CONTROL OF LASER RADIATION PARAMETERS

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Effect of small misalignments and the parameter spread on the optical modes of a multicore fibre laser with a spatial filter

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***Abstract*. A multicore fibre laser is considered in which a ring waveguide (RW) of length equal to 1/2 or 1/4 of the Talbot length is used as a spatial filter separating a required collective mode. Explicit expressions are obtained for the eigenvalues of the modes in such lasers by using the Gaussian approximation for the fields of individual microcores. The effect of regular and random displacements of microcores and of the angular misalignment of a multicore fibre (MCF) relative to the RW is considered within the framework of the theory of weak perturbations. A change in the mode loss spectrum caused by a random phase incursion in the MCF is calculated using the perturbation theory. The explicit expressions derived for the perturbed eigenvalues are compared with the results of numerical simulation of the MCF laser with the RW of length equal to 1/4 of the Talbot length. Comparison of the results allows the use of these expressions for estimating the eigenvalues of Talbot resonators with the perturbed parameters of a laser set.**

Keywords: *fibre laser, radiation phase locking, resonator, Talbot effect.*

1. Introduction

Fibre lasers are widely used in optical communication systems and have a number of properties that make them promising for industrial applications. The use of multichannel (multicore) fibres (MCFs) makes these lasers more compact because their length can be reduced due to a more efficient absorption of pump radiation from diode lasers. A perspective design of a MCF laser was proposed in Ref. [1]. The active channels are arranged on a circle near the fibre cladding (Fig. 1). Phase locking of radiation from all the channels can significantly enhance the output brightness. The relevant studies [2–4] have shown that the most promising is phase locking achieved by using an intracavity spatial ring waveguide (RW) filter of width close to the channel diameter. If channels are arranged periodically over the azimuth, then the output radiation emerging from them can be reproduced after the double passage over the RW with an optical length that is a multiple of the Talbot length (see, for example, [5]).

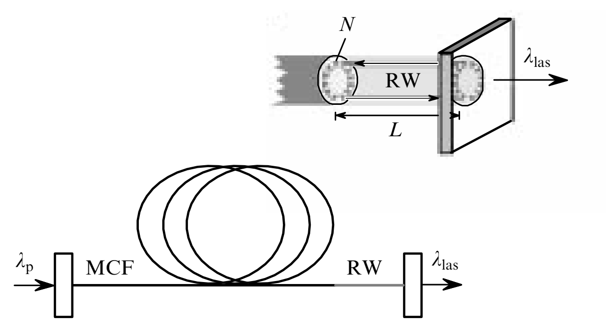

Figure 1. Design of a MCF laser with a RW spatial filter. λ_{p} and λ_{las} are the pump and lasing wavelengths; L is the RW length; N is the number of microchannels.

The use of such a spatial filter was discussed in many papers (see review [6]). However, the Talbot effect in a circular geometry possesses a number of specific features. In addition, the method of MCF drawing prevents the fabrication of identical fibres. A small size of channels and a small length of filters make the alignment of the system quite difficult. The aim of this paper is to estimate, using the perturbation theory, the misalignments and parameter spread of the channels, which are crucial for phase locking of the laser under study. The analytic expressions that we obtained are compared with the results of numerical solution of a scalar quasi-optical equation.

2. The ideal design

Consider the propagation of a scalar wave field in the absence of perturbations in a ring waveguide whose thickness is much smaller than the ring radius. We assume that the output field of the MCF represents a system of identical beams, which are arranged periodically over the azimuth, each of the beams corresponding to the field of the fundamental mode of a channel [single-mode microfibre (MF)]. The field of one mode was described in numerical calculations by known expressions [7]. The propagation of radiation in the RW is described by a parabolic equation. The numerical method of direct simulation of radiation in a composite fibre is described in Ref. [8].

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The radial operator for a thin ring can be reduced to the second derivative with respect to a variable changing within the ring thickness. In this case, a three-dimensional parabolic equation formally coincides with a two-dimensional quasi-optical equation for an angle-periodical plane geometry. If we assume additionally that the RW maintains only one radial mode, then, by separating variables, we obtain a parabolic equation describing the radiation propagation with one-dimensional diffraction over the azimuth. Therefore, the problem of radiation propagation in the RW maintaining one radial mode is reduced to a one-dimensional diffraction problem with a periodic condition imposed on the field, which corresponds to the passage over the azimuth by 2π.

Radiation is incident on the RW from N single-mode MFs with a fixed transverse structure described by the function $f(r)$. We consider only real and normalised functions $f(r)$ ($\int f^2(r)\mathrm{d}r = 1$), which vanish at a distance of smaller than the structure period. The field distribution at the MF output is approximated in analytic calculations by a Gaussian beam. Although the field does not completely vanish at a finite distance, it is negligible at the centre of an adjacent MF under conditions considered here.

