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Second-harmonic generation of multimode laser radiation with the amplitude or arbitrary modulation of the field in the beam cross-section

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***Abstract*. Second-harmonic generation (SHG) of multimode laser radiation is studied theoretically in the fixed-field approximation. It is shown that for the fixed angular spectrum, the phase modulation of the fundamental-wave cross-section considerably decreases the SHG efficiency compared to that for the amplitude modulation.**

Keywords: *second-harmonic generation, multimode laser radiation, phase modulation.*

It was shown in paper [1] that the phase distortions of a fundamental wave in the beam cross-section have a predominant effect on the SHG efficiency, whereas the effect of amplitude distortions is much weaker. However, the results [1] were obtained numerically and, therefore, are not descriptive enough. In this paper, the above statement is proved analytically and some numerical estimates are performed, although in the fundamental-wave fixed-field approximation.

We will simulate multimode fundamental radiation by an infinitely broad statistically homogeneous wave with random distortions of the field in the beam cross-section. We assume that in one case the distortions are random ones, while in the other case, only the amplitude distortions take place. We assume that in both cases the angular spectrum of the fundamental radiation is the same and has the Gaussian shape. We will calculate the SHG efficiency taking into account only aperture effects upon the ooe interaction. All statistical averagings are preformed for an ensemble.

For the purely amplitude modulation of the field (hereafter, called variant A), we have

$$A_1 = A_1^*, \tag{1}$$

where A_1 is the complex amplitude of the electric field of the fundamental wave. For an arbitrary modulation of the field (hereafter, called variant B), we have

$$\overline{A_1^2} = 0. \tag{2}$$

In both variants, the field can be expanded in the cross-section in the Fourier integral (here, for brevity, we perform the expansion only in the coordinate x; the expansion in the coordinate y will be taken into account in the final result):

$$A_1(x) = \int S_1(k_{1x}) \exp(\mathrm{i}k_{1x}x)\mathrm{d}k_{1x}, \tag{3}$$

where $S_1(k_{1x})$ is the spectral amplitude; k_{1x} is the projection of the wave vector k_1 on the x axis. For a statistically homogeneous field, the relation

$$\overline{S_1(k_{1x})S_1^*(k'_{1x})} = G_1(k_{1x})\delta(k_{1x} - k'_{1x}). \tag{4}$$

takes place [2]. Because we assume that the angular spectrum $G_1(k_{1x})$ has a Gaussian shape, we have

$$G_1(k_{1x}) = S_0^2 \exp(-k_{1x}^2/\Delta k^2), \tag{5}$$

where Δk is the width of the angular spectrum.

We also assume below that the random quantity S_1 is described by a normal distribution, so that upon averaging of the product of four arbitrary spectral amplitudes S_1, S'_1, S''_1 and S'''_1, the relation

$$\overline{S_1S'_1S''_1S'''_1} = \overline{S_1S'_1}\;\overline{S''_1S'''_1} + \overline{S_1S''_1}\;\overline{S'_1S'''_1} + \overline{S_1S'''_1}\;\overline{S'_1S''_1}. \tag{6}$$

is fulfilled [2].

From expressions (1) and (2), the relations

$$S_1(k_{1x}) = S_1^*(-k_{1x}) \quad \text{(variant A)}, \tag{7}$$

and

$$\overline{S_1(k_{1x})S_1(k'_{1x})} = 0 \quad \text{(variant B)}. \tag{8}$$

can be obtained for the spectral amplitudes.

For the spectral amplitude of the second harmonic in the case of phase matching, we have [1]

$$S_2(k_{2x}) \sim \int_0^z \mathrm{d}\xi \int S_1(k_{1x})S_1(k_{2x} - k_{1x}) \exp(-\mathrm{i}\beta_2 k_{2x}\xi)\mathrm{d}k_{1x}$$
$$\sim z\,\mathrm{sinc}\left(\frac{\beta_2 k_{2x} z}{2}\right) \int S_1(k_{1x})S_1(k_{2x} - k_{1x})\mathrm{d}k_{1x}, \tag{9}$$

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where z is the length of a nonlinear crystal and β_2 is the anisotropy angle.

