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Self-action of a light beam in a photorefractive crystal in an alternating electric field upon synchronous intensity modulation

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***Abstract*.** **The propagation of a light beam in a photorefractive crystal in an alternating meander electric field is studied. A nonlinear equation is obtained for a spatial charge field formed in the crystal upon modulation of the light intensity synchronously with the applied electric field. It is established theoretically that the spatial charge field has two components, which make contributions to different mechanisms of a photoresponse providing self-focusing and the appearance of spatial solitons, as well as the trajectory self-bending. The conditions are found at which the soliton regime is realised and parameters affecting this regime in a greater extent are determined.**

Keywords: *photorefractive crystal, self-action of light, spatial soliton.*

Self-action effects are observed in photorefractive crystals at low intensities of light beams [1 – 5]. In the case of a drift mechanism of the photorefractive response, when a permanent external field (local nonlinearity) is applied to the crystal, self-focusing and self-defocusing occur and spatial solitons appear [3, 4, 6 – 9]. An alternating external electric field is commonly used to increase the response of crystals with low electrooptical constants such as sillenites $Bi_{12}SiO_{20}$, $Bi_{12}GeO_{20}$, $Bi_{12}TiO_{20}$, semiconductors GaAs, InP, GaP, et al. [10].

In this case, the distribution of perturbations of optical properties formed in the medium is determined by the gradient of the light-field intensity (nonlocal nonlinearity), resulting in the efficient energy transfer between the components of the spatial spectrum. The most dramatic manifestation of the self-action of light in a medium with a nonlocal response is the self-bending of light beams [1, 7 – 9]. However, upon the time modulation of the light intensity imposed synchronously with an alternating voltage applied to the crystal, along with the nonlocal component of a response, a local component also appears, which is caused by the screening of the external field by an accumulating spatial charge.

In paper [11], a decrease in the diffraction divergence of the light beam modulated at frequency 50 Hz was demonstrated in a $Ba_2NaNb_5O_{15}$ crystal to which an external field was applied at the same frequency, and the possibility of the appearance of the soliton regime was pointed out.

In this paper, we considered the field of a spatial charge produced in a photorefractive crystal in a meander electric field upon the propagation of a light beam modulated synchronously with the external field. It is shown that the relation between the local and nonlocal components of the response is determined by the external-field amplitude and the degree of intensity modulation. The conditions are considered under which a bright soliton is formed in the crystal.

Consider a photorefractive crystal (Fig. 1) in which a light beam with the one-dimensional intensity distribution $I(x, t)$ on the input face propagates along the z axis. An alternating external meander field with the amplitude E_{m} and period T is applied to the electrodes along the x axis. We assume that the input distributions of the light intensity for the positive and negative half-periods of the applied voltage are $I^{+}(x) = I(x)(1 + m)$ and $I^{-}(x) = I(x)(1 - m)$, respectively (m is the degree of intensity modulation).

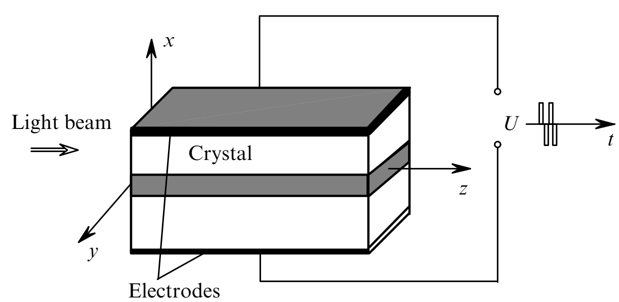

Figure 1. Configuration of a crystal with the applied meander voltage $U(t)$.

Photoexcitation of charge carriers and their redistribution over impurity centres in the external field result in the formation of the spatial-charge field with the dominating component $E_x(x, z)$. This field changes the refractive index of the crystal by the value $\Delta n(x, z) \sim E_x(x, z)$ due to the linear electrooptical effect and causes the self-action of the initial light beam.

