Quantum Electronics **32** (2) 160 – 164 (2002)

PACS numbers: 42.55.Lt; 42.60.Da; 42.62.Eh
DOI: 10.1070/QE2002v032n02ABEH002149

Frequency characteristic of a uniformly rotating laser gyroscope with differently amplified counterpropagating waves

E.A.Bondarenko

Abstract. **It is shown that the frequency characteristic of a uniformly rotating laser gyroscope with differently amplified counterpropagating waves is described by the expression containing components that are commuting or noncommuting with respect to the angular velocity.**

Keywords: *ring gas laser, laser gyroscope, frequency characteristic.*

1. Introduction

Among laser gyroscopes of different types, a gyroscope based on a ring He – Ne gas laser containing equal amounts of isotopes ^{20}Ne and ^{22}Ne can be distinguished. This laser uses the n-mirror resonator ($n = 3$, 4), which provides output radiation that is linearly polarised in the sagittal plane. The laser, which emits, as a rule, at 0.6328 μm, is pumped by a dc discharge using a symmetric scheme (one cathode – two anodes).

This paper is devoted to the calculation of the frequency characteristic of a uniformly rotating laser gyroscope of this type with differently amplified counterpropagating waves (the different amplification being caused, for example, by different Q factors of the resonator for counterpropagating waves).

The calculation was stimulated by a search for the answer to the question of whether the different amplification of counterpropagating waves in a laser gyroscope, which are coupled through backscattering and inhomogeneously distributed losses, can result in the appearance of a noncommuting component of the frequency characteristic, which, being an even function of the angular velocity, does not change its sign upon rotation in the opposite direction?

Analysis of the results of papers known to the author, in which the frequency characteristic of a uniformly rotating laser gyroscope was studied, does not give a definite answer to this question (in the form of the calculated expression).

E.A.Bondarenko ‘Ritm’ Interindustry Research Institute of Problems of Mechanics, Kiev Polytechnical Institute, National Technical University of Ukraine, prosp. Pobedy 37, KPI-4030, korp. 28, 03056 Kiev, Ukraine; e-mail: ritm@ukrpost.net

Received 26 December 2001
Kvantovaya Elektronika **32** (2) 160 – 164 (2002)
Translated by M.N.Sapozhnikov

In this paper, the frequency characteristic was calculated in the approximation of weakly coupled counterpropagating waves with an accuracy to the terms of the second-order smallness over coupling coefficients. The semiclassical equations for the laser gyroscope were used, which are valid for a small excess of the pump energy over the threshold. The frequency characteristic was calculated under the following assumptions: (i) a laser gyroscope operates far from the boundaries of the capture zone; (ii) currents in the discharge arms of the laser gyroscope are balanced; and (iii) the laser gyroscope resonator is tuned to the centre of the emission line.

2. Basic relations

The frequency characteristic was calculated using differential equations (6.45) – (6.47) from paper [1], which are equivalent to equations (5.55) – (5.57) from paper [2]. In the case of different amplification of counterpropagating waves, these equations can be written in the form

$$\dot{I}_1 = (\alpha_1 - \beta I_1 - \theta I_2)I_1 - 2r_2(I_1 I_2)^{1/2}\cos(\psi + \varepsilon_2),$$

$$\dot{I}_2 = (\alpha_2 - \beta I_2 - \theta I_1)I_2 - 2r_1(I_1 I_2)^{1/2}\cos(\psi - \varepsilon_1), \qquad (1)$$

$$\dot{\psi} = \omega + r_2\left(\frac{I_2}{I_1}\right)^{1/2}\sin(\psi + \varepsilon_2) + r_1\left(\frac{I_1}{I_2}\right)^{1/2}\sin(\psi - \varepsilon_1),$$

