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# Choice of optimal conditions for light diffraction and two-wave interaction in a cubic photorefractive piezoelectric $Bi_{12}SiO_{20}$ crystal

V.V.Shepelevich, N.N.Egorov, P.I.Ropot, A.A.Firsov

***Abstract*.** **The maximum and minimum diffraction efficiencies and the effective amplification of a hologram are studied experimentally in a 2.1 mm thick, $(\bar{1}\bar{1}0)$ cut $Bi_{12}SiO_{20}$ crystal as functions of the orientation angle formed by the vector of a holographic grating with the crystallographic direction [001]. A comparison is made between the orientation dependences of the polarisation azimuths at which the extrema of the diffraction efficiency and of the effective amplification are achieved. It is shown that the polarisation azimuths of the reading light corresponding to the maximum diffraction efficiency and the maximum effective amplification coincide within the orientation angles 0 – 90° and 270 – 360° and differ by 90° in the range 90 – 270°.**

***Keywords***: *piezoelectric effect, photoelasticity, optical activity, diffraction efficiency, effective amplification.*

## 1. Introduction

Optimisation of the conditions of light diffraction and two-wave interaction in a $Bi_{12}SiO_{20}$ (BSO) crystal has been discussed in earlier studies of holographic processes proceeding in this crystal (see, for example, [1, 2]). Analytic expressions for the diffraction efficiency of a hologram were obtained for fixed orientations of the holographic-grating vector in papers [3 – 6], and the polarisation dependence of the diffraction efficiency was investigated numerically and confirmed experimentally in paper [7]. The effect of the orientation of the holographic-grating vector on the diffraction of light in cubic optically active photorefractive crystals with parameters close to those of a BSO crystal was theoretically studied in paper [8].

The orientation dependence of the gain of a signal light wave upon two-wave interaction in a cubic photorefractive crystal of the class 43m (CdTe : Ge), which does not exhibit optical activity, was studied theoretically and experimentally in paper [9]. The dependence of energy transfer between light waves in a cubic optically active crystal (class 23) on the orientation angle was theoretically studied in paper [10]. However, the inverse piezoelectric effect and photoelasticity were neglected in all these papers.

**V.V.Shepelevich, N.N.Egorov, P.I.Ropot, A.A.Firsov** N.K.Krupskaya Mozyr State Pedagogical Institute, ul. Studencheskaya 28, 247760 Mozyr, Gomel'skaya oblast, Belarus; e-mail: mozvuz@mail.gomel.by

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The influence of the piezoelectric effect on diffraction and energy transfer between light waves in cubic photorefractive crystals was studied for the first time in paper [11]. However, the optical activity of these crystals was ignored in this paper. The orientation dependence of the diffraction efficiency of holograms in a BSO crystal was studied theoretically and experimentally in papers [12, 13], taking into account the piezoelectric effect and optical activity.

The influence of the inverse piezoelectric effect and optical activity on the orientation dependence of the signal-wave gain upon two-wave interaction in a BSO crystal was studied theoretically and experimentally for two fixed polarisations of light waves (in the plane of incidence and perpendicular to it) in paper [14].

The orientation dependence of the maximum gain in a BSO crystal of a fixed thickness was studied theoretically in paper [15]. The results of this paper were experimentally confirmed in papers [16, 17]. However, the orientation dependences of the maximum and minimum diffraction efficiencies of transmission holograms in a BSO crystal, as well as the conditions for optimisation of the diffraction efficiency and the gain were not compared in these papers.

In this paper, we study theoretically and experimentally the orientation dependences of the maximum and minimum diffraction efficiencies of a hologram, as well as of the maximum and minimum gain upon two-wave interaction in a piezoelectric BSO crystal. We also compare the conditions for optimisation of the signal-wave gain with those for the diffraction efficiency of a hologram.