For brevity, we omitted constant factors in expression (9) because we are interested only in the ratio of the SHG efficiencies for variants A and B.

By using expressions (3)–(8), we obtain for the angular spectrum of the second harmonic (for both variants)

$$\overline{S_2(k_{2x})S_2^*(k'_{2x})} = G_2(k_{2x})\delta(k_{2x} - k'_{2x}). \tag{10}$$

(Expression (10) reflects an obvious result that the second-harmonic field is statistically homogeneous over the cross-section if the fundamental-wave field is also homogeneous.)

For variant A, we have

$$G_2(k_{2x}) \sim z^2 \exp\left[-\left(\frac{\beta_2 k_{2x} z}{2\sqrt{\pi}}\right)^2\right] \times S_0^4 \Delta k\left[\left(\frac{\pi}{2}\right)^{1/2} \exp\left(-\frac{k_{2x}^2}{2\Delta k^2}\right) + \Delta k\frac{\pi}{2}\delta(k_{2x})\right], \tag{11}$$

and for variant B,

$$G_2(k_{2x}) \sim z^2 \exp\left[-\left(\frac{\beta_2 k_{2x} z}{2\sqrt{\pi}}\right)^2\right] \times S_0^4 \Delta k\left(\frac{\pi}{2}\right)^{1/2} \exp\left(-\frac{k_{2x}^2}{2\Delta k^2}\right). \tag{12}$$

Expressions (11) and (12) were derived using the approximate relation [3]

$$\mathrm{sinc}^2\, x \approx \exp(-x^2/\pi). \tag{13}$$

Because the fields have infinite cross-sections in our model, the SHG efficiency is defined as the ratio of the field intensities for the second and fundamental harmonics. Because $I \sim \overline{AA^*}$, it follows from expressions (3) and (4) that [2]

$$I \sim \int G(k_x)\mathrm{d}k_x. \tag{14}$$

Expression (14) is obviously related both to the fundamental- and second-harmonic fields. By using (11), (12), and (14), and taking into account the dependence on the coordinate y omitted above for brevity, we obtain the following expressions for the SHG efficiency for variants A and B, respectively,

$$\eta_{\mathrm{A}} \sim z^2 S_0^2 \pi \Delta k^2\left[\left(1 + \frac{\beta_2^2 z^2 \Delta k^2}{2\pi}\right)^{-1/2} + \frac{1}{2}\right], \tag{15}$$

$$\eta_{\mathrm{B}} \sim z^2 S_0^2 \pi \Delta k^2\left(1 + \frac{\beta_2^2 z^2 \Delta k^2}{2\pi}\right)^{-1/2}. \tag{16}$$

Finally, the ratio of these efficiencies is

$$\frac{\eta_{\mathrm{A}}}{\eta_{\mathrm{B}}} = 1 + \frac{1}{2}\left(1 + \frac{\beta_2^2 z^2 \Delta k^2}{2\pi}\right)^{1/2}. \tag{17}$$

Within the framework of the assumptions adopted above, expression (17) is exact because we omitted identical constant factors in (15) and (16) during calculations.

According to (17), the SHG efficiency for variant A is higher by a factor of 1.5 than that for variant B even for the 90° phase matching, while for the critical phase matching ($\beta_2 z\Delta k \gg \pi$), the SHG efficiency in the case of amplitude modulation of the field can be infinitely higher than that for an arbitrary modulation.

Note that for both variants studied here, the width and shape of the angular spectrum were assumed identical, which cast doubt on the correctness of calculations of the SHG efficiency by the method of ray tubes [4, 5]. However, it has been shown in paper [6] that the method of ray tubes can be applied only when the phase modulation of the fundamental radiation dominates.

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