where

$$\omega = M_{\rm g}\Omega + \sigma_2 - \sigma_1 = M\Omega; \qquad (2)$$

$M_{\rm g} = 8\pi A/\lambda L$ is the geometrical scale factor of the laser gyroscope; $I_{1,2}$ are the dimensionless intensities of the counterpropagating waves (the wave with the subscript 1 propagates in the positive direction of the laser gyroscope rotation); ψ is the instantaneous phase difference of the counterpropagating waves; $\dot{\psi}$ is the instantaneous circular beat frequency of the counterpropagating waves; $\alpha_{1,2}$, β, and θ are the Lamb coefficients characterising the excess of amplification over losses for each of the counterpropagating waves, their self-saturation, and mutual saturation, respectively; $r_{1,2}$ and $\varepsilon_{1,2}$ are the moduli and arguments of complex integral coupling coefficients for the counterpropagating waves (for coupling through backscattering from the resonator mirrors and through the losses inhomogeneously distributed along the axial contour); ω is the splitting of the circular frequencies of the counterpropagating waves caused by the rotation of the laser gyroscope in the inertial space with the angular velocity Ω and calculated by neglecting the coupling between the waves; A is the area of the axial contour; L is the perimeter of the axial contour; λ is the emission wavelength; $\sigma_{1,2}$ are the Lamb coefficients determining a small correction to the geometrical scale factor; and M is the scale factor of the laser gyroscope taking into account the influence of the active medium.

Let us represent coefficients $\alpha_{1,2}$ in equations for $I_{1,2}$ in the form

$$\alpha_{1,2} = \alpha \mp \delta, \tag{3}$$

where the parameter δ characterises the difference of the amplification of the counterpropagating waves. It follows from (3) that

$$\alpha = \frac{\alpha_2 + \alpha_1}{2}, \quad \delta = \frac{\alpha_2 - \alpha_1}{2}.$$

Let us introduce the parameters $\alpha_p = \alpha$ and $\alpha_m = \alpha_p(\beta - \theta)/(\beta + \theta)$, which characterise the reciprocal relaxation times of the sum and difference of the intensities of counterpropagating waves, respectively, and the dimensionless parameter $D = \delta/\alpha_m$, which satisfies the condition

$$|D| < 1, \tag{4}$$

of the stable generation of two counterpropagating waves by neglecting their coupling (see comments to expression (8.7) in paper [1]).

3. Statement of the problem

We will describe the frequency characteristic of a uniformly rotating laser gyroscope by the expression

$$\frac{dN}{dt} = \frac{k_f}{2\pi}\,\omega_{beat}, \tag{5}$$

where k_f is the 'frequency multiplication' coefficient; dN/dt is the repetition rate of information pulses N from the inertial output of the laser gyroscope; and ω_{beat} is the circular beat frequency of counterpropagating waves.

We assume that the parameter ω_{beat} is related to the instantaneous circular frequency $\dot{\psi}$ of counterpropagating waves by the expression

$$\omega_{beat} = \langle \dot{\psi} \rangle, \tag{6}$$

in which $\langle \ldots \rangle$ is the time averaging operator.

The expression for ω_{beat} required for the calculation of the frequency characteristic of the laser gyroscope should be obtained, according to (6), by solving the system of equations (1) taking into account (3) and (4). The calculations will be performed with an accuracy to the second-order terms in $r_{1,2}$.

4. Calculation of the frequency characteristic of a laser gyroscope

We will solve the system of equations (1) by the method of successive approximations. In the zero order in $r_{1,2}$ (neglecting coupling between counterpropagating waves), the first two equations of the system (1) have the form

$$\dot{I}_{10} = (\alpha_1 - \beta I_{10} - \theta I_{20})I_{10},$$
$$\dot{I}_{20} = (\alpha_2 - \beta I_{20} - \theta I_{10})I_{20}. \tag{7}$$

In the stationary regime ($\dot{I}_{10} = \dot{I}_{20} = 0$), we obtain

$$I_{10} = (1 - D)U, \quad I_{20} = (1 + D)U, \tag{8}$$

where $I_{10,20}$ are the constant components of the intensities of counterpropagating waves and $U = \alpha/(\beta + \theta)$.

Using the relations

$$I_1 = I_{10} + u - v, \quad I_2 = I_{20} + u + v \tag{9}$$

we introduce the new variables u and v, which represent the components (u is the half-sum and v is the half-difference) of the intensities of counterpropagating waves, which depend on their coupling through backscattering and inhomogeneously distributed losses.