## 2. Theory

In this paper, we do not focus attention on the kinetics of processes proceeding in a crystal during the hologram recording, and introduce the electric field of a crystal lattice phenomenologically, its strength not being related to the chosen model of a dynamic hologram. By solving a system of equations of coupled waves [13, 18], we can determined the diffraction efficiency $\eta$ and the relative intensity $\gamma$ of a signal wave, which is also called the effective amplification [19]. The study of functions $\eta(\psi)$ and $\gamma(\psi)$, where $\psi$ is the initial azimuth of polarisation of the reading light (see, for example, [18]) showed that their maxima for linearly polarised waves are achieved at

$$\psi_{\eta,\gamma}^{\max} = \begin{cases} \dfrac{1}{2}\arctan\dfrac{F}{G} + \dfrac{\alpha d}{2} + \dfrac{\pi}{2} & \text{for } H_{\eta,\gamma} > 0, \\ \dfrac{1}{2}\arctan\dfrac{F}{G} + \dfrac{\alpha d}{2} & \text{for } H_{\eta,\gamma} < 0, \end{cases} \qquad (1)$$

where $\psi_{\eta}^{\max}$ and $\psi_{\gamma}^{\max}$ the azimuths at which the maximum diffraction efficiency $\eta$ and the maximum effective amplification $\gamma$ are achieved, respectively. In (1),

$$H_{\eta} = (r - A)\frac{\sin \alpha d}{\alpha}\left[r\left(1 - 6\sin^2\theta\right) + 2(B + C)\sin^2\theta - B\right]; \qquad (2)$$

$$H_{\gamma} = \left[\kappa E d\left(\rho_0^2 - 1\right)(r - A)\cos\theta - 2\rho_0\right]\frac{\sin \alpha d}{\alpha}G; \qquad (3)$$

$$F = 2\left[r\left(1 - 3\cos^2\theta\right) + (B + C)\cos^2\theta - \frac{C}{2}\right]\sin\theta; \qquad (4)$$

$$G = \left[r\left(1 - 6\sin^2\theta\right) + 2(B + C)\sin^2\theta - B\right]\cos\theta; \qquad (5)$$

$\kappa = \pi n^3/2\lambda$; $n$ is the refractive index of the unperturbed crystal; $\lambda$ is the wavelength of the reading light; $d$ is the crystal thickness; $E$ is the amplitude of the electric field of a spatial charge; $\alpha$ is the specific rotary power of the crystal; $\theta$ is the orientation angle formed by the vector $\boldsymbol{K}$ of the holographic grating with the crystallographic direction [001]; $r$ is the electrooptical coefficient of a clamped crystal; $\rho_0 = (I_{\mathrm{R0}}/I_{\mathrm{S0}})^{1/2}$; $I_{\mathrm{R0}}$ and $I_{\mathrm{S0}}$ are the initial intensities of reading light waves; and the quantities $A$, $B$, and $C$ are defined in paper [20]. Expressions $\psi_{\eta,\gamma}^{\min}$ are obtained from (1) by replacing the sign $>$ by $<$ and vice versa.

By substituting the azimuth (1) into expressions for the diffraction efficiency $\eta$ and the relative intensity $\gamma$ of the signal wave, we can determine the maxima and minima $\eta_{\psi}^{\max}$, $\eta_{\psi}^{\min}$, $\gamma_{\psi}^{\max}$, $\gamma_{\psi}^{\min}$ of these quantities achieved by a proper choice of the azimuth $\psi$ of polarisation of the reading light [20, 21].

Expressions (1) for $\psi_{\eta}^{\max}$ and $\psi_{\gamma}^{\max}$ are also valid in the absence of optical activity or piezoelectric effect and have a simpler form. Note that the 'exclusion' of the optical activity (for example, in crystal of the class $\bar{4}3$m, where the optical activity is forbidden by the symmetry rules) should be accompanied by the formal substitution $(\sin \alpha d)/\alpha d \to 1$, while the neglect of the piezoelectric effect is equivalent to the equality $A = B = C = 0$. In this case, the crystal lattice is 'controlled' only by the electrooptical effect, which is described by the electrooptical coefficient $r$ [22].

