of a predictor–corrector algorithm [12] with a checking of the accuracy of computations by the Runge method [13] and a verification of the condition $\sigma_{11} + \sigma_{22} + \sigma_{33} = 1$. The programme was verified by comparing the results of computations with the known analytical results from the SIT theory [10] (the discrepancy was less than one percent) and, wherever possible, with the analytical results presented in paper [5].

The results of computations are presented in the form of dependences of E_p and E_s on w for fixed values of s. The dependences of the characteristics of the pump and signal pulses, such as the area under the envelope (Θ_p and Θ_s), the peak value of the envelope (E_{pm} and E_{sm}), and the temporal position of these peak values (w_{pm} and w_{sm}) are presented, as well as the dependences of the local gain G of the signal pulse on s, where

$$\Theta_i = \int_{+\infty}^{-\infty} E_i(w, s) dw \quad (i = p, s), \quad G = \frac{d}{ds}\ln E_{sm}.$$

In this case, the area $\Theta_p(0)$ of the input pump pulse (4) is πA_p.

3. Parameters

Let us make estimates for saturated indium vapours by identifying the levels 1, 2, 3 with the levels $5P_{1/2}$ and $5P_{3/2}$ and $6S_{1/2}$, respectively ($\lambda_p = 4101.8$ Å and $\lambda_s = 4511.3$ Å, where $\lambda_{p,s} = 2\pi/\omega_{p,s}$). It was shown in Ref. [4] that the level degeneracy in the quantum number M of the projection of the total angular momentum on the z axis can be taken into account by assuming that all the atoms may occupy only states with $M = 1/2$.

Using the experimental values of the oscillator strengths, transition frequencies [14], and the data on the saturated vapour pressure of indium [15], we obtain $\alpha = 2.25$, $\beta = 0.91$, $x_p = 4.9 \times 10^{10}\sqrt{T}/N$, $T_p = 5.4 \times 10^{-9}/\sqrt{T}$, and $N = 7.2 \times 10^{(32.3 - 12019.2/T)}/T$, $T_p = 5.4 \times 10^{-9}/\sqrt{T}$. In addition, we have $I_p = 1.1 \times TE_p^2$ and $I_s = 0.45 \times TE_p^2$ for the intensities of the pumping and signal pulses, respectively. Here, x_p is measured in centimetres, T_p is measured in seconds, N is in cm^{-3} and T is in Kelvins.

An important characteristic of the pump and signal pulses is the velocity of their propagation (V_p and V_s, respectively) in the stationary reference frame x, t. Because of a possible deformation of the pulse envelope, we treat the propagation velocity as the rate of displacement of a characteristic point on the pulse, for example, its maximum. Using (3), we can show easily that

$$\frac{c}{V_{s,p}} = 1 + \frac{T_p c}{x_p v_{s,p}}, \quad (5)$$

where $v_{s,p}$ is the velocity of the characteristic point moving in the s, w reference frame.

4. Results of computations

According to the SIT theory, the evolution of a pulse in a resonance medium is determined by its initial area, the areas that are odd multiples of π being critical for the soliton formation process. For this reason, the specific values of A_p for computations in (4) were chosen in such a way that the area $\Theta_p(0)$ of the input pump pulse was within the characteristic intervals defined by the SIT theory. It was assumed in the computations that $w_0 = 7$, which provided