The problem of propagation of a periodic set of beams has been analysed many times. We will follow the approach described in Ref. [9], where this problem was reduced to the solution of a system of equations for the field amplitudes of individual beams. The field produced by N radiation sources (fibres) at the RW entrance can be written in the form

$$E(\rho) = \sum_n C(R_n) f(\rho - R_n).$$

Here, ρ is the tangential variable; $R_n = 2\pi R_c n/N$ is the coordinate of the nth channel; R_c is the average radius of the ring; and $C(R_n)$ is the field amplitude. The projection of the total field on $f(r)$ at the distance z from the entrance at the location of the MF can be written in the form

$$C(R_n, z) = \sum_{n'} M(R_n, R_{n'}) C(R_{n'}). \tag{1}$$

In this case, matrix elements have the integral representation

$$M(R_n, R_{n'}) = \left(-\frac{\mathrm{i}k_0}{2\pi z}\right)^{1/2} \times \int \mathrm{d}\rho \mathrm{d}\rho' f(\rho - R_n) f(\rho' - R_{n'}) \exp\left[\frac{\mathrm{i}k_0}{2z}(\rho - \rho')^2\right], \tag{2}$$

where $k_0 = 2\pi n_0/\lambda$; n_0 is the refractive index of the medium; $z = 2L$ is the radiation propagation length; and L is the RW length. In the geometry considered here, radiation is reflected from the remote end of the RW and returns back to the MCF, by exciting MF modes. After the double passage over the MF, the field is again radiated to the RW. Assuming that the passage of radiation over channels in the passive MCF results only in the phase incursion, which is the same for all the channels, we can write the condition of the field reproduction after the round trip in the resonator in the form

$$\gamma C(R_n) = \sum_{n'} M(R_n, R_{n'}) C(R_{n'}), \tag{3}$$

where γ is the eigenvalue. In the case of a periodic arrangement of radiation sources [because the matrix $M(R_n, R_{n'})$ is a difference matrix], Eqn (3) has solutions in the form $C_m(R_n) = \exp(\mathrm{i}q_m R_n)$, where $q_m = 2\pi m/(Nb)$; m is the number of a collective mode; and $b = R_n/n$ is the period of the MF grating. By substituting the solutions to the system of equations (3), we can easily obtain the expression for the eigenvalues γ_m of the system (3) (see [9])

$$\gamma_m = M(q_m) = \sum_{n'-n} \exp[\mathrm{i}q_m(R_{n'} - R_n)] \int f_{\mathrm{F}}(q) f_{\mathrm{F}}(-q) \times \exp\left(-\frac{\mathrm{i}q^2 z}{2k_0}\right) \exp[\mathrm{i}q(R_{n'} - R_n)] \frac{\mathrm{d}q}{2\pi}, \tag{4}$$

where

$$f_{\mathrm{F}}(q) = \int f(\rho) \exp(\mathrm{i}q\rho) \mathrm{d}\rho$$

is the Fourier transform of the function f. For the RW length equal to half the Talbot length $z_{\mathrm{T}} = k_0 b^2/\pi = 2n_0 b^2/\lambda$, the expression

$$M(q) = \sum_k \exp\left(\frac{\mathrm{i}z_{\mathrm{T}} q^2}{2k_0} - \mathrm{i}bqk\right) \times \int f(\rho) f\left(\rho - \frac{z_{\mathrm{T}} q}{k_0} + kb\right) \mathrm{d}\rho. \tag{5}$$

was obtained in Ref. [9]. For the RW length equal to one fourth of the Talbot length, we can obtain

$$M(q) = \sum_k \exp\left[\frac{\mathrm{i}z_{\mathrm{T}} q^2}{4k_0} - \mathrm{i}bq\left(k + \frac{1}{2}\right)\right] \times \int f(\rho) f\left[\rho - \frac{z_{\mathrm{T}} q}{2k_0} + b\left(k + \frac{1}{2}\right)\right] \mathrm{d}\rho. \tag{6}$$

In particular, by approximating the field distribution for one channel by a Gaussian beam

$$f(\rho) = \exp\left(-\frac{\rho^2}{2a^2}\right)(a\sqrt{\pi})^{-1/2} \tag{7}$$

we can find the explicit expressions for the eigenvalues

$$\gamma_m^{(z_{\mathrm{T}}/2)} = \sum_{k=0,\pm 1} \exp\left[-\left(\frac{b}{a}\right)^2 \left(\frac{k-1}{2} + \frac{m}{N}\right)^2 + 2\pi\mathrm{i}\left(\frac{k-1}{2} + \frac{m}{N}\right)^2 - \frac{\pi(k-1)^2 \mathrm{i}}{2}\right] \tag{8}$$

for $L = z_{\mathrm{T}}/2$ and

$$\gamma_m^{(z_{\mathrm{T}}/4)} = \exp\left(-\frac{\pi\mathrm{i}}{4}\right) \times \exp\left[-\left(\frac{b}{2a}\right)^2 \left(\frac{m}{N} - \frac{1}{2}\right)^2 + \pi\mathrm{i}\left(\frac{m}{N} - \frac{1}{2}\right)^2\right] \tag{9}$$

for $L = z_{\mathrm{T}}/4$.