## 3. Analysis of the theoretical results

Analysis of expressions (1) shows that the azimuths $\psi_{\eta}^{\max}$ and $\psi_{\gamma}^{\max}$ of the incident light corresponding to the maximum diffraction efficiencies and the effective amplification depend on the crystal thickness and the orientation angle $\theta$ and can be different due to different expressions for $H_{\eta}$ and $H_{\gamma}$. Below, we assume that the crystal thickness is fixed. It is obvious that expression (5) for $G$ vanishes at some orientation angles. In this case, the azimuth $\psi_{\eta,\gamma}^{\max}$ can jump by $\pi/2$. One can see that such a jump does occur for $\psi_{\eta}^{\max}$, because $H_{\eta}$ in (2) does not change its sign in the general case when the function $G$ (5) passes through zero. On the contrary, the function $H_{\gamma}$ (3) changes its sign, resulting in the change in $\psi_{\gamma}^{\max}$ by $\pi/2$ due to the additional conditions in expression (1). The double 'jump' by $\pi/2$ does not change the azimuth, and therefore the function $\psi_{\gamma}^{\max}(\theta)$ does not experience jumps except trivial shifts by $\pi$ used for convenience in the construction of the dependence $\psi_{\gamma}^{\max}(\theta)$.

Therefore, we found a substantial difference in the behaviour of the function $\psi^{\max}(\theta)$ for the diffraction efficiency and the effective amplification. This difference can be observed by comparing dependences $\psi_{\eta}^{\max}(\theta)$ (Fig. 1a) and $\psi_{\gamma}^{\max}(\theta)$ (Fig. 1b) shown together with functions $\eta_{\psi}^{\max}(\theta)$ and $\gamma_{\psi}^{\max}(\theta)$, respectively. Hereafter, we assume that the crystal field is $E = 1.93$ kV cm$^{-1}$ in the calculation of the diffraction efficiency and $E = 0.76$ kV cm$^{-1}$ in the calculation of the effective amplification and the initial ratio of the intensities of light waves is $I_{\mathrm{R0}}/I_{\mathrm{S0}} = 2$. The specific rotary power of the crystal was assumed equal to 0.4 rad mm$^{-1}$.

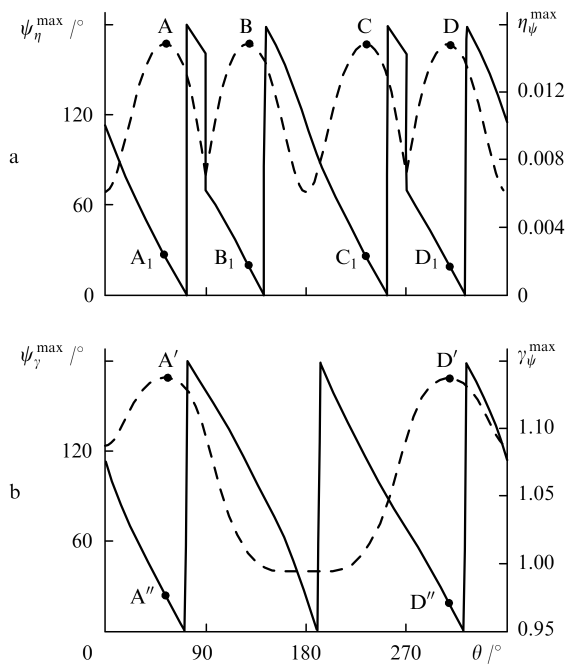


**Figure 1.** The maximum diffraction efficiency $\eta_{\psi}^{\max}$ (a, dashed curve), the maximum effective amplification $\gamma_{\psi}^{\max}$ (b, dashed curve), and the azimuths of polarisation of linearly polarised light waves $\psi_{\eta}^{\max}$ (a, solid curve) and $\psi_{\gamma}^{\max}$ (b, solid curve) at which the values $\eta_{\psi}^{\max}$ and $\gamma_{\psi}^{\max}$ are achieved as functions of the orientation angle $\theta$ for a 2.1 mm thick BSO crystal with the Bragg angle equal to 7°.

