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Second harmonic generation in periodically poled crystals in the fixed-intensity approximation

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***Abstract*. The SHG efficiency is calculated in periodically poled crystals in the fixed-intensity approximation. It is shown that, as in homogeneous crystals, the solution obtained in this approximation coincides, with an accuracy of the factor $(l/L)^6$, with the exact solution for a nonlinear regime even when the quasi-phase-matching condition is exactly fulfilled. The relative accuracy of this approximation increases with increasing mismatch (l is the crystal length and L is the nonlinear length). The fixed-intensity approximation can be used for such crystals up to $l \approx L$ even in the case of exact quasi-phase matching, while the fixed-field approximation is valid only for $l < 0.3L$.**

Keywords: *second harmonic generation, periodically poled structure, quasi-phase matching.*

1. Introduction

Periodically poled crystals (PPCs) attract considerable scientific and practical interest as efficient converters of laser radiation to optical harmonics and parametric waves. In such crystals, the so-called quasi-phase matching is realised [1]. The PPC represents an artificially constructed system of sequentially arranged domains with the antiparallel direction of spontaneous polarisation. The domain size in the propagation direction of radiation is exactly equal to the so-called coherence length at which the generalised phase shift between the fundamental wave and the second-harmonic wave achieves 90° [2].

The PPCs have a number of advantages over homogeneous crystals. For example, they can be manufactured by using nonlinear media, which do not possess the conventional phase matching (including isotropic media), and new components of the nonlinear tensor of the type d_{33} (which are not available in conventional crystals). In addition, there are no restrictions on the polarisation of the waves in PPCs, several optical harmonics can be simultaneously generated in one crystal, parametric generation of light with multiple frequencies can be performed, etc. [1].

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In the fixed-field approximation, which is widely used in SHG calculations in PPCs, the coherence length of the crystal depends only on the mismatch of the wave vectors, while the complex amplitude of the laser radiation field is assumed constant. In other words, both the real amplitude and phase of the fundamental wave are assumed constant. This approximation substantially simplifies calculations; however, information on the nonlinear character of the interaction is lost and a number of important features of SHG disappear [2]. The correctness and the region of application of this approximation for calculating PPCs are not *a priori* obvious.

Of course, this problem can be solved by direct computer-aided numerical calculations of a sequence of nonlinear truncated equations for SHG in each domain [2]. It is also interesting to obtain exact solutions for SHG in PPCs (for a crystal as a whole) in terms of special functions, which are used in the SHG theory in homogeneous crystals [3]. In this paper, we use for analytic calculations the so-called *fixed-intensity approximation* for the fundamental radiation in which only the real amplitude of the fundamental radiation is assumed constant, but not its phase [4] (see also [2]). In this approximation, the coherence length depends not only on the mismatch, but also on the so-called *nonlinear length*, i.e., on the nonlinearity parameters and the amplitude of the fundamental radiation field [2].

2. Basics equations and their solution in the fixed-intensity approximation

An ideal PPC has the 'lattice' period $\Lambda = 2ml_c$, where $l_c = \pi/\Delta k$ is the coherence length in the domain in the fixed-field approximation; $\Delta k = k_2 - 2k_1$ is the wave detuning for phase matching of type I in the domain; $k_{1,2}$ is the wave numbers for the fundamental (1) and second (2) harmonics; and m is the quasi-phase-matching order. As a rule, $\Delta k \gg 2\sigma_1 U$ in PPCs, where $U^2 = [a_1(0)]^2 + [a_2(0)]^2\sigma_2/\sigma_1$ is a constant, which is one of the two exact integrals of the system of nonlinear truncated equations [2]; $\sigma_{1,2}$ are nonlinear coupling coefficients; and $a_{1,2}(0)$ are the amplitudes of the waves at the domain exit.

It was shown in our paper [3] that, by making the appropriate change of variables in equations describing SHG in conventional homogeneous crystals, we can obtain similar equations that describe SHG in PPCs as a whole by making the change of variables

$$\xi \to \frac{2\xi}{m\pi}, \quad \Delta_1 \to -\frac{m\pi\beta}{2},$$

where $\xi = \sigma_1 U z$ is the reduced length in the interaction direction z; $\Delta_1 = \Delta k/2\sigma_1 U$ is the reduced wave detuning in the domain; $\beta = \Delta_1 \Delta k'/\Delta k$ is the reduced generalised mismatch (in the crystal as a whole); $\Delta k' = \Delta k - K_m$ is the generalised SHG mismatch in the PPC as a whole; and $K_m = 2\pi m/\Lambda$ is the wave number (the modulus of the reciprocal lattice vector) of the PPC, i.e., $\Delta k' = \Delta k - 2\pi m/\Lambda$.