In a crystal with one partially compensated photoactive centre and one type of carriers, the spatial-charge field is determined by the known system of equations [12]. In an

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external field with a period satisfying the conditions $\tau_r \ll T \ll \tau_{di}$ (τ_r is the recombination time of charge carriers and τ_{di} is the Maxwellian relaxation time), the field $E_x(x,z)$ experiences negligible oscillations upon switching the external-voltage polarity. In this case, we can use the method of field averaging over the period T [13, 14] and assume that the spatial intensity distribution during the positive and negative half-periods have the same form $\tilde{I}(x,z)$, by changing only in the amplitude:

$$I(x,z) = \tilde{I}(x,z)(1 \pm m). \tag{1}$$

When the adiabaticity condition $\partial n/\partial t = 0$ is fulfilled at the low light intensity ($n_e \ll N_a$, $SI \ll 1/\tau_r$) in the absence of saturation of traps ($\partial E_x/\partial x \ll eN_a/\varepsilon$), the equation for the distribution of the spatial-charge field $\tilde{E}_{sc}(x,z)$ averaged over the period of the applied field has the form

$$L_d^2 \frac{d^2\tilde{E}_{sc}}{dx^2} + \left[2\frac{\tilde{E}_{sc}L_e}{E_m} + \frac{3L_d^2}{I_d+\tilde{I}}\frac{d\tilde{I}}{dx}\right]\frac{d\tilde{E}_{sc}}{dx}$$
$$-\left[1 - \frac{1}{I_d+\tilde{I}}\left(\frac{\tilde{E}_{sc}L_e}{E_m}\frac{d\tilde{I}}{dx} + 2L_d^2\frac{d^2\tilde{I}}{dx^2}\right)\right]\tilde{E}_{sc} \tag{2}$$
$$= \frac{E_m}{I_d+\tilde{I}}\left[m\tilde{I} + L_e\left(1+\frac{L_d^2}{L_e^2}\right)\frac{d\tilde{I}}{dx} - \frac{L_d^4}{L_e}\frac{d^3\tilde{I}}{dx^3}\right],$$

where $L_e = \mu\tau_r E_m$ and $L_d = (k_B \Theta \mu\tau_r/e)^{1/2}$ are the drift and diffusion lengths; μ and n_e are the mobility and concentration of charge carriers; k_B is the Boltzmann constant; Θ is the absolute temperature; N_a is the concentration of acceptors; ε is the static dielectric constant of the crystal; e is the elementary electric charge; $I_d = \beta/S$ is the 'dark' illumination determined by the rate β of thermal ionisation of donors and their photoionisation cross section S.

According to nonlinear equation (2), the contribution of diffusion to the spatial-charge field is determined by the relation between the diffusion length and the characteristic scale b of the spatial inhomogeneity of the light field. The diffusion length L_d of photorefractive crystals with typical values of the parameter $\mu\tau_r = 10^{-12} - 10^{-10}$ m^2 V^{-1} does not exceed 1.6 μm. If the light-beam diameter is greater than 15 μm, the diffusion can be neglected, and $\tilde{E}_{sc}$ can be written in the linear approximation as a sum of the local and nonlocal components

$$\tilde{E}_{sc} = -E_m \frac{m\tilde{I}}{I_d+\tilde{I}} - \frac{E_m L_e}{I_d+\tilde{I}}\frac{d\tilde{I}}{dx}. \tag{3}$$

Note that the contribution of the local component to the field $\tilde{E}_{sc}$ can be controlled by varying the degree of intensity modulation, while the value of the nonlocal component is determined by the ratio L_e/b.