By substituting (9) into (1), we obtain the system of differential equations for u, v, and ψ:

$$\dot{u} + \alpha_p u + D\alpha_m v = -EUr_p\cos(\psi + \varphi_p),$$
$$\dot{v} + \alpha_m v + D\alpha_p u = EUr_m\cos(\psi + \varphi_m), \tag{10}$$
$$\dot{\psi} = \omega + E^{-1}\{[1 - D(E^2U)^{-1}(Du - v)]r_p\sin(\psi + \varphi_p)$$
$$+ [D - (E^2U)^{-1}(Du - v)]r_m\sin(\psi + \varphi_m)\}.$$

The right-hand sides of these equations contain terms that are necessary and sufficient for calculating the frequency characteristic with the given accuracy. Here,

$$E = (I - D^2)^{1/2}; \tag{11}$$

$$r_p = [r_1^2 + r_2^2 + 2r_1r_2\cos(\varepsilon_1 + \varepsilon_2)]^{1/2};$$
$$r_m = [r_1^2 + r_2^2 - 2r_1r_2\cos(\varepsilon_1 + \varepsilon_2)]^{1/2};$$
$$\tan\varphi_p = \frac{r_2\sin\varepsilon_2 - r_1\sin\varepsilon_1}{r_2\cos\varepsilon_2 + r_1\cos\varepsilon_1}; \tag{12}$$
$$\tan\varphi_m = \frac{r_2\sin\varepsilon_2 + r_1\sin\varepsilon_1}{r_2\cos\varepsilon_2 - r_1\cos\varepsilon_1}.$$

It follows from (12) that

$$r_p\sin\varphi_p = r_2\sin\varepsilon_2 - r_1\sin\varepsilon_1,$$
$$r_p\cos\varphi_p = r_2\cos\varepsilon_2 + r_1\cos\varepsilon_1,$$
$$r_m\sin\varphi_m = r_2\sin\varepsilon_2 + r_1\sin\varepsilon_1, \tag{13}$$
$$r_m\cos\varphi_m = r_2\cos\varepsilon_2 - r_1\cos\varepsilon_1.$$

We will find $\langle\dot{\psi}\rangle$ from the third equation of system (10) by seeking its solution in the form

$$\psi = \omega t + \psi_1 + \psi_2 \,, \tag{14}$$

where the subscripts 1 and 2 show that the corresponding variable (which is small compared to ωt) is a quantity of the first and second order of smallness in $r_{1,2}$, respectively. It follows from (14) that

$$\dot{\psi} = \omega + \dot{\psi}_1 + \dot{\psi}_2 \,. \tag{15}$$

To calculate the frequency characteristic of the rotating laser gyroscope, it is necessary to determine $\langle\dot{\psi}_1\rangle$ and $\langle\dot{\psi}_2\rangle$ and then, according to (6), to use the expression

$$\omega_{\text{beat}} = \langle\dot{\psi}\rangle = \omega + \langle\dot{\psi}_1\rangle + \langle\dot{\psi}_2\rangle. \tag{16}$$

Let us find first the quantity $\langle\dot{\psi}_1\rangle$. By substituting (14) into the third equation of system (10) and retaining in it the terms of the first order in $r_{1,2}$, we obtain the differential equation for ψ_1

$$\dot{\psi}_1 = E^{-1}[r_\text{p}\sin(\omega t + \varphi_\text{p}) + Dr_\text{m}\sin(\omega t + \varphi_\text{m})] \,. \tag{17}$$

By averaging (17) over time, we find $\langle\dot{\psi}_1\rangle = 0$.

The differential equation for ψ_2 can be found similarly:

$$\dot{\psi}_2 = E^{-1}\big\{\psi_1[r_\text{p}\cos(\omega t + \varphi_\text{p}) + Dr_\text{m}\cos(\omega t + \varphi_\text{m})] - \big(E^2U\big) - 1(Du - v)[Dr_\text{p}\sin(\omega t + \varphi_\text{p}) + r_\text{m}\sin(\omega t + \varphi_\text{m})]\big\}. \tag{18}$$

The quantity $\langle\dot{\psi}_2\rangle$ is determined by substituting the expressions for ψ_1, u, and v into the right-hand side of (18) and by averaging the result over time.