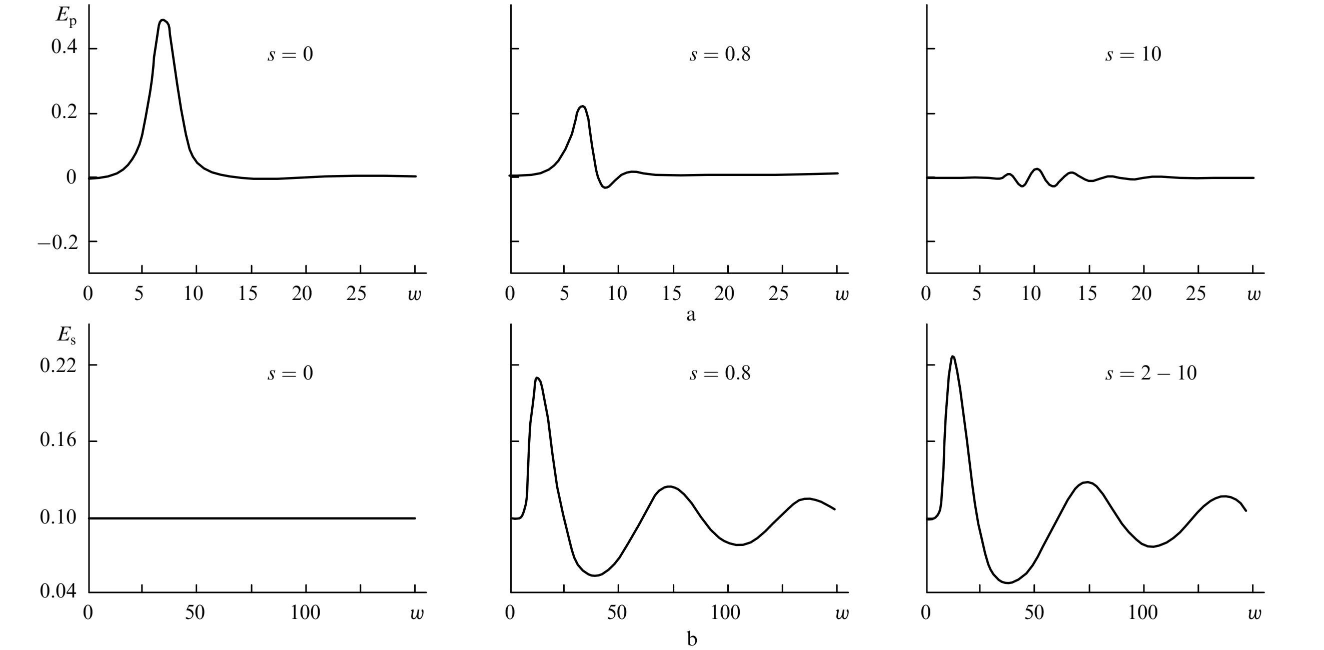

Figure 1. Pump (a) and signal (b) pulse envelopes for various distances during the formation of a 0π-pulse in the pump channel.

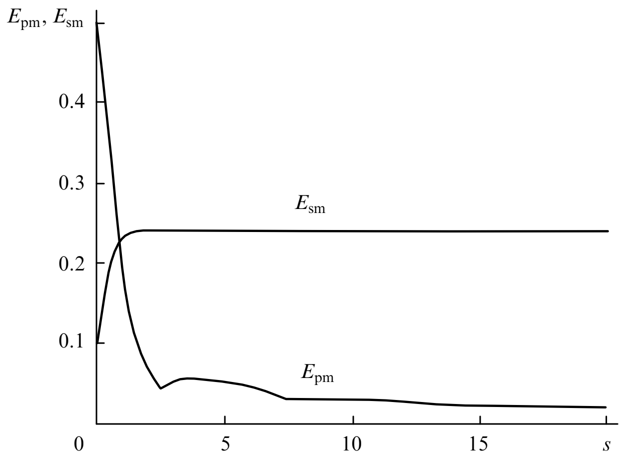

Figure 2. Dependences of the peak values of the pump and signal pulse envelopes on the distance during the formation of a 0π-pulse in the pump channel.

the absence of a pump pulse at the input ($s = 0$) at the initial instant $w = 0$.

4.1 The case $\Theta_p(0) < \pi$

Let us put $A_p = 0.5$ and $A_s = 0.1$ in (4). Then, $\Theta_p(0) = 0.5\pi$, and according to the SIT theory, the pump pulse should be transformed into a 0π-pulse in the absence of signal radiation. The functions $E_p(w, s)$ and $E_s(w, s)$ are presented in Fig. 1, while Fig. 2 shows the peak values E_{pm} and E_{sm} of the pulse envelopes as functions of s. One can see from Fig. 1 that at distances $s \geqslant 0.8$, the pump pulse assumes an oscillating shape characteristic of a 0π-pulse, and its peak value decreases with increasing s (Fig. 2). One can see from Figs 1b and 2 that the signal pulse passes through the formation stage at distances s between 0 and 2, and propagates further almost without any change in its shape and peak value. An analysis of the dependence of the gain G on s shows that the region of exponential signal gain is absent in this case.