The truncated equations with these new variables for the complex amplitudes $A_{1,2}$ of plane interacting waves for SHG in PPCs in the absence of absorption have the form [3]

$$\frac{dA_1}{dz} = -i\sigma_1 A_1^* A_2 \exp[-i(\Delta k - 2\pi m/\Lambda)z],$$
$$\frac{dA_2}{dz} = -i\sigma_2 A_1^2 \exp[i(\Delta k - 2\pi m/\Lambda)z]. \quad (1)$$

Following [2, 4], we differentiate equations of system (1) with respect to z,

$$\frac{d^2 A_1}{dz^2} = -i\sigma_1\left[A_2 \frac{dA_1^*}{dz} + A_1^* \frac{dA_2}{dz} - i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)A_1^* A_2\right] \times \exp\left[-i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)z\right],$$
$$\frac{d^2 A_2}{dz^2} = -i\sigma_2\left[2A_1 \frac{dA_1}{dz} + i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)A_1^2\right] \times \exp\left[i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)z\right], \quad (2)$$

and introduce the wave intensities

$$I_1 = A_1 A_1^* = a_1^2, \qquad I_2 = A_2 A_2^* = a_2^2. \quad (3)$$

By using (3), we rewrite (2) in the form

$$\frac{d^2 A_1}{dz^2} + i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\frac{dA_1}{dz} - \sigma_1(\sigma_1 I_2 - \sigma_2 I_1)A_1 = 0,$$
$$\frac{d^2 A_2}{dz^2} - i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\frac{dA_2}{dz} + 2\sigma_1\sigma_2 A_2 I_1 = 0. \quad (4)$$

The fixed-intensity approximation for the fundamental radiation means that $I_1(z) \equiv I_1(0) = I_{10} = a_1^2(0)$, and the equation for A_2 in (4) takes the form

$$\frac{d^2 A_2}{dz^2} - i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\frac{dA_2}{dz} + 2\sigma_1\sigma_2 A_2 I_{10} = 0. \quad (5)$$

Let us introduce the boundary conditions at the entrance to the PPC as a whole:

$$A_2(0) = 0, \quad \frac{dA_2}{dz} = -i\sigma_2 A_1^2(0). \quad (6)$$

The solution of equation (5) can be represented in the form

$$A_2(z) = -i\sigma_2 A_1^2(0) z \exp\left[i\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\frac{z}{2}\right]\operatorname{sinc}(Qz), \quad (7)$$

where

$$Q = \left\{\left[\frac{1}{2}\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\right]^2 + 2\sigma_1\sigma_2 I_{10}\right\}^{1/2}. \quad (8)$$

For a sufficiently large generalised (i.e., for the crystal as a whole) wave mismatch $(\Delta k - 2\pi m/\Lambda) \gg (2\sigma_1\sigma_2 I_{10})^{1/2}$, the fixed-intensity approximation coincides with the fixed-field approximation [2].

Let us introduce the real amplitude and phase of the second-harmonic field and rewrite equation (7), taking into account that $-i = \exp(-i\pi/2)$, in the form

$$a_2(z)\exp[i\varphi_2(z)] = \sigma_2 a_1^2(0) z \operatorname{sinc}(Qz) \times \exp\left\{i\left[2\varphi_1(0) - \frac{\pi}{2} + \frac{1}{2}\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\right]\right\}. \quad (9)$$

This gives

$$a_2(z) = \sigma_2 a_1^2(0) z \operatorname{sinc}(Qz), \quad (10)$$

$$\varphi_2(z) = 2\varphi_1(0) - \frac{\pi}{2} + \frac{1}{2}\left(\Delta k - \frac{2\pi m}{\Lambda}\right). \quad (11)$$

In the fixed-field approximation, we have instead of (10) [2]

$$a_2(z) = \sigma_2 a_1^2(0) z \operatorname{sinc}\left[\frac{1}{2}\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\right]. \quad (12)$$