The expression for ψ_1 obtained from (17) has the form

$$\psi_1 = -\frac{1}{\omega}E^{-1}[r_\text{p}\cos(\omega t + \varphi_\text{p}) + Dr_\text{m}\cos(\omega t + \varphi_\text{m})]. \tag{19}$$

The expressions for u and v can be obtained by solving the first two differential equations of system (10), by setting $\psi = \omega t$ in their right-hand sides. In the stationary regime, we have

$$u = A_u(\omega)\sin\omega t + B_u(\omega)\cos\omega t, \qquad v = A_v(\omega)\sin\omega t + B_v(\omega)\cos\omega t, \tag{20}$$

where

$$A_{u,v}(\omega) = \frac{a_{u,v}(\omega)}{G(\omega)}; \quad B_{u,v}(\omega) = \frac{b_{u,v}(\omega)}{G(\omega)}; \tag{21}$$

$$a_u(\omega) = EU\big(A_0 + A_1u\omega + A_2^u\omega^2 + A_3^u\omega^3\big);$$
$$b_u(\omega) = EU\big(B_0^u + B_1^u\omega + B_2^u\omega^2 + B_3^u\omega^3\big);$$
$$a_v(\omega) = EU\big(A_0^v + A_1^v\omega + A_2^v\omega^2 + A_3^v\omega^3\big);$$
$$b_v(\omega) = EU\big(B_0^v + B_1^v\omega + B_2^v\omega^2 + B_3^v\omega^3\big). \tag{22}$$

Here,

$$A_0^u = E^2\alpha_\text{p}\alpha_\text{m}^2(P_\text{s} + DM_\text{s}); \; B_0^u = -E^2\alpha_\text{p}\alpha_\text{m}^2(P_\text{c} + DM_\text{c});$$
$$A_0^v = -E^2\alpha_\text{p}^2\alpha_\text{m}(DP_\text{s} + M_\text{s}); \; B_0^v = E^2\alpha_\text{p}^2\alpha_\text{m}(DP_\text{c} + M_\text{c});$$
$$A_1^u = -\alpha_\text{m}[D\alpha_\text{p}(DP_\text{c} + M_\text{c}) + \alpha_\text{m}(P_\text{c} + DM_\text{c})];$$
$$B_1^u = -\alpha_\text{m}[D\alpha_\text{p}(DP_\text{s} + M_\text{s}) + \alpha_\text{m}(P_\text{s} + DM_\text{s})];$$
$$A_1^v = \alpha_\text{p}[\alpha_\text{p}(DP_\text{c} + M_\text{c}) + D\alpha_\text{m}(P_\text{c} + DM_\text{c})]; \tag{23}$$
$$B_1^v = \alpha_\text{p}[\alpha_\text{p}(DP_\text{s} + M_\text{s}) + D\alpha_\text{m}(P_\text{s} + DM_\text{s})];$$
$$A_2^u = (\alpha_\text{p}P_\text{s} - D\alpha_\text{m}M_\text{s}); \quad B_2^u = -(\alpha_\text{p}P_\text{c} - D\alpha_\text{m}M_\text{c});$$
$$A_2^u = (D\alpha_\text{p}P_\text{s} - \alpha_\text{m}M_\text{s}); \quad B_2^v = -(D\alpha_\text{p}P_\text{c} - \alpha_\text{m}M_\text{c});$$
$$A_3^u = -P_\text{c}; \quad B_3^u = -P_\text{s}; \quad A_3^v = M_\text{c}; \quad B_3^v = M_\text{s}.$$

The parameters

$$P_\text{s} = r_\text{p}\sin\varphi_\text{p}, \quad P_\text{c} = r_\text{p}\cos\varphi_\text{p}, \quad M_\text{s} = r_\text{m}\sin\varphi_\text{m}, \quad M_\text{c} = r_\text{m}\cos\varphi_\text{m} \,, \tag{24}$$

appearing in expressions (23) can be calculated from relations (13).

The expression for the parameter $G(\omega)$ in (21) has the form

$$G(\omega) = E^4\alpha_\text{p}^2\alpha_\text{m}^2 + \big(\alpha_\text{p}^2 + \alpha_\text{m}^2 + 2D^2\alpha_\text{p}\alpha_\text{m}\big)\omega^2 + \omega^4. \tag{25}$$

Thus, we have all the necessary components for determining $\langle\dot{\psi}_2\rangle$. [We do not present here the expression for $\langle\dot{\psi}_2\rangle$; it will be taken into account in the final expression for ω_beat, which we will obtain from (16).]