The linear electrooptical effect causes the change in the refractive index of the crystal in the region of the light-beam propagation:

$$\Delta n(x,z) = -\frac{n_0^3 r_{eff}\tilde{E}_{sc}}{2}, \tag{4}$$

where n_0 is the refractive index of the unperturbed medium and r_{eff} is the effective electrooptical coefficient. We will describe its evolution with the help of the standard truncated wave equation for the complex amplitude $A(x,z)$ of the light field written in the paraxial approximation

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k_0 n_0}\frac{\partial^2}{\partial x^2}\right)A(x,z) = -ik_0\Delta n A(x,z), \tag{5}$$

where $k_0 = 2\pi/\lambda$ is the wave number in vacuum. In the linear approximation, when the spatial-charge field is determined by equation (3), by introducing the normalised amplitude $U(x,z) = A(x,z)/\sqrt{I_d}$ into equation (5), we can represent it in the form

$$i\frac{\partial U}{\partial z} - \frac{1}{2k_0 n_0}\frac{\partial^2 U}{\partial x^2} - \alpha\frac{|U|^2 U}{1+|U|^2} - \gamma\frac{U}{1+|U|^2}\frac{\partial |U|^2}{\partial x} = 0, \tag{6}$$

where $\alpha = \pi n_0^3 r_{eff} m E_m/\lambda$ characterises the local nonlinearity of the medium and $\gamma = \pi n_0^3 r_{eff} L_e E_m/\lambda$ describes the nonlocal nonlinearity of the diffusion type [8]. Note that in the case under study, when an alternating field is applied to the crystal and the input-beam intensity is synchronously modulated, the local component of the response has the same character as the photovoltaic nonlinearity, which was studied in detail in papers [8, 15].

We can neglect the diffusion nonlinearity in photorefractive crystals with small drift lengths ($L_e \ll b$) (assuming that $\gamma = 0$) and consider the solution of the soliton type

$$U(x,z) = u(x)\exp(-i\nu z), \tag{7}$$

where the real function $u(x)$ specifies the field distribution over the transverse coordinate, while the positive constant ν characterises a nonlinear phase incursion upon propagation of the beam along the z axis. By using the dimensionless transverse coordinate $\xi = (2k_0 n_0 \nu)^{1/2}x$ and the nonlinearity parameter $\delta = \alpha/\nu$, we obtain from (6) the equation for $u(\xi)$

$$u'' - u + \delta\frac{u^3}{1+u^2} = 0, \tag{8}$$

from which a solution in the form of a bright soliton can be obtained [15].

To obtain a soliton with the unit amplitude $A(0,z)$, the nonlinearity parameter $\delta = \pi n_0^3 r_{eff} L_e E_m/\lambda\nu$ should be related to the 'dark' illumination by the expression

$$\delta = \frac{1}{1 - I_d \ln(1 + 1/I_d)}. \tag{9}$$

By using the method described in paper [15], we calculated numerically the envelope $u(\xi)$ of the soliton beam for $I_d = 10^{-2}, mE_m = 5$ kV cm^{-1}, and $\lambda = 633$ nm for a $Ba_2NaNb_5O_{15}$ crystal with parameters $n_0 = 2.221$ and $r_{eff} = 48$ pm V^{-1} [16]. We determined from it the shape of the input light-field distribution $A(x,0)$ required for obtaining the soliton regime, which is shown in Figs 2 and 4 by dotted curves.

To analyse the influence of the experimental conditions on the propagation of the light beam with such input intensity distribution, we used the wave equation (5), which was solved by the finite-difference method according to the Douglas scheme [17], and the linear approximation (3) for the spatial-charge field. The solid curves in Fig 2 correspond to the output distribution of the light field with small drift lengths ($\mu\tau_r = 10^{-15}$ m^2 V^{-1}) in a crystal of length 10 mm along the z axis. In the case of a perfect matching between