Fig. 1a shows distinct jumps of the azimuth $\psi_{\eta}^{\max}$ by $\pi/2$ at points $\theta = 90$ and 270°, whereas no such jumps are observed for $\psi_{\gamma}^{\max}$ in Fig. 1b. The dependences of $\psi_{\eta}^{\min}$ and $\psi_{\gamma}^{\min}$ on the orientation angle $\theta$ are presented in Fig. 2 together with functions $\eta_{\psi}^{\min}(\theta)$ and $\gamma_{\psi}^{\min}(\theta)$. Fig. 3 shows the dependences of the diffraction efficiency $\eta$ of the hologram (Fig. 3a) and the effective amplification $\gamma$ (Fig. 3b) on the orientation angle $\theta$ for different polarisation azimuths $\psi$ together with dependences $\eta_{\psi}^{\max}(\theta)$, $\eta_{\psi}^{\min}(\theta)$ and $\gamma_{\psi}^{\max}(\theta)$, $\gamma_{\psi}^{\min}(\theta)$ (thick curves).

One can see from Fig. 3a that the functions $\eta_{\psi}^{\max}(\theta)$ and $\eta_{\psi}^{\min}(\theta)$ are the upper and lower envelopes of the family of curves $\eta(\theta)$ plotted at fixed azimuths $\psi$. The maximum diffraction efficiency at the fixed polarisation ($\psi = 0$) is nearly twice as large as the corresponding maximum at $\psi = 90°$. The absolute maxima of the diffraction efficiency are achieved at the following values of the orientation and polarisation angles: $\theta \approx 53°$, $\psi \approx 26.5$ (point A), $\theta \approx 127°$, $\psi \approx 21.6°$ (point B), $\theta \approx 233°$, $\psi \approx 26.5°$ (point C), and $\theta \approx$

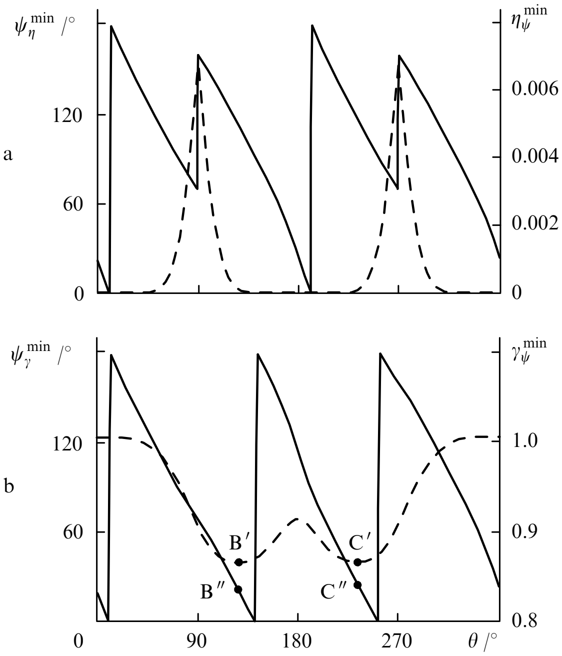


**Figure 2.** The minimum diffraction efficiency $\eta_\psi^{\min}$ (a, dashed curve), the minimum effective amplification $\gamma_\psi^{\min}$ (b, dashed curve), and the azimuths of polarisation of linearly polarised light waves $\psi_\eta^{\min}$ (a, solid curve) and $\psi_\gamma^{\min}$ (b, solid curve) at which the values $\eta_\psi^{\min}$ and $\gamma_\psi^{\min}$ are achieved as functions of the orientation angle $\theta$ for a 2.1 mm thick BSO crystal with the Bragg angle equal to 7°.