5. The result of calculation of the frequency characteristic of a laser gyroscope

The frequency characteristic of a laser gyroscope uniformly rotating in inertial space with the angular velocity Ω is calculated from expression (5). We should substitute into the right-hand side of (5) the following expression for the angular beat frequency ω_beat of counterpropagating waves:

$$\omega_\text{beat} = \left[1 - \frac{R_\text{p}^2}{2E^2\omega^2} + \frac{H(\omega)}{2E^2G(\omega)}\right]\omega + D\alpha_\text{m}r_1r_2\sin\varepsilon_{12}\,\frac{E^2\alpha_\text{p}\alpha_\text{m} - \omega^2}{G(\omega)}, \tag{26}$$

where

$$R_\text{p} = \big(R_1^2 + R_2^2 + 2R_1R_2\cos\varepsilon_{12}\big)^{1/2};$$
$$R_\text{m} = \big(R_1^2 + R_2^2 - 2R_1R_2\cos\varepsilon_{12}\big)^{1/2};$$
$$R_1 = (1 - D)r_1; \quad R_2 = (1 + D)r_2; \quad \varepsilon_{12} = \varepsilon_1 + \varepsilon_2; \tag{27}$$

$$H(\omega) = (\alpha_p^2 + D^2\alpha_p\alpha_m + \omega^2)R_m^2 + D\alpha_m(\alpha_p + \alpha_m)(R_2^2 - R_1^2).$$

Expression (26) together with (27) is the main result of this work. In particular, it follows from (26) that the frequency characteristic of the laser gyroscope under study contains the component commuting with respect to the angular velocity (the first term) and the noncommuting component (the second term).

Expression (26) is valid for a sufficiently broad range of values of the parameter D satisfying the condition $D < 1$, and it can be useful when the different amplification of counterpropagating waves is deliberately created to study the operation of a laser gyroscope in this regime [4] (see the numerical example below).

However, for practical designs of laser gyroscopes the regime is typical at which $|D| \ll 1$ ($D = 0$ in an ideal gyroscope). This circumstance allows us to consider the expression for $\omega_{\rm beat}$ in the first order in D. In this approximation,

$$E = 1,\ R_p^2 = r_p^2 + 2D(r_2^2 - r_1^2),\ R_m^2 = r_m^2 + 2D(r_2^2 - r_1^2),$$

$$R_2^2 - R_1^2 = r_2^2 - r_1^2 + 2D(r_2^2 + r_1^2), \tag{28}$$

$$\omega_{\rm beat} = \left[1 - \frac{r_p^2}{2\omega^2} + \frac{r_m^2}{2(\alpha_m^2 + \omega^2)}\right]\omega + D(r_2^2 - r_1^2)\left\{-\frac{1}{\omega^2} + \frac{1}{\alpha_m^2 + \omega^2}\left[1 + \frac{\alpha_m(\alpha_p + \alpha_m)}{2(\alpha_p^2 + \omega^2)}\right]\right\}\omega + D\alpha_m r_1 r_2 \sin\varepsilon_{12}\,\frac{\alpha_p\alpha_m - \omega^2}{(\alpha_p^2 + \omega^2)(\alpha_m^2 + \omega^2)}. \tag{29}$$

The first term in the right-hand side of expression (29) is well-known in the literature and was probably first presented in paper [5] (see expression (10) in [5] and comments to it). The second term in the right-hand side of expression (29) is known from paper [6] (see expression (23) in [6]).

6. Numerical example

Let us estimate quantitatively the nonlinearity of the frequency characteristic of a laser gyroscope as a whole and the noncommuting component of this nonlinearity. For convenience, we will consider the relative quantities rather than the absolute ones.

Let us rewrite expression (26) for $\omega_{\rm beat}$ in the form

$$\omega_{\rm beat} = [1 + K_{\rm nl}(\omega)]\omega \quad (\omega = M\Omega). \tag{30}$$

Here, the parameter $K_{\rm nl}(\omega)$ characterises the relative nonlinearity of the frequency characteristic.

In turn, $K_{\rm nl}(\omega)$ can be written as

$$K_{\rm nl}(\omega) = K_{\rm sym}(\omega) + K_{\rm asym}(\omega), \tag{31}$$

where

$$K_{\rm sym}(\omega) = -\frac{R_p^2}{2E^2\omega^2} + \frac{H(\omega)}{2E^2G(\omega)}, \tag{32}$$

$$K_{\rm asym}(\omega) = D\alpha_m r_1 r_2 \sin\varepsilon_{12}\,\frac{E^2\alpha_p\alpha_m - \omega^2}{\omega G(\omega)}. \tag{33}$$

One can see from (32) that $K_{\rm sym}(\omega)$ is an even function of ω and characterises the symmetric component of the relative nonlinearity of the frequency characteristic.