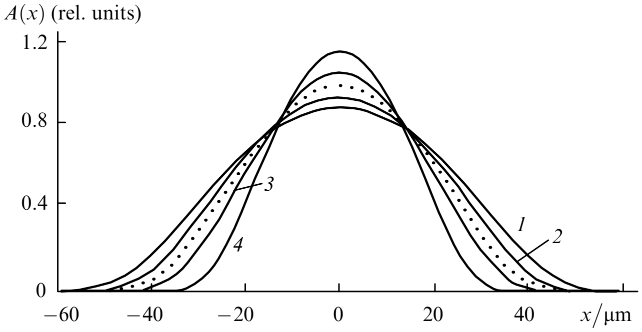

Figure 2. Distribution of the light field at the crystal input (dotted curve) and output (solid curves) for the 'dark' illumination $I_d = 0.01$ and $\mu\tau_r = 10^{-15}$ m^2 V^{-1} and $mE_m = 3$ (*1*), 4 (*2*), 6 (*3*), and 8 kV cm^{-1} (*4*). For $mE_m = 5$ kV cm^{-1}, the output intensity distribution coincides with the input one.

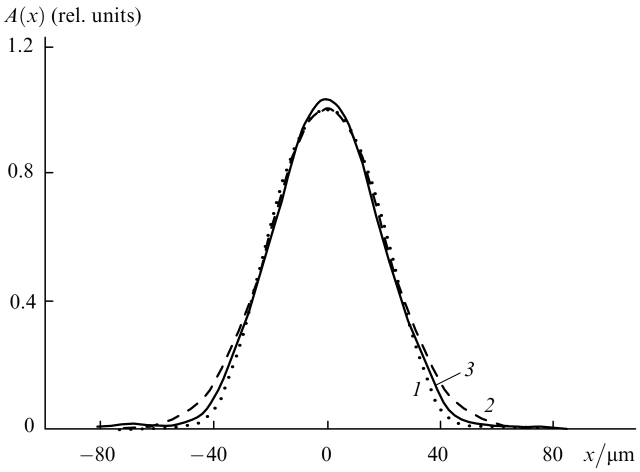

Figure 3. Gaussian distribution of the light field with the waist $x_0 = 11$ μm at the crystal input (*1*) and output (*2*). Curve *3* is the soliton profile $A(x, 0)$ for $mE_m = 5$ kV cm^{-1}, $I_d = 0.01$, and $\mu\tau_r = 10^{-15}$ m^2 V^{-1}.

the chosen and calculated parameters ($I_d = 10^{-2}$, $mE_m = 5$ kV cm^{-1}), the output intensity distribution almost completely coincided with the input distribution. However, an increase in the amplitude E_m of the field applied to the crystal or an increase in the degree of modulation m resulted in self-focusing of the initial beam, which was especially noticeable for $mE_m > 6$ kV cm^{-1}. When the parameter mE_m was decreased to 4 kV cm^{-1} (Fig. 2, curve *2*), the beam was not completely focused. A small change in the 'dark' illumination also resulted in the deviation from the soliton regime.

Fig. 3 shows the Gaussian distribution of the input light field with the half-width (HWHM) $x_0 = 11$ μm (curve *1*), which is most close to a perfect soliton distribution $A(x, 0)$ (curve *3*), and the output light-field distribution (curve *2*). Although the calculated curves are somewhat different, this difference can hardly be detected experimentally. Thus, the realisation of the soliton regime of propagation of the light beam in a $Ba_2NaNb_5O_{15}$ crystal with small values of $\mu\tau_r$ strongly depends on the parameters mE_m and I_d.

Fig. 4 shows the results of analysis of the effect of the nonlocal component of the response on the propagation of the light beam with the input intensity distribution corresponding to a spatial soliton for $mE_m = 5$ kV cm^{-1} and $I_d = 10^{-2}$. The increase in the drift length L_e up to 0.5 μm due to the increase in $\mu\tau_r$ up to 10^{-13} m^2 V^{-1} for $m = 1$ and $E_m = 5$ kV cm^{-1} does not distort noticeably the beam profile (Fig. 4, curve *1*). However, the beam trajectory bends, and the output field distribution shifts relative to the beam profile in a crystal with a purely local response by ~ 3 μm. The increase in the drift length L_e up to 1 μm ($\mu\tau_r = 2 \times 10^{-12}$ m^2 V^{-1}) results in the beam shift by 10 μm and in the distortion of its profile (Fig. 4, curve *2*). The right wing of the beam profile becomes steeper, resulting in the increase in the local component of the spatial-charge field in this region and in a greater distortion of the beam profile. As a result, self-focusing develops and the modulation instability appears, which was studied in detail for the response of the mixed type in paper [9].