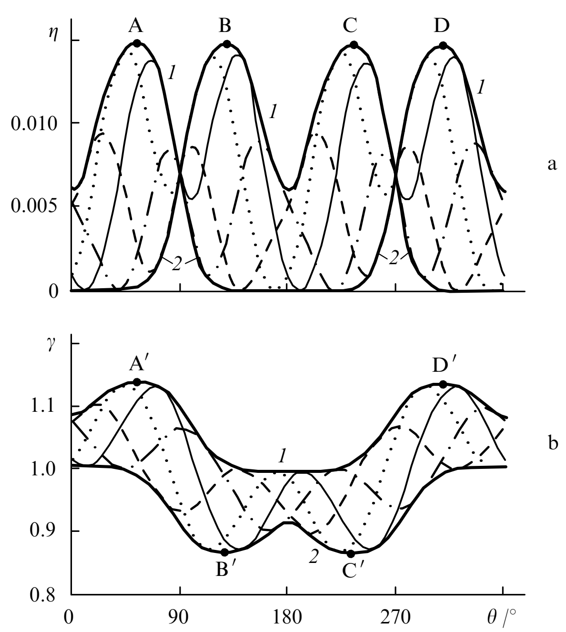


**Figure 3.** Dependences of the diffraction efficiency $\eta$ (a) and of the effective amplification $\gamma$ (b) on the orientation angle $\theta$ for azimuths of linearly polarised light waves $\psi = 0$ (solid curve), 45° (dotted curve), 90° (dashed curve), and 135° (dot-and-dash curve), and also the maximum ($\eta_\psi^{\max}, \gamma_\psi^{\max}$; *1*) and minimum ($\eta_\psi^{\min}, \gamma_\psi^{\min}$; *2*) envelopes for a 2.1 mm thick BSO crystal with the Bragg angle equal to 7°.

307°, $\psi \approx 21.6°$ (point D). Note that the crystal orientations at which the absolute maxima of the diffraction efficiency are achieved differ from traditional orientations ($\boldsymbol{K} \perp [001]$ and $\boldsymbol{K} \| [001]$). Strictly speaking, these orientations do not coincide with the direction $[1\bar{1}1]$ as well (see also [21, 23]).

Analysis of Fig. 3a also shows that the straight lines $\theta = \pi k/2$, where $k = 0, \pm 1, \pm 2, \pm 3, \ldots$, are the symmetry axis for the functions $\eta_\psi^{\max}(\theta)$ and $\eta_\psi^{\min}(\theta)$. One can see from Fig. 3b that the functions $\gamma_\psi^{\max}(\theta)$ and $\gamma_\psi^{\min}(\theta)$ are envelopes of the curves $\gamma(\theta)$ constructed at fixed initial polarisation azimuths $\psi$. As in a previous case, the maximum effective amplification $\gamma_\psi^{\max}$ at $\psi = 0$ exceeds the amplification at $\psi = 90°$. The values of the orientation and polarisation angles $\theta$ and $\psi$ corresponding to the absolute maxima of the upper envelope $\gamma_\psi^{\max}(\theta)$ are: $\theta \approx 53°$, $\psi \approx 26.5°$ (point A′), and $\theta \approx 307°$, $\psi \approx 21.6°$ (point D′). It also follows from Fig. 3b that the vertical straight lines $\theta = k\pi$ are the symmetry axes for the upper and lower envelopes.

Note that the polarisation and orientation angles at which the absolute maxima of the diffraction efficiency are achieved (points A, B, C, and D in Figs 1a and 3a) coincide with the corresponding angles at which either the absolute maxima (points A′, D′ in Figs 1b and 3b) or absolute minima (points B′, C′ in Figs 2b and 3b) of the effective amplification are achieved ($\psi_\eta^{\max}(A_1) = \psi_\gamma^{\max}(A'') = 26.5°$, $\psi_\eta^{\max}(B_1) = \psi_\gamma^{\min}(B'') = 21.6°$, $\psi_\eta^{\max}(C_1) = \psi_\gamma^{\min}(C'') = 26.5°$, $\psi_\eta^{\max}(D_1) = \psi_\gamma^{\max}(D'') = 21.6°$, see Figs 1 and 2).