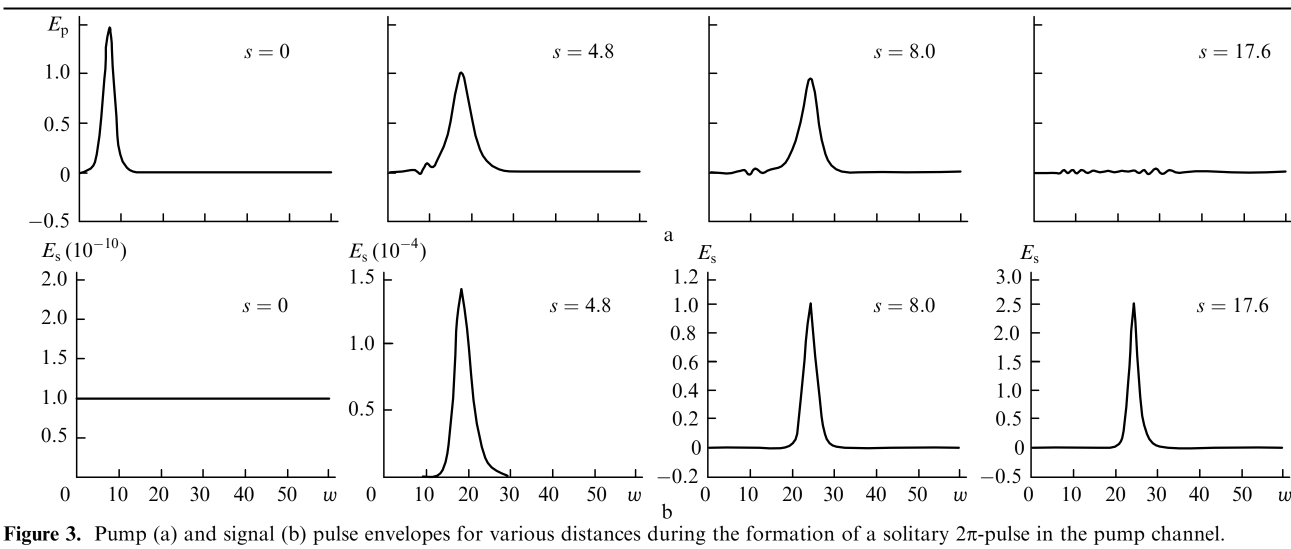

Figure 3. Pump (a) and signal (b) pulse envelopes for various distances during the formation of a solitary 2π-pulse in the pump channel.

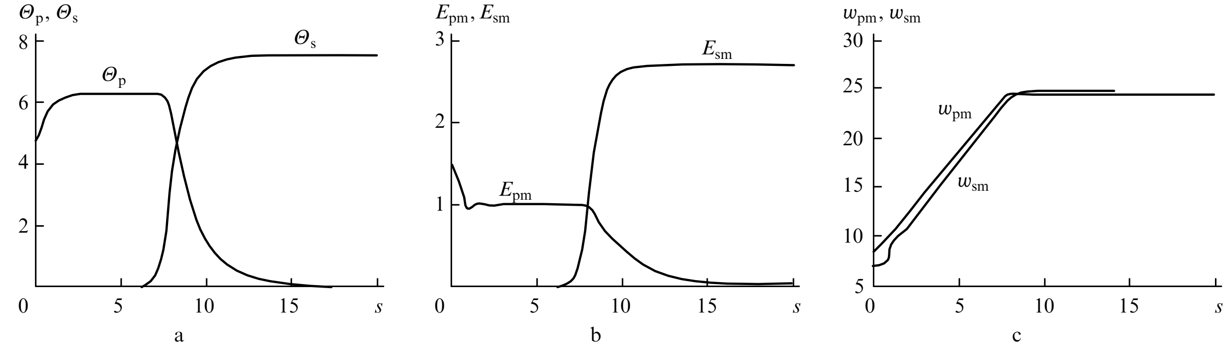

Figure 4. Dependence of the (a) area, (b) peak value, and (c) temporal position of the pump and signal pulses on the distance during the formation of a 2π-pulse in the pump channel.