Let us find the solution for the phase $\varphi_1(z)$ of the fundamental radiation in the fixed-intensity approximation. By introducing the real amplitude and phase, we obtain from the first equation of system (1)

$$\frac{da_1}{dz} + i a_1\left(\frac{d\varphi_1}{dz}\right) = -i\sigma_1 a_1 a_2 \exp\left\{i\left[\varphi_2 - 2\varphi_1 - \left(\Delta k - \frac{2\pi m}{\Lambda}\right)z\right]\right\}, \quad (13)$$

which gives for the phase

$$\frac{d\varphi_1}{dz} = -\sigma_1 a_2 \cos\left[\varphi_2 - 2\varphi_1 - \left(\Delta k - \frac{2\pi m}{\Lambda}\right)z\right]. \quad (14)$$

By using relations (10) and (11), we transform (14) to the relation

$$\frac{d\varphi_1}{dz} = -\sigma_1\sigma_2 I_{10} z \operatorname{sinc}(Qz) \times \sin\left[2\varphi_1(0) - 2\varphi_1(z) - \left(\Delta k - \frac{2\pi m}{\Lambda}\right)\frac{z}{2}\right]. \quad (15)$$

The solution of this equation has the form

$$\varphi_1(z) = \varphi_1(0) + \frac{(\Delta k - 2\pi m/\Lambda)z}{8 + (\Delta k - 2\pi m/\Lambda)^2/(\sigma_1\sigma_2 I_{10})}[1 - \operatorname{sinc}(2Qz)]. \quad (16)$$

This means that the phase velocity of the fundamental radiation wave and, hence, the refractive index of the PPC depend on the radiation intensity, i.e., the self-action of the light wave takes place in a quadratically nonlinear medium.

By introducing the effective length of nonlinear interaction $L = (\sigma_1\sigma_2 I_{10})^{-1/2}$, we write expression (8) for the parameter Q in the form

$$Q = \frac{1}{2}\left(\Delta k - \frac{2\pi m}{\Lambda}\right)\left\{1 + 8\left[\left(\Delta k - \frac{2\pi m}{\Lambda}\right)L\right]^{-2}\right\}^{1/2}. \quad (17)$$

It follows from relation (10) that the generalised coherence length L_c (i.e., the coherence SHG length for the entire PPC rather than for a domain), in the fixed-intensity approximation (i.e., the distance at which the second harmonic amplitude does not decrease) for SHG in the PPC is determined by the expression

$$L_c = \frac{\pi}{2Q} = \frac{\pi}{\Delta k - 2\pi m/\Lambda} \times \left\{1 + \frac{8}{[(\Delta k - 2\pi m/\Lambda)L]^2}\right\}^{-1/2} \quad (18)$$

Note that $L_c \gg l_c$, where l_c is the coherent interaction length in the domain in the fixed-field approximation.

According to (18), the period $a_2(z)$ of the spatial beats of the real amplitude in the PPC, i.e., the quantity $4L_c$ depends on the nonlinear length L and, hence, on the intensity I_{10}. Recall that the coherence SHG length $L_c = \pi(\Delta k - 2\pi m/\Lambda)^{-1}$ in the PPC is independent of the intensity in the fixed-field approximation.

It is interesting to compare expression (8) for L_c obtained in the fixed-intensity approximation with the exact expression for the coherence length, which is equal to one fourth of the period of spatial beats, in the 'nonlinear' regime (when the reaction of the fundamental radiation to the second harmonic is taken into account) [2]:

$$L_c^{\text{pres}} = KL\sqrt{\beta}, \quad (19)$$

where

$$K = \int_0^1 \left[(1 - y^2)(1 - \beta^2 y^2)\right]^{-1/2} dy \quad (20)$$

is the complete elliptic integral of the first kind [5] and

$$\beta = 1 + \left(\frac{\Delta k - 2\pi m/\Lambda}{4}\right)^2 L^2. \quad (21)$$

The calculation of the coherence length by exact expression (19) shows that it virtually coincides with the coherence length (19) calculated in the fixed-intensity approximation when the condition

$$\frac{L_c}{L} < \frac{\pi}{2}. \quad (22)$$

is fulfilled.