Expressions (5) and (6) were derived by assuming that the function $f(r)$ strictly vanishes at the finite distance.

Because this condition is not satisfied for the Gaussian approximation, the terms of the order of $\exp\{-[b/(2a)]^2\}$ in (8) appearing due to the overlap of the fields from the adjacent MFs should be omitted. For $z = z_{\rm T}$, the eigenvalues γ_0 (inphase mode) and $\gamma_{N/2}$ (antiphase mode) are equal to 1 and $-\rm i$, respectively. This means that both modes have no losses (exact self-reproduction) and differ by their frequencies. For odd N and $z = z_{\rm T}$, only the inphase mode is reproduced, which is of an obvious practical interest. Modes with $m = (N \pm 1)/2$, for which $|\gamma_{(N\pm1)/2}| = \exp\{-[b\times(2Na)^{-1}]^2\}$, have minimal losses. Therefore, in this case the degeneration in losses is lifted, and the discrimination of the undesired modes is determined by the quantity $|\gamma_{(N\pm1)/2}|$.

The antiphase mode with an additional phase plate, which straightens the phase front, also can be of interest. For the RW of length $z_{\rm T}/4$ and even N, only the antiphase mode has the eigenvalue $\gamma_{N/2} = \exp(-{\rm i}\pi/4)$ whose modulus is equal to unity. The modulus of the eigenvalue for two modes nearest to the antiphase mode is $|\gamma_{N/2\pm1}| = \exp\{-[b/(2Na)]^2\}$.

Therefore, one mode can be separated by two methods. We can use either a RW of length $z_{\rm T}/2$ and a MCF with an odd number of channels (inphase mode) or a RW of length $z_{\rm T}/4$ and a MCF with an even number of channels (antiphase mode). The selectivity proves to be same in both these cases. Note also that the modes with numbers m and $N - m$ have equal eigenvalues. Physically, it is obvious because the field envelopes $C_m(R_n)$ and $C_{N-m}(R_n)$ differ only in the direction of the phase variation.

The expressions presented above describe the ideal design of a laser shown in Fig. 1. The misalignment of the MCF and RW and the spread of channel parameters (MF parameters) eliminate the Talbot effect, reduce the eigenvalue of the selected mode, and can induce lasing at the adjacent modes. Phase locking of a MCF laser emitting an antiphase mode was experimentally demonstrated in Ref. [5]. In the next sections, we will consider analytically the effect of misalignments and of the spread of MF parameters for two RW lengths ($L = z_{\rm T}/2$ and $z_{\rm T}/4$) and compare it with numerical calculations performed for a real laser built in Ref. [5].

3. Analysis of weak perturbations

When deviations from an ideal design are small, the effects produced by them are added independently. Therefore, we can analyse separately the following perturbations: the parallel displacement of the RW relative to the MCF, the angular misalignment of the RW, the random displacements of the MFs relative to their regular arrangement in the MCF plane, and the random spread of the radiation propagation constants in MFs resulting in phase fluctuations of the field propagated through the MCF. Our analysis is based on a standard theory of weak perturbations for the diffraction operator of radiation propagation in the RW maintaining one radial mode. We considered the perturbation of the parameters of the inphase ($L = z_{\rm T}/2$) and antiphase ($L = z_{\rm T}/4$) modes.

3.1 Parallel displacement

In the case of the parallel displacement of the RW across the axis, the displacement of channels depends on their positions and the direction of the RW displacement. It is obvious that when the number of channels is large, one should not expect a strong dependence of the eigenvalue change on the displacement direction. In the case of a one-dimensional analytic model, the azimuthal displacement of the MF upon parallel displacement can be written as a function of the MF number n in the form

$$\Delta_n = S \sin\frac{2\pi n}{N}, \tag{10}$$

where S is the maximum displacement of the MF axis. This perturbation will cause a change in the coupling matrix $M(R_n, R_{n'})$ in Eqn (3):

$$M(R_n, R_{n'}) = \int \frac{{\rm d}q}{2\pi} f_{\rm F}(-q) f_{\rm F}(q) \times \exp\left[-\frac{{\rm i}q^2 z}{2k_0} + {\rm i}q(R_{n'} - R_n) + {\rm i}q(\Delta_{n'} - \Delta_n)\right]. \tag{11}$$