4.2 The case $\pi < \Theta_p(0) < 3\pi$

Let us put $A_p = 1.5$ and $A_s = 10^{-10}$ in (4). Then, $\Theta_p(0) = 1.5\pi$, and according to [10], the pump pulse should be transformed into a solitary 2π-pulse in the absence of the signal. The functions E_p and E_s are presented in Fig. 3.

Fig. 4 shows the dependences of $\Theta_{s,p}$, $E_{sm,pm}$, and $w_{sm,pm}$ on s. It follows from Fig. 4a that for values of s between 0 and 3, the input pump pulse is transformed into a steady-state 2π-pulse. The transformation is accompanied by a decrease in its peak value to ~ 1 (Fig. 4b). The pump pulse propagates in this form up to distances $s = 8$. On the time scale w, the pulse duration is found to be 4.1. According to the SIT theory, the propagation velocity of a 2π-pulse of such a duration in the s, w reference frame should be equal to 0.442. An analysis of the w_{pm} curve (Fig. 4c) shows that the pump pulse peak moves with a velocity 0.452. This value is close to that predicted by the SIT theory, which serves as a further confirmation of the propagation of a pump pulse in the form of an isolated soliton. One can see from Fig. 3a that a small precursor pulse precedes the 2π-pulse.

At distances $s > 8$, the area Θ_p and the peak value E_{pm} of the pump pulse begin to decrease (see Figs 3a and 4a, b). This is due to the disintegration of the 2π-pulse after the transfer of its energy to the signal pulse. We can conclude easily from Figs 4b, c and expression (5) that the pump pulse propagates with a velocity c at large distances ($s > 8$) and loses its energy almost entirely.

Let us now describe the signal pulse. One can see from Fig. 4b that its peak value becomes comparable with the peak value of the pump pulse for $s > 8$. The slope of the curves in Fig. 4c shows that the signal pulse propagates with the velocity of the pump pulse, while a comparison of curves in Figs 3a and 3b shows that the signal pulse is localised in the region of the pump pulse.

Fig. 5 shows the dependence of the local gain G of the signal pulse on s. The constant value of G for $s = 2.5 - 8$ indicates that the signal pulse grows exponentially with distance in this region with a gain $G = 2.8$. Note that the possibility of an exponential signal amplification in the field of a steady-state 2π-pump pulse was predicted during an analysis of the linear stage of the double resonance [5].

According to Ref. [5], the signal gain in this case is described by the expression

$$G = \frac{2\sqrt{\alpha} - 1}{2\sqrt{\pi}} \tau_p \int_{-\infty}^{+\infty} \frac{\exp(-\varepsilon^2)}{1 + \tau_p^2 \varepsilon^2/4} d\varepsilon, \tag{6}$$

where τ_p is the duration of the 2π-pump pulse. For the

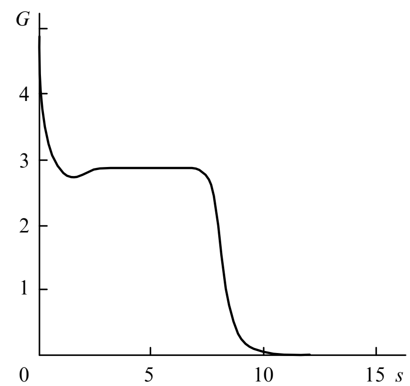

Figure 5. Dependence of local gain for the signal pulses on the distance during the formation of a 2π-pulse in the pump channel.

value of the parameter α chosen by us and for $\tau_p = 4.1$, which corresponds to a 2π-pulse formed at the pump frequency, expression (6) gives $G = 2.7$. The difference between the values of the gain obtained by numerical and analytical methods of description is explained by the fact that, strictly speaking, the analytical theory is valid in the limit $\beta \to 1$. At distances $s > 12$, the signal amplification virtually stops and its propagation velocity becomes equal to c in x, t coordinates.