The parameter $K_{\rm asym}(\omega)$ (33) is an odd function of ω and characterises the asymmetric component of the relative nonlinearity of the frequency characteristic. It also follows from (33) that $K_{\rm asym}(\omega)$ changes its sign at points $\omega_\otimes = \pm E(\alpha_p\alpha_m)^{1/2}$ or, in terms of the angular velocity, at points

$$\Omega_\otimes = \pm E(\alpha_p\alpha_m)^{1/2}M^{-1}. \tag{34}$$

The parameters $K_{\rm nl}(\omega)$ and $K_{\rm asym}(\omega)$ belong to the metrological parameters of laser gyroscopes [7]. Let us estimate these parameters as functions of the angular velocity Ω for different values of D.

Let us first write the expression for calculating the parameter D from the given coefficient $F = I_{20}/I_{10}$ characterising the ratio of the constant components of intensities of counterpropagating waves. According to (8), this expression has the form

$$D = \frac{F-1}{F+1}. \tag{35}$$

In particular, it follows from (35) that, when the amplification of counterpropagating waves is the same ($F = 1$), we have $D = 0$.

As an example, we consider a laser gyroscope that was studied theoretically and experimentally in paper [8]. The gyroscope cavity has the form of an equilateral triangle with the nominal parameter $I_0 = 210$ mm. (We will use the parameter L equal to 215.5 mm. In this case, the calculated 'arc pulse value' of the laser gyroscope will correspond to the value 3.147″ reported in [8]. In addition, by introducing a small error to calculations, we will neglect the dispersion coefficients $\sigma_{1,2}$ in (2), thereby assuming that $M = M_g$. Then, for the given value of L, the scale factor is $M = 411793$.) Let us assume that there is a device inside the gyroscope cavity that allows us to redistribute asymmetrically the losses for counterpropagating waves, by retaining their average value invariable.

Let us first calculate the parameters α_p and α_m. According to [8], $\alpha_p = (c/L)\gamma(N-1)$, where c is the speed of light; L is the perimeter of the axial contour; γ are losses per transit; and N is the relative excess of the pump energy over the threshold. We assume that $N = 1.45$ and $= 1.8\times10^{-3}$. Then, $\alpha_p = 2\pi\times179465\ {\rm s}^{-1}$, which corresponds to $\Omega_{\alpha_p} = 156.9\ {\rm s}^{-1}$. For the laser gyroscope under study,

$$\frac{\beta-\theta}{\beta+\theta} = \frac{2.228 - 1.564}{2.228 + 1.564} = 0.175,$$

so that $\alpha_m = 2\pi\times31425\ {\rm c}^{-1}$ ($\Omega_{\alpha_m} = 27.5^\circ{\rm s}^{-1}$).

Let us specify now the values of coupling parameters for counterpropagating waves. According to [8], $(L/c)r_1 = (L/c)r_2 = 3\times10^{-6}$, which gives $r_1 = r_2 = 2\pi\times665\ {\rm s}^{-1}$ ($\Omega_{r_1} = \Omega_{r_2} = 0.58^\circ\ {\rm s}^{-1}$). Let $\varepsilon_{12} = 175^\circ$. Then, according to (12) we have $r_p = 2\pi\times58\ {\rm c}^{-1}$ ($\Omega_{r_p} = 0.05^\circ\ {\rm s}^{-1}$) and $r_m = 2\pi\times1328\ {\rm c}^{-1}$ ($\Omega_{r_m} = 1.16^\circ\ {\rm s}^{-1}$).

Expressions (31)–(35) in combination with the above parameters of the laser gyroscope give the numerical

Table 1. Dependence of parameters of the relative nonlinearity of the frequency characteristic of the laser gyroscope on the angular rotation velocity for different amplifications of counterpropagating waves..