This matrix is no longer a function of the difference in the channel numbers, and therefore the system (3) cannot be solved. By using the perturbation theory up to the second order inclusive, because the first nonvanishing term proves to be proportional to S^2, we can obtain the following expressions for the eigenvalues of the inphase ($L = z_{\rm T}/2$) and antiphase ($L = z_{\rm T}/4$) modes for even N, respectively,

$$\frac{\gamma_0}{\gamma_0^{(0)}} = 1 - \frac{S^2}{4a^2}\left[1 - \frac{\gamma_1^{(0)}}{\gamma_0^{(0)}} + \frac{2\gamma_1^{(0)}}{\gamma_0^{(0)} - \gamma_1^{(0)}} \frac{b^2}{(aN)^2}\right], \tag{12}$$

$$\gamma_{N/2}^{(z_{\rm T}/4)} = (-{\rm i})^{1/2}\left\{1 - \frac{S^2}{4a^2}\left[1 - \frac{\gamma_{1+N/2}^{(0)}}{\gamma_{N/2}^{(0)}} + \frac{2\gamma_{1+N/2}^{(0)}}{\gamma_{N/2}^{(0)} - \gamma_{1+N/2}^{(0)}} \frac{b^2}{(2aN)^2}\right]\right\}, \tag{13}$$

where the superscript (0) denotes the unperturbed eigenvalue. In a real laser, displacements in the radial direction make a comparable contribution to the losses until these displacements are much less than the RW thickness. In this case, to obtain the total losses, it is sufficient to square the modulus of eigenvalues (12) and (13).

3.2 Angular misalignment

The tilt of the optical axis of the MCF with respect to the RW is equivalent to an optical wedge deflecting the beams returned from the MCF. The phase factor corresponding to the action of such a wedge on the circular set of MFs can be written in the form $\exp[{\rm i}k_0\psi R_{\rm c}\sin(\rho/R_{\rm c})]$, where ψ is the axis tilt, and then

$$M(R_n, R_{n'}) = \left(-\frac{{\rm i}k_0}{2\pi z}\right)^{1/2} \int {\rm d}\rho {\rm d}\rho' f(\rho - R) f(\rho' - R_{n'}) \times \exp\left[\frac{{\rm i}k_0}{2z}(\rho - \rho')^2 + {\rm i}k_0\psi R_{\rm c}\sin\frac{R_{n'}}{R_{\rm c}}\right]. \tag{14}$$

The quantity $k_0\psi R_{\rm c} \ll 1$ can be used as a small parameter of the perturbation theory. Upon the expansion of the exponential in (14), the terms will appear that are proportional to $\sin(2\pi n/N)$ and $\sin^2(2\pi n/N)$ and couple the mth unperturbed mode with the adjacent $(m \pm 1)$th and $(m \pm 2)$th modes. As a result, the system becomes sensitive to such perturbations. To describe the effect of the axis tilt more accurately, we will take into account in the perturbation theory the coupling between nearest modes, not assuming that this coupling is weak. Note that the modes adjacent to the inphase or antiphase mode have the same losses. However, because the perturbing term has a certain symmetry $[\sin(2\pi n/N)]$, a mode is excited which represents an odd combination of two adjacent modes. Therefore, the problem can be reduced to the calculation of two modes interacting with each other.

This problem has an analogue in quantum mechanics: the mixing of two levels by the resonance perturbation. After the appropriate transformations, we can obtain the following expression for the perturbed eigenvalues of the interacting modes

$$\tilde{\gamma}_{0,1} = \frac{1}{2}\left\{\left[(\gamma_0+\gamma_1)\left(1-\frac{\varphi^2}{4}\right) - \frac{\gamma_1\gamma_2}{\gamma_0-\gamma_2}\frac{\varphi^2}{2}\right] \pm |\gamma_0-\gamma_1|\left(1-\frac{\varphi^2}{\varphi_c^2}\right)^{1/2}\right\}. \quad (15)$$

Here, $\varphi = k_0\psi R_c$; $\gamma_0 = |\gamma_{N/2}| = 1$ is the moduli of the unperturbed eigenvalues of the modes being reproduced; γ_1 and γ_2 are the moduli of the unperturbed eigenvalues of the adjacent modes [along with the mixing of two resonantly interacting modes, the perturbation of these modes by more remote modes is also taken into account in (15)];

$$\varphi_c = (1-\gamma_1)\left[\frac{(1+\gamma_1)^2}{2} - \frac{\gamma_1\gamma_2(1-\gamma_1)}{1-\gamma_2}\right]^{-1/2}$$

is the critical parameter determining the axis tilt after which the renormalised modes have equal losses:

$$|\tilde{\gamma}_{0,1}| = \gamma_1^{1/2}\left(1-\frac{\varphi^2}{4}\frac{\gamma_2}{\gamma_0-\gamma_2}\right).$$