Relation (22) is not fulfilled for small generalised mismatches $\Delta k - 2\pi m/\Lambda$ or at low intensities (when $L_c \gg L$), but in any case the fixed-intensity approximation gives more correct results than the fixed-field approximation. Let us prove this statement by the example of the exact fulfilment of the phase-matching condition in the PPC ($\Delta k = 2\pi m/\Lambda$), when inequality (22) is not only invalid but has the opposite sign. In this case, $\beta = 1$ and $K = L_c = \infty$, and the expression for the SHG intensity conversion follows directly from (1) [2]:

$$\eta_{\text{int}} = \frac{I_2(l)}{I_{10}} = \tanh^2(l/L), \quad (23)$$

where l is the crystal length.

In the fixed-field and fixed-intensity approximations, we have

$$\eta_{\text{f}} = \left(\frac{l}{L}\right)^2, \quad (24)$$

and

$$\eta_{\text{int}} = \frac{1}{2}\sin^2\left(\frac{\sqrt{2}l}{L}\right). \quad (25)$$

respectively.

By expanding expressions (23) and (25) into power series in the parameter $l/L < 1$ and comparing the obtained expressions, we can show that the SHG efficiency (25) calculated in the fixed-intensity approximation coincides with exact expression (23) with an accuracy of $(l/L)^6$. Note that the fixed-intensity approximation can be used up to $\Delta k = 2\pi m/\Lambda$ even in the case of the exact quasi-phase matching in the PPC ($l \approx L$), whereas the fixed-field approximation is valid only for $l < 0.3L$. The accuracy of the fixed-intensity approximation increases with increasing generalised mismatch.

Finally, we present the expression for the SHG efficiency in the PPC calculated in the fixed-intensity approximation using (10):

$$\eta_{\text{int}} = \frac{I_2(l)}{I_{10}} = \left(\frac{l}{L}\right)^2 \text{sinc}^2(Ql), \quad (26)$$

where the parameter Q is expressed by (10). Because Q depends on L, i.e., on the intensity, the positions of the zeroes $\eta(\Delta k')$ of the quasi-phase-matching curve and the amplitudes of the secondary maxima will also depend on the intensity: as the intensity increases, the zeroes approach the coordinate origin, while the intensity of the secondary maxima increase.

3. Conclusions

The calculation of the SHG efficiency in PPCs in the fixed-intensity approximation has shown that, as in homogeneous crystals, the solution obtained in this approximation coincides, with an accuracy of the factor $(l/L)^6$, with the exact solution for a nonlinear regime even when the quasi-phase-matching condition is exactly fulfilled. The relative accuracy of this approximation increases with increasing mismatch. The fixed-intensity approximation can be used up to $l \approx L$ even in the case of exact quasi-phase matching in the PPC, whereas the fixed-field approximation is valid only for $l < 0.3L$.

The question of the correctness of using the fixed-field approximation in the SHG calculations in each individual domain of the PPC is beyond the scope of this paper. On the one hand, an increase in the amplitude of the harmonic field in each domain, irrespective of its number, is rather small compared to the amplitudes of the fundamental and second harmonics (note that this statement is invalid for several first domains where the increase in the harmonic field is compared by the order of magnitude with the field itself (see also [3]), but on the other hand, the neglect of even a small phase shift of the fundamental radiation can be incorrect due to the accumulation of this effect at many domains.

References

1. Dmitriev V.G., in *Fundamental'naya optika i spektroskopiya. Sbornik lektsii UNTs* (Fundamental Optics and Spectroscopy, Collection of Lectures, Educational and Scientific Center) (Moscow: P. N. Lebedev Physics Institute, 2001) No. 3, pp 7 – 59.
2. Dmitriev V.G., Tarasov L.V. *Prikladnaya nelineinaya optika* (Applied Nonlinear Optics) (Moscow: Radio i Svyaz', 1982).
3. Dmitriev V.G., Yur'ev Yu.V. *Kvantovaya Elektron*., **25**, 1033 (1998) [*Quantum Electron*., **28**, 1007 (1998)].
4. Tagiev Z.A. *Cand. Sci. Dissertation* (Moscow: Moscow State University, 1976).
5. Janke E., Emde F., Lösch F. *Tables of Higher Functions* (New York: McGraw-Hill, 1960; Moscow: Nauka, 1977).