$\Omega/^\circ\ s^{-1}$	60	120	180	240	300	360
$\omega/2\pi/Hz$	68632	137264	205896	274529	343161	411793
$F = 1.01 \quad D = 5 \times 10^{-3} \quad \Omega_{\otimes} = \pm 65.7^\circ\ s^{-1}$						
K_{nl}	1.5×10^{-4}	4.4×10^{-5}	2.0×10^{-5}	1.2×10^{-5}	7.4×10^{-6}	5.2×10^{-6}
K_{asym}	3.9×10^{-10}	-5.7×10^{-10}	-3.3×10^{-10}	-1.9×10^{-10}	-1.1×10^{-10}	-7.0×10^{-11}
ρ	2.5×10^{-6}	-1.3×10^{-5}	-1.6×10^{-5}	-1.6×10^{-5}	-1.5×10^{-5}	-1.3×10^{-5}
$F = 2.0 \quad D = 0.33 \quad \Omega_{\otimes} = \pm 61.9^\circ\ s^{-1}$						
K_{nl}	1.6×10^{-4}	4.5×10^{-5}	2.0×10^{-5}	1.2×10^{-5}	7.4×10^{-6}	5.2×10^{-6}
K_{asym}	8.5×10^{-9}	-3.9×10^{-8}	-2.2×10^{-8}	-1.2×10^{-8}	-7.4×10^{-9}	-4.7×10^{-9}
ρ	5.4×10^{-5}	-8.9×10^{-4}	-1.1×10^{-3}	-1.1×10^{-3}	-1.0×10^{-3}	-9.0×10^{-4}
$F = 3.0 \quad D = 0.5 \quad \Omega_{\otimes} = \pm 56.9^\circ\ s^{-1}$						
K_{nl}	1.6×10^{-4}	4.5×10^{-5}	2.0×10^{-5}	1.2×10^{-5}	7.4×10^{-6}	5.2×10^{-6}
K_{asym}	-2.0×10^{-8}	-6.1×10^{-8}	-3.4×10^{-8}	-1.9×10^{-8}	-1.1×10^{-8}	-7.0×10^{-9}
ρ	-1.3×10^{-4}	-1.4×10^{-3}	-1.7×10^{-3}	-1.6×10^{-3}	-1.5×10^{-3}	-1.4×10^{-3}
$F = 4.0 \quad D = 0.6 \quad \Omega_{\otimes} = \pm 52.5^\circ\ s^{-1}$						
K_{nl}	1.6×10^{-4}	4.5×10^{-5}	2.0×10^{-5}	1.2×10^{-5}	7.4×10^{-6}	5.2×10^{-6}
K_{asym}	-5.5×10^{-8}	-7.5×10^{-8}	-4.0×10^{-8}	-2.2×10^{-8}	-1.3×10^{-8}	-8.3×10^{-9}
ρ	-3.4×10^{-4}	-1.7×10^{-3}	-2.0×10^{-3}	-1.9×10^{-3}	-1.9×10^{-3}	-1.6×10^{-3}
$F = 5.0 \quad D = 0.67 \quad \Omega_{\otimes} = \pm 48.9^\circ\ s^{-1}$						
K_{nl}	1.7×10^{-4}	4.5×10^{-5}	2.0×10^{-5}	1.2×10^{-5}	7.4×10^{-6}	5.2×10^{-6}
K_{asym}	-8.7×10^{-8}	-8.5×10^{-8}	-4.5×10^{-8}	-2.5×10^{-8}	-1.5×10^{-8}	-9.3×10^{-9}
ρ	-5.2×10^{-4}	-1.9×10^{-3}	-2.2×10^{-3}	-2.1×10^{-3}	-2.0×10^{-3}	-1.8×10^{-3}

estimates of parameters K_{nl} and K_{asym} and of their ratio $\rho = K_{asym}/K_{nl}$. The results are summarised in Table 1.

References

1. Menegozzi L.N., Lamb W.E. Jr. *Phys. Rev.*, **8**, A2103 (1973).
2. Aronowitz F., in *Optical Gyros and their Application* (RTO AGARDograph 339, 1999) p. 3-1.
3. Aronowitz F., Collins R.J. *J. Appl. Phys.*, **41**, 130 (1970).
4. Lee P.H., Atwood J.G. *IEEE J. Quantum Electron.*, **2**, 235 (1966).
5. Landa P.S., Lariontsev E.G. *Radiotekhnika i Elektronika*, **15**, 1214 (1970).
6. Birman A.Ya., Naumov P.B., Savushkin A.F. *Kvantovaya Elektron.*, **8**, 2454 (1981) [*Sov. J. Quantum Electron.*, **11**, 1498 (1981)].
7. *IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Laser Gyros* (IEEE Std 647-1981).
8. Aronowitz F., Lim W.L. *IEEE J. Quantum Electron.*, **13**, 338 (1977).