4.3 The case $3\pi < \Theta_p(0) < 5\pi$

Let us put $A_p = 4$ and $A_s = 10^{-10}$ in (4). Then, $\Theta_p(0) = 4\pi$, and two steady-state 2π-pulses should be formed in the pump channel in the absence of signal radiation [10]. The functions E_p and E_s are presented in Fig. 6.

It follows from Fig. 6 that during its propagation, the pump pulse is transformed into two individual pulses marked as *1* and *2* in the order of their emergence, and called first and second pulse in the following. Accordingly, we mark by $\Theta_p^{(1,2)}$, $E_{pm}^{(1,2)}$, $w_{pm}^{(1,2)}$ in Fig. 7 the quantities Θ_p, E_{pm}, w_{pm} defined for pulses 1 and 2. In accordance with Fig. 7a, $\Theta_p^{(1)} = 2\pi$ at distances $s = 4 - 25$, and $\Theta_p^{(2)} = 2\pi$ at distances $s = 4 - 12$. This means that each of the two pump pulses is a steady state 2π-pulse at such distances.

The durations of the first and second pump pulses at the regions where they have an area equal to 2π are 0.7 and 2.1, respectively. According to the predictions of the SIT theory, the velocities of 2π-pulses of such durations should be 8.6 and 1.2 in the s, w reference frame. This is confirmed by the slopes of the dependences $w_{pm}^{(1)}$ and $w_{pm}^{(2)}$ on s (Fig. 7c). For the first 2π-pulse, $E_{pm}^{(1)}$ is about three times as large as $E_{pm}^{(2)}$

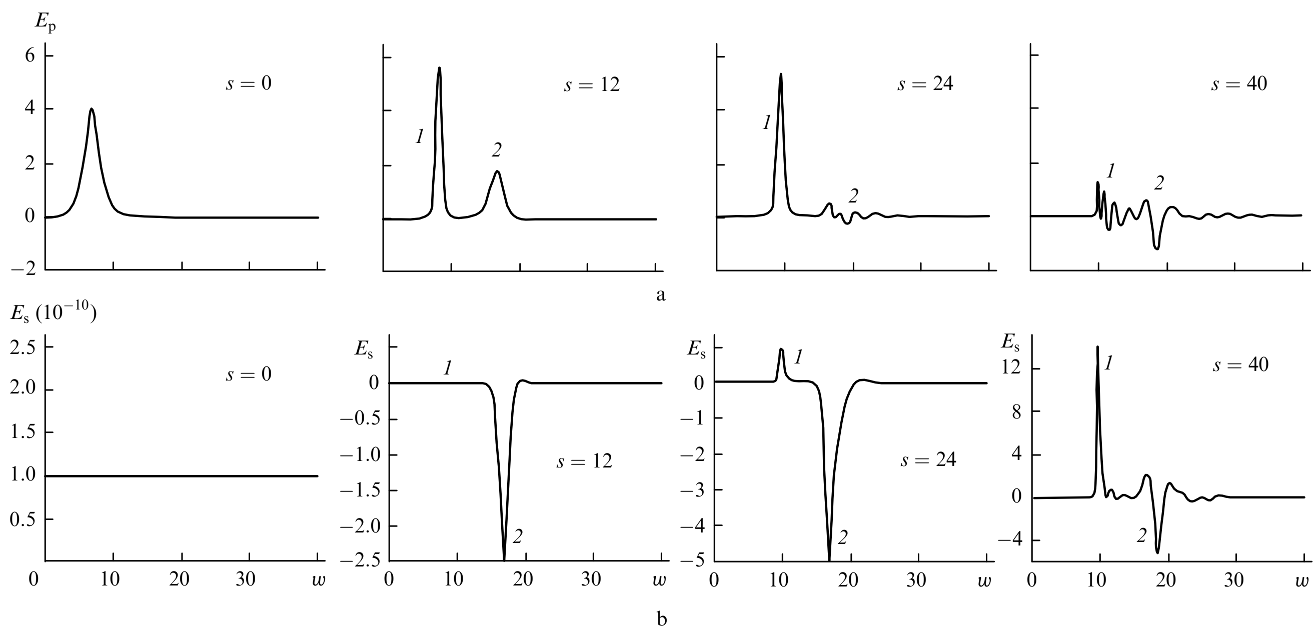

Figure 6. Pump (a) and signal (b) pulse envelopes for various distances during the formation of two 2π-pulses in the pump channel.