To confirm the properties established above, we performed the following experiments. A light beam from a 50-mW He–Ne laser was divided with a beamsplitter into two beams (reference and probe beams) with an intensity ratio of 2 : 1. In each of the beams, a $\lambda/4$ phase plate, a polariser, and an aperture were placed. The beams intersected inside a $(\bar{1}\bar{1}0)$ BSO crystal plate. The crystal plate could be rotated around the horizontal axis coinciding with the bisectrix of the angle between the beams. As in theoretical calculations, the Bragg angle in the crystal was 7°, the specific rotary power was $\alpha = 0.4$ rad mm$^{-1}$, and the crystal thickness was $d = 2.1$ mm. The rest of the physical parameters of the BSO crystal were taken from paper [15].

Fig. 4 shows the extreme diffraction efficiencies $\eta_\psi^{\max}(\theta)$ and $\eta_\psi^{\min}(\theta)$ (Fig. 4a) and effective amplifications $\gamma_\psi^{\max}(\theta)$ and $\gamma_\psi^{\min}(\theta)$ (Fig. 4b) measured at azimuths found from Figs 1 and 2. One can see that the experimental dependences $\eta_\psi^{\max}(\theta)$ and $\eta_\psi^{\min}(\theta)$, as well $\gamma_\psi^{\max}(\theta)$ and $\gamma_\psi^{\min}(\theta)$ are in good agreement with theoretical curves.

## 4. Conclusions

We have found theoretically and experimentally that the maximum diffraction efficiency is achieved in a 2.1 mm thick BSO crystal for the following orientation and polarisation angles: $\theta \approx 53°$, $\psi \approx 26.5°$; $\theta \approx 127°$, $\psi \approx 21.6°$; $\theta \approx 233°$, $\psi \approx 26.5°$; $\theta \approx 307°$, $\psi \approx 21.6°$. The maxima of the effective amplification are observed at $\theta \approx 53°$, $\psi \approx 26.5°$ and $\theta \approx 307°$, $\psi \approx 21.6°$. The absolute minima of the effective amplification are achieved at $\theta \approx 127°$, $\psi \approx 21.6°$ and $\theta \approx 233°$, $\psi \approx 26.5°$.

Note that we have ignored in this paper absorption of light and circular dichroism. The consideration of this factors can modify somewhat the conditions of optimisation of the diffraction efficiency and the effective amplification.

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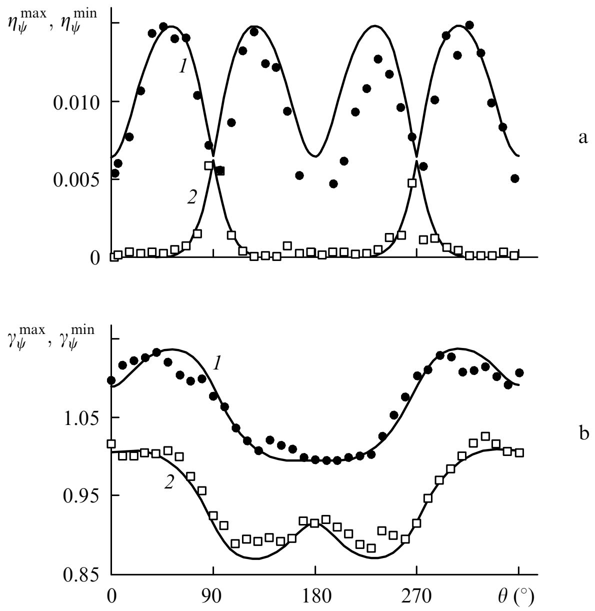


**Figure 4.** Experimental (circles and squares) and theoretical (curves) dependences of the extreme diffraction efficiencies $\eta_\psi^{max}$ (*1*) and $\eta_\psi^{min}$ (*2*) (a) and of the effective amplification $\gamma_\psi^{max}$ (*1*) and $\gamma_\psi^{min}$ (*2*) (b) on the orientation angle $\theta$ for $E = 1.93$ (a) and 0.76 kV cm$^{-1}$ (b).

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