For $\varphi \geqslant \varphi_c$, the contributions of the mode being reproduced and of the adjacent mode are of the same order, resulting in a change in the field amplitudes $C(R_n)$ in the MF. The contrast of the field amplitudes is

$$K = \max\left|\frac{C(R_n)}{C(R_{n+N/2})}\right| = \left[\frac{\gamma_0+2\gamma_1+2(\gamma_0-\gamma_1)\left(\varphi_c^{-2}-\varphi^{-2}\right)^{1/2}}{\gamma_0+2\gamma_1-2(\gamma_0-\gamma_1)\left(\varphi_c^{-2}-\varphi^{-2}\right)^{1/2}}\right]^{1/2}.$$

3.3 Random MF displacement

The position of a MF, its size, and shape can vary during the manufacturing of MCFs. Consider first a random displacement of the MF relative to its ideal position. The spread in the MF size or shape leads to a random change in the propagation constant of the fibre mode, resulting in turn in a random phase incursion, which will be considered in the next section.

It is reasonable to assume that random displacements of the MF over two coordinates are statistically independent and, in the case of small displacements, cause the same decrease in the mode eigenvalue. In this case, the results of the theory developed here for the one-dimensional geometry can be applied to real lasers by squaring the modulus of the found eigenvalue. It is convenient to consider displacement perturbations using the Fourier representation [see (4)]. By denoting the random displacement of the MF centre as Δ_n, we can write the Fourier transform of the MF field as

$$F[f(\rho-R_n-\Delta_n)] = F[f(\rho-R_n)]\exp(-\mathrm{i}q\Delta_n).$$

It follows from this expression that a random spread in the positions of the MF centres results in the modification of the elements of the coupling matrix by a simple multiplication of the integrand in (11) by an exponential factor. If the displacements Δ_n are independent and are described by a normal distribution, we can obtain explicit expressions for the eigenvalues averaged over an ensemble of samplings:

$$\gamma = M(q) = \sum_k \exp\left(\frac{\mathrm{i}b^2q^2}{2\pi} - \mathrm{i}bqk\right) \times \int\left\langle f(\rho)f\left(\rho-\Delta+\frac{b^2q}{\pi}-kb\right)\right\rangle \mathrm{d}\rho \quad (16)$$

for $L = z_{\mathrm{T}}/2$ and

$$\gamma = M(q) = \sum_k \exp\left[\frac{\mathrm{i}b^2q^2}{4\pi} - \mathrm{i}bq\left(k+\frac{1}{2}\right)\right] \times \int\left\langle f(\rho)f\left[\rho-\Delta+\frac{b^2q}{2\pi}-b\left(k+\frac{1}{2}\right)\right]\right\rangle \mathrm{d}\rho \quad (17)$$

for $L = z_{\mathrm{T}}/4$. Here, Δ is a random variable and the angle brackets mean averaging over an ensemble of samplings. Recall that q runs the values $q_m = 2\pi m/(Nb)$, where m is the number of a collective mode, which varies from zero to $N-1$.

In the case of the Gaussian approximation of the MF field, the expressions for the eigenvalues have the form

$$\gamma_m^{(z_{\mathrm{T}}/2)} = \frac{1}{(1+\sigma^2/a^2)^{1/2}}\sum_{k=0,\pm1}\exp\left[\left(\frac{k-1}{2}+\frac{m}{N}\right)^2 \times\left(2\pi\mathrm{i}-\frac{b^2}{a^2+\sigma^2}\right) - \frac{\pi(k-1)^2\mathrm{i}}{2}\right] \quad (18)$$

for $L = z_{\mathrm{T}}/2$ and

$$\gamma_m^{(z_{\mathrm{T}}/4)} = \frac{\exp(-\mathrm{i}\pi/4)}{(1+\sigma^2/a^2)^{1/2}} \times\exp\left[\left(\frac{m}{N}-\frac{1}{2}\right)^2\left(\pi\mathrm{i}-\frac{b^2/4}{a^2+\sigma^2}\right)\right] \quad (19)$$

for $L = z_{\mathrm{T}}/2$. Here, σ is the dispersion of a random MF displacement.