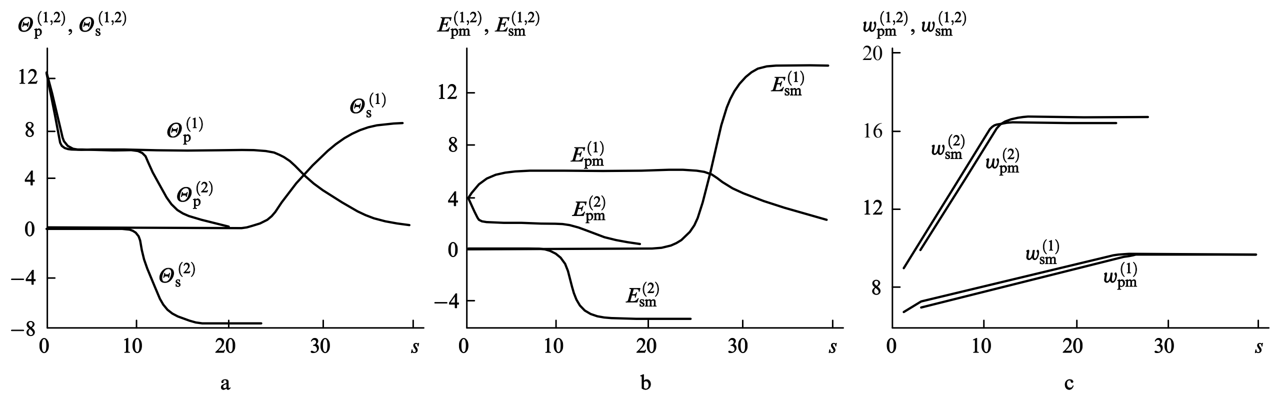

Figure 7. Dependence of the (a) area, (b) peak value, and (c) temporal position for the first and second pump pulses, as well as the first and second signal pulses on distance.

for the second pulse. The energy carried by a 2π-pulse through a unit cross-sectional area is proportional to the peak value of its envelope. Therefore, the first 2π-pulse transports about three times more energy per unit area than the second pulse. At large distances, both 2π-pulses disintegrate after transfer of energy to the signal radiation. One can see from Fig. 7 that the first 2π-pulse is preserved right up to distances $s = 25$, while the second pulse exists in a region about half as long.

Computations show that each 2π-pump pulse generates its own signal pulse in the region of its localisation. These pulses are marked in Fig. 6b as *1* and *2* according to the order of their emergence and are called first and second pulse, respectively. Accordingly, the notation $\Theta_s^{(1,2)}$, $E_{sm}^{(1,2)}$, $w_{sm}^{(1,2)}$ has been used in Fig. 7 to denote Θ_s, E_{sm}, w_{sm} defined for the first and second pulses. Note that the envelopes of the first and second signal pulses have opposite signs, i.e., these pulses have a phase difference equal to π.

Fig. 8 shows the dependences of the local gains $G^{(1)}$ and $G^{(2)}$ of the first and second signal pulses on s. The constant values of $G^{(1)}$ and $G^{(2)}$ (the plateaus in the curves) over a certain interval of s indicate that the signal pulse grows exponentially with s over this interval. This is in accord with the results of the analytical theory of the linear stage of the transient double resonance [5]. For $\tau_p = 0.7$ and 2.1 ($\alpha = 2.25$), expression (6) gives $G^{(1)} = 1.0$ and $G^{(2)} = 2.3$, while the numerical experiment gives the values $G^{(1)} = 0.9$ and $G^{(2)} = 2.0$. These small discrepancies are due to the difference of the parameter β from unity. An interesting

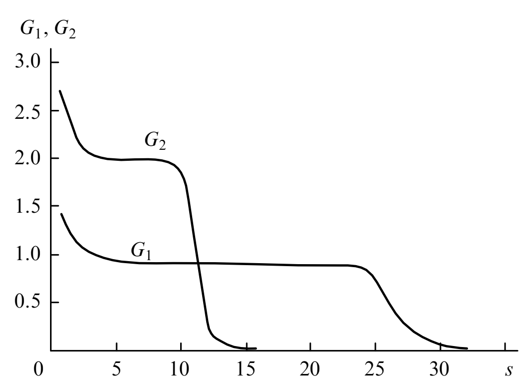

Figure 8. Dependence of local gain for the first and second signal pulses on the distance.

feature of the process under study is that the second 2π-pump pulse, being much weaker than the first one, ensures a larger gain for the signal pulse at the initial stage than the first pump pulse ($G^{(1)} > G^{(2)}$).