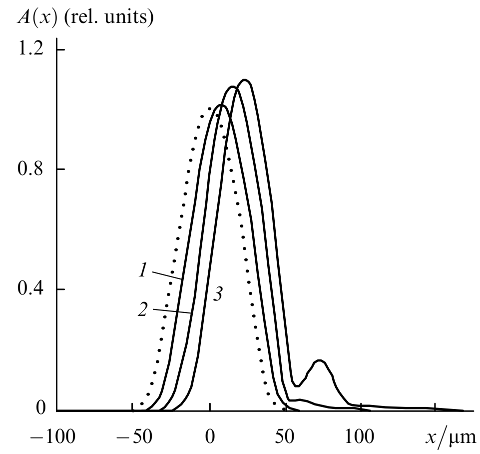

Figure 4. Distribution of the light field at the crystal input (dotted curve) and output (solid curves) for $m = 1, E_m = 5$ kV cm^{-1} and the 'dark' illumination $I_d = 0.01$ and $\mu\tau_r = 10^{-12}$ (*1*), 2×10^{-12} (*2*), and 3×10^{-12} m^2 V^{-1} (*3*). For $\mu\tau_r < 10^{-14}$ m^2 V^{-1}, the output intensity distribution coincides with the input one.

This process develops faster with increasing the drift length (Fig. 4, curve *3*). In this case, it follows from calculation at $L_e = 1.5$ μm that the beam filamentation begins at $z > 8$ mm. The same results are obtained if the drift length is increased by the external field E_m when the parameter $\mu\tau_r$ remains constant. For a crystal with $\mu\tau_r = 5 \times 10^{-13}$ m^2 V^{-1}, the drift length $L_e = 0.5$ μm is obtained for $E_m = 10$ kV cm^{-1}. Note that the local photoresponse can be invariable in this case because the decrease in the degree of modulation.

The beam disintegration into separate filaments in the case of the mixed response is distinctly observed in the simulation of the propagation of the beam with a rectangular profile of the type $A(x, 0) = \{\tanh[(a_0 - x)/x_0] + \tanh[(a_0 + x)/x_0]\}/2$, where a_0 is the beam width and x_0 is half-width of the slope. Fig. 5 demonstrates the features of the propagation of such a beam for $a_0 = 120$ μm, $x_0 = 20$ μm, $mE_m = 5$ kV cm^{-1}, $I_d = 0.01$, and $\mu\tau_r = 2 \times 10^{-12}$ m^2 V^{-1}. The figure shows a change in the beam intensity over the crystal length 15 mm, the scale along the transverse axis x being enlarged 14 times for clarity compared to that along the coordinate z. One can see that in this case, the modulation instability beginning from the energy concentration in the diffraction outburst near the left slope and resulting in the beam filamentation develops at $z > 5$ mm.

Figure 5. Intensity distribution of the 'rectangular' light beam propagating in a crystal of length 15 mm along the z axis for $m = 0.5$, $E_m = 5$ kV cm^{-1}, $I_d = 0.01$, and $\mu\tau_r = 2 \times 10^{-12}$ m^2 V^{-1}. The scale along the transverse axis x is 14 times enlarged.

The electrooptical constants of sillenite crystals $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ are an order of magnitude lower that those of a $Ba_2NaNb_5O_{15}$ crystal. However, by changing the parameters of the light-field distribution and the external-field amplitude, one can observe the above effects of self-action of light beams in these crystals as well.

Thus, the regime of propagation of bright spatial solitons can be provided in $Ba_2NaNb_5O_{15}$ crystals or sillenites by a proper choice of the amplitude of the applied external meander field, the degree of synchronous intensity modulation, and the parameters of the input-light distribution.

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