3.4 Random phase incursion

As we note above, a random spread of field-phase incursions after the passage through MFs is caused by the difference between their sizes or shape. For weak phase perturbations, the system of linear equations for the wave-field amplitudes at the MF end has the form

$$\gamma C(R_n) = (E+V_n)\sum_{n'=0}^{N-1} M_{nn'}C(R_{n'}).$$

Here, E is the unit matrix; the matrix $M_{nn'}$ corresponds to the ideal system; $V_n = \exp(\mathrm{i}\phi_n) - 1$ are the elements of the diagonal matrix, which contain the phase differences φ_n appearing after the double passage of the field trough the MCF. For doubly degenerate modes with the numbers m and $N - m$, it is sufficient to use the first-order perturbation theory, which gives

$$\gamma_m^{(1)} = \frac{\gamma_m^{(0)}}{N}\left[\sum_k V_k + \left(\sum_k V_k^2\right)^{1/2}\right]. \tag{20}$$

After averaging over a random spread, we find

$$\frac{\gamma_m^{(0)} + \gamma_m^{(1)}}{\gamma_m^{(0)}} = \mathrm{e}^{-\sigma^2/2} + \left[\frac{(1 - \mathrm{e}^{-\sigma^2/2})^2 - \mathrm{e}^{-\sigma^2}(1 - \mathrm{e}^{-\sigma^2})}{N}\right]^{1/2}$$

$$= 1 - \frac{\sigma^2}{2} \pm \mathrm{i}\left(\frac{\sigma^2}{N}\right)^{1/2}. \tag{21}$$

The latter equality is obtained in the case of a small dispersion of the phase incursion. For nondegenerate modes with the numbers $m = 0$ and $m = N/2$, the perturbation theory in the second order gives the expression

$$\gamma_m^{(1)} + \gamma_m^{(2)} = \frac{\gamma_m^{(0)}}{N}\sum_k V_k$$

$$+ \frac{\gamma_m^{(0)}}{N^2}\sum_k V_k^2 \sum_{m'} \frac{\gamma_{m'}^{(0)}}{\gamma_m^{(0)} - \gamma_{m'}^{(0)}}. \tag{22}$$

For a inphase mode ($m = 0$), the second-order perturbation theory gives, after averaging over random phases, the general expression ($L = z_{\mathrm{T}}/2$)

$$\frac{\gamma_0}{\gamma_0^{(0)}} = \mathrm{e}^{-\sigma^2/2} + \frac{1 + \mathrm{e}^{-2\sigma^2} - 2\mathrm{e}^{-\sigma^2/2}}{N}$$

$$\times \sum_m \frac{\gamma_m^{(0)}}{\gamma_0^{(0)} - \gamma_m^{(0)}} = 1 - \frac{\sigma^2}{2} - \frac{\sigma^2}{N}\sum_m \frac{\gamma_m^{(0)}}{\gamma_0^{(0)} - \gamma_m^{(0)}}. \tag{23}$$

By using the Gaussian approximation and expression (8), we can obtain the approximate expression

$$\gamma_0 \approx 1 - \frac{\sigma^2}{2} - \frac{\sigma^2}{N}\left(\frac{2\gamma_1^{(0)}}{1 - \gamma_1^{(0)}} - \frac{2\mathrm{i}\gamma_1^{(0)}}{1 + \mathrm{i}\gamma_1^{(0)}} - \frac{\mathrm{i}}{1 + \mathrm{i}}\right). \tag{24}$$

For the RW of length $L = z_{\mathrm{T}}/4$, taking into account expressions (9) for the eigenvalues, we obtain

$$\sqrt{\mathrm{i}}\,\gamma_{N/2}^{(z_{\mathrm{T}}/4)} \approx 1 - \frac{\sigma^2}{2} - \frac{\sigma^2}{N}\frac{2\gamma_{1+N/2}^{(0)}}{\gamma_{N/2}^{(0)} - \gamma_{1+N/2}^{(0)}}. \tag{25}$$

Expressions (24) and (25) give a possiblity to estimate the maximum number of MFs in a one-dimensional set $N_{\max}$ all lasers of which are included into phased lasing. A criterion based on a comparison of the correction the eigenvalue with the difference of the nearest eigenvalues [10] gives for the system under study for $L = z_{\mathrm{T}}/2$ the value $N_{\max} \sim [b^2/(\sigma a^2)]^{2/3}$ and for $L = z_{\mathrm{T}}/4$, we have $N_{\max} \sim [b^2 \times (4\sigma a^2)^{-1}]^{2/3}$. When the number of MFs in the ring exceeds $N_{\max}$, the entire set can be divided into groups with the average number of MFs equal to $N_{\max}$ [10].

4. Comparison of the results of analysis with numerical calculations

Numerical studies are based on the three-dimensional diffraction program [see (8)], which describes the propagation of monochromatic radiation in the approximation of scalar paraxial optics. We considered a construction realised experimentally [5] and representing two spliced fibre pieces, one of them including a circular set consisting of 18 MFs of diameter 8 μm arranged at a radius of 131 μm, and another containing the RW with the ring thickness slightly exceeding the microchannel diameter (Fig. 1).