However, the second 2π-pulse disintegrates for $s > 12$ after transfer of its energy to the second signal pulse (see Fig. 7b), which in turn leads to a stabilisation of the peak value of the second signal pulse. However, the first 2π-pump pulse continues to amplify its signal pulse exponentially up to $s \sim 25$ (see Fig. 8). Consequently, the steady-state peak value of $E_{\mathrm{sm}}^{(1)}$ for $s > 30$ is almost three times larger (in magnitude) than the steady-state value of $E_{\mathrm{sm}}^{(2)}$ (see curves *3* and *4* in Fig. 7b). Thus, the large energy of the first 2π-pump pulse ensures a high peak value of the signal radiation pulse at sufficiently large distances.

5. Estimates

We will perform the dimensional estimates for the case of formation of two 2π-pump pulses in the saturated indium vapour at $T = 1020$ K. The condition $A_{\mathrm{p}} = 4$ indicates that there is a pump pulse with an envelope in the form of a hyperbolic secant at the entrance to the resonance medium, the duration and the maximum intensity of this pulse being 3.4×10^{-10} s and 17.6 kW cm^{-2}, respectively. The input pump pulse is transformed into two individual 2π-pulses at a distance of about 3 cm from the input surface, The first 2π-pulse has a maximum intensity of 36 kW cm^{-2}, which is about double the intensity of the input pump pulse, and a duration 1.2×10^{-10} s. The maximum intensity of the second 2π-pulse is 4 kW cm^{-2}, and its duration is 3.6×10^{-10} s. Using Eqn (5), we find from Fig. 7c that the propagation velocities of the first and second 2π-pulses are 0.55 and 0.15 c, respectively. Before disinitegrating, the first and the second 2π-pulses traverse distances equal to 16 and 9 cm respectively in the medium.

It is convenient to start the description of signal radiation from the second signal pulse, which is at the stage of amplification right up to distances ~ 11 cm and whose intensity attains the value 10 kW cm^{-2} at the end of this path. At distances exceeding 11cm, the peak value of the envelope of this pulse remains unchanged. The second signal pulse undergoes exponential amplification at distances between 2 and 8 cm for a dimensional gain factor 2.6 cm^{-1}. The first signal pulse is amplified to distances right up to 30 cm from the input surface. The maximum energy flux density attained at the end of this path is 66 kW cm^{-2}, which is about four times the corresponding value for the input pump pulse. The duration of this pulse is less than one-fifth of the duration of the pump pulse at the entrance to the resonance medium. At distances between 4 and 16 cm, the first signal pulse is at the stage of exponential growth with a gain coefficient 1.3 cm^{-1}.

6. Conclusions

Our investigations have shown that in all the cases considered by us, the pump pulse energy is transformed almost entirely into the signal radiation energy. However, the exponential signal amplification is due to the presence of signal 2π-pulses in the pump channel. The 2π-pulse with the lower energy ensures a larger gain for the signal pulse at the stage of exponential growth of the latter. Having absorbed a significant part of the pump pulse energy, the signal pulse may have a much higher peak value of the energy flux density than the input pump pulse, but its duration may be much shorter.

Dimensional estimates indicate that the characteristics of laser radiation and of the resonance medium required for an experimental observation of the effects under study are attainable in actual practice. The proposed theory is confined to the case of strict equality of the carrier frequency of each pulse to the central frequency of the corresponding quantum transition. Note that the absence of resonance at the linear stage substantially modifies the interaction of pulses. In particular, the signal radiation experiences the additional amplitude and frequency modulation [4, 5]. Further investigations will be aimed at an abandoning of the strict resonance condition.

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