The resonator is formed by mirrors located close to the ends of the fibres, and the RW length is $L = z_{\mathrm{T}}/4$. We calculated the field after the double passage over the RW. The resulting field distribution was projected on a set of modes of microchannels. The coefficients obtained in this way were multiplied by the field in each MF, and the calculation of the double passage over the RW was repeated. These iterations were continued until the divergence was obtained, i.e., until the reproduction of the complex coefficients of the expansion in the MF modes with the prescribed accuracy. In this case, as in usual iteration calculations of an optical resonator, a complex eigenvalue was obtained, whose amplitude determined the losses after the passage and the phase determined an exact resonance frequency. The propagation of radiation in the MF was not considered.

We assumed that the round trip in each MF could only result in a change in the complex amplitude of an MF mode. In accordance with the perturbation under study, we introduced the MF displacement in the output plane of the MCF, the phase incursion simulating a wedge in the case of the tilted axis, or a random phase incursion.

Figs 2 and 3 show the moduli and phases of the eigenvalues calculated for experimental conditions by expressions (8) and (9) for 18 modes for the RW length $L = z_{\mathrm{T}}/2$ and $z_{\mathrm{T}}/4$. The radius a of a Gaussian beam approximating a MF mode in analytic calculations was set equal to 3 μm and the period of arrangement of the MFs on the ring was $b = 2\pi R_{\mathrm{c}}/N = 45.7$ μm. Because the distance between MFs is noticeably greater than the beam diameter, the RW spatial filter efficiently discriminates fractional modes for $L = z_{\mathrm{T}}/2$. However, two modes (inphase, $m = 0$, and antiphase, $m = 9$) have no losses and are excited in the same way. For the RW with

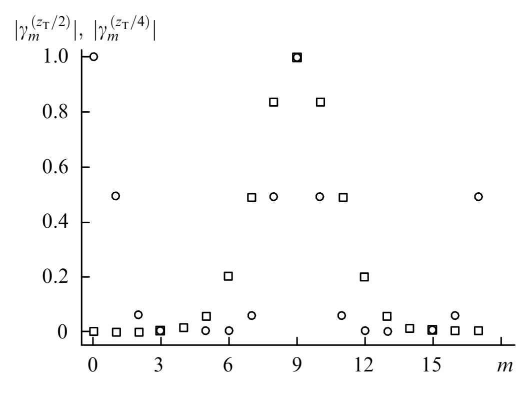

Figure 2. Moduli of eigenvalues $\gamma_m^{(z_{\mathrm{T}}/2)}$ (○: inphase mode) and $\gamma_m^{(z_{\mathrm{T}}/4)}$ (□: antiphase mode).

$L = z_{\mathrm{T}}/4$, only one antiphase mode has no losses; however, the discrimination of the nearest modes is not so strong in this case. To achieve phase locking with the parameters presented above, the RW length should be equal to $z_{\mathrm{T}}/4$. For this reason, we performed numerical calculations for verifying the analytic expressions using this RW length.

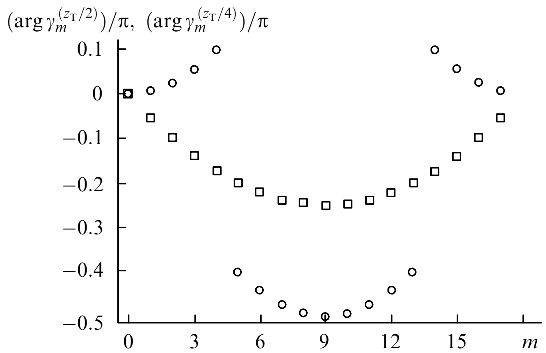

Figure 3. Phases of eigenvalues $\gamma_m^{(z_{\mathrm{T}}/2)}$ (○: inphase mode) and $\gamma_m^{(z_{\mathrm{T}}/4)}$ (□: antiphase mode).

In numerical calculations, the problem of a proper choice of the RW width appears. When the RW width coincides with the MF size, the construction will be obviously too sensitive to the MF displacement in the radial direction. A RW of large thickness will maintain several radial modes, which eliminates the Talbot effect. We have chosen a compromising variant and the RW thickness was set equal to 12 μm (the numerical aperture was 0.16). In this case, the radial structures of the MF modes and of the RW mode do not completely coincide. However, the calculation shows that the extent of reproduction of the antiphase-mode structure after the double passage over the RW is rather high, namely, the eigenvalue modulus is 0.936.

We neglected the radial structure of the field in analytic calculations and considered diffraction in the azimuthal direction. To take into account the difference between the analytic and numerical results, we compared the results of analysis with the eigenvalue moduli calculated numerically and normalised to 0.936. The results of this comparison are presented in tables.

4.1 Parallel displacement

Table 1.

$S/\mu\mathrm{m}$	$\lvert\gamma\rvert$	$\lvert\gamma_{\mathrm{norm}}\rvert$	$\lvert\gamma_{\mathrm{anal}}\rvert^2$
0	0.936	1	1
0.5	0.907	0.97	0.97
1	0.81	0.86	0.89
1.5	0.72	0.77	0.766

Comparison of the analytic and numerical results demonstrates their good agreement, which confirms our assumption about the independent contributions from radial and angular displacements in the mode losses.

4.2 Angular misalignment

For the numerical simulations, $\varphi_{\mathrm{c}} = 0.13$, so that all the calculations were performed for $\varphi > \varphi_{\mathrm{c}}$. Note that, although a change in the eigenvalue is small, the perturbation theory cannot be applied in this case. A strong mixing of the modes results in the inhomogeneous distribution of the field in the MF, which is characterised by the contrast K. The approximation in which the perturbation caused by the next to the nearest mode is taken into account in the perturbation theory proves to be adequate to the real situation. The analytic estimate gives satisfactory agreement with the numerical calculation, both for the eigenvalue modulus and for the contrast of the field distribution in the MF.

Table 2.

ψ/mrad	φ	$\lvert\gamma_{\mathrm{norm}}\rvert$	$\lvert\gamma_{\mathrm{anal}}\rvert$	K	K_{anal}
0.2	0.157	0.954	0.91	1.57	1.75
0.3	0.235	0.915	0.9	2.46	2.79
0.4	0.314	0.894	0.89	3.75	3.48

4.3 Random MF displacement

Table 3.

$\sigma/\mu\mathrm{m}$	$\lvert\gamma_{\mathrm{norm}}\rvert$	$\lvert\gamma_{\mathrm{anal}}\rvert$	$\sigma^{2\mathrm{d}}/\mu\mathrm{m}$	$\lvert\gamma_{\mathrm{norm}}\rvert^{2\mathrm{d}}$
	0.94		–	–
1	0.906	0.95	–	–
	0.93		–	–
	0.906			0.872
$\sqrt{2}$	0.87	0.9	1	0.888
	0.823			0.860
	0.855		–	–
2	0.839	0.83	–	–
	0.682		–	–
	0.694		–	–
	0.74			0.684
$2\sqrt{2}$	0.737	0.73	2	0.708
	0.662			0.715

The effect caused by random MF displacements was simulated numerically in two ways. In the first case, we considered only angular random MF displacements, in complete agreement with the analytic theory. For each average displacement, an eigenvalue was found for several particular realisations. Comparison of the second and third columns in Table 3 demonstrates that the value of $\lvert\gamma_{\mathrm{anal}}\rvert$ obtained by averaging over many realisations falls inside the spread interval $\lvert\gamma_{\mathrm{norm}}\rvert$ for all calculations. In addition, we determined the eigenvalue numerically when random displacements in different directions on the plane were assumed to occur with identical dispersions. It was expected that for such perturbations (the corresponding values in the two last columns of Table 3 have the superscript 2d), we have $\langle\lvert\gamma\rvert^{2\mathrm{d}}\rangle \simeq \langle\lvert\gamma\rvert\rangle^2$, or in other words, $\langle\lvert\gamma\rvert^{2\mathrm{d}}\rangle\rvert_{\sigma} \simeq \langle\lvert\gamma\rvert\rangle\rvert_{\sigma\sqrt{2}}$. Comparison of the second, third, and fifth columns shows that these relations are fulfilled within a random spread.

4.4 Random phase incursion

We calculated the eigenvalues for three particular realisations of random quantities for prescribed dispersion of the phase incursion. Within the dispersions chosen, the variations of the eigenvalue modulus depending on the sampling are not too large and are in reasonable agreement with the analytic values.

Table 4.

σ/rad	$\lvert\gamma_{\text{norm}}\rvert$	$\lvert\gamma_{\text{anal}}\rvert$
	0.982	
0.2	0.978	0.96
	0.984	
	0.929	
0.4	0.855	0.84
	0.855	

5. Conclusions

We have analysed in detail the collective modes of a one-dimensional circular set of identical radiation sources and obtained explicit expressions for the eigenvalues of Talbot resonators of lengths $z_{\mathrm{T}}/2$ and $z_{\mathrm{T}}/4$ by using the Gaussian approximation of the fields of individual radiation sources. We have considered within the framework of the theory of weak perturbations the effect of regular and random displacements of the positions of radiation sources, as well as of the angular MCF misalignment relative to the RW playing the role of a spatial filter separating a desired collective mode. By using the perturbation theory, we have also calculated a change in the mode losses caused by the random phase incursion in the MCF. The explicit expressions derived for the perturbed eigenvalues have been compared with the results of numerical simulations of a MCF laser with the RW of length $z_{\mathrm{T}}/4$. In this laser, the generation of an antiphase mode have been observed experimentally in Ref. [5]. A comparison of the results have shown that the expressions derived by us can be used for estimating the eigenvalues of Talbot resonators with perturbed parameters of a laser set.

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