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Phase locking of a multicore fibre laser with a large spread of microcore optical paths

D.V.Vysotsky, N.N.Elkin, A.P.Napartovich, V.N.Troshchieva

***Abstract*. The possibility of phase locking of a multicore laser with a spread of the optical paths of microcores amounting to several tens of laser radiation wavelengths is studied. It is shown that phase locking under such conditions can be obtained by using an intracavity filter in the form of a ring waveguide whose length is equal to 1/4 of the Talbot distance. It is shown that a global coupling between the fibre microcores can be obtained using this filter without the loss of the radiation intensity.**

Keywords: *fibre laser, radiation phase locking, resonator, Talbot effect.*

1. Introduction

Diode-pumped fibre lasers are used in many commercial devices as low-cost, powerful, and convenient bright laser radiation sources. The use of a multichannel fibre (MCF) is promising for the development of compact high-power fibre lasers. The MCF proposed in Ref. [1] consists of N single-mode microfibres (microchannels) arranged on a circle inside a multimode fibre. The microchannels are doped with Nd^{3+} ions and are pumped by radiation from a diode laser propagating in a multimode fibre (Fig. 1).

In the absence of phase locking of the channels in a fibre laser, the divergence of its output radiation is determined by the microchannel aperture. The phase-locked lasing is characterised by the phase difference between the adjacent microchannels equal to $2\pi m/N$ for the mth supermode. The far-zone field of an inphase supermode with the zero phase difference between the channels ($m = 0$) is described by the zero-order Bessel function. In this case, the axial brightness is N^2 times higher than upon independent lasing in individual microchannels.

To perform phase locking of the output radiation from a MCF laser, it is necessary to use additional phase elements. As shown in paper [2], phase locking of radiation from several lasers can be achieved by using the Talbot effect [3, 4], i.e., the reproduction of the periodic distribution of a field after its propagation by a certain distance. The distance at which the self-reproduction takes place (the Talbot distance) for the one-dimensional distribution of the field or a quadratic lattice is equal to $z_{\mathrm{T}} = 2\Lambda^2/\lambda$, where λ is the radiation wavelength and Λ is the field distribution period.

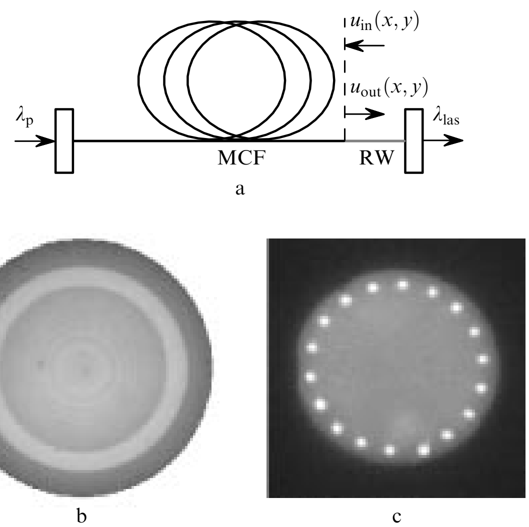

Figure 1. Scheme of the MCF laser with a ring waveguide (RW) and an external mirror (a) and the output apertures of the RW (b) and MCF (c); λ_{p} and λ_{las} are the pump and lasing wavelengths.

The applicability of the theory of the Talbot effect to a system with a circular array of emitters requires a separate study. The different methods for phase locking of radiation emitted by MCF lasers with circularly arranged micro-channels were studied both theoretically and experimentally earlier. An MCF laser with a plane external mirror located at a distance of $z_{\mathrm{T}}/4$ or $z_{\mathrm{T}}/2$ from the MCF was studied in Ref. [5]. The authors of Ref. [5] observed the far-zone field that corresponded to the generation of an antiphase super-mode and adjacent supermodes against the noise back-ground with a broad divergence.

The theoretical analysis of a resonator consisting of a MCF and an external mirror performed in Ref. [6] showed that diffraction in the radial direction rapidly destroys the periodic spatial distribution of the field. It was found that single-mode lasing could be obtained by either increasing the distance between the MCF and the mirror, which results in large losses, or by using a spherical mirror, which forms a conic resonator. However, the latter method proved to be inefficient in practice.

D.V.Vysotsky, N.N.Elkin, A.P.Napartovich, V.N.Troshchieva State Research Center of Russia, Troitsk Institute for Innovation and Fusion Research, 142190 Troitsk, Moscow oblast, Russia; tel. (095) 334-04-50; e-mail: apn@triniti.ru; web-site: www.triniti.ru

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The authors of paper [7] proposed to use a ring waveguide (RW) between a MCF and an external mirror to reduce losses. The RW represents a multimode fibre containing a ring with an enhanced refractive index and the radius and thickness that are matched with the MCF. By using this method, the authors of Ref. [7] obtained phase-locked lasing with the distribution of output radiation close to the antiphase distribution.

The selection of a supermode in a MCF laser with a spatial Talbot filter was analysed in papers [5–7] assuming all the channels had identical parameters. However, the identity of the channels is violated during the manufacturing of the MCF laser. The main problem is a spread in the optical paths of individual microchannels, which is difficult to reduce. A dispersion of the rate constants of radiation propagation, which appeared mainly due to different diameters of microchannels produced after the MCF drawing resulted in the dispersion of optical paths of microchannels of the order of several tens of wavelengths for fibres of length 30–150 cm used in experiments [5, 7].

A similar situation was observed earlier for generation in a wide-aperture laser with a set of retroreflectors and an angular radiation selector [8]. Radiation transfer between subresonators formed by a common plane mirror and individual retroreflectors can enhance the field coherence. In Ref. [9], the possibility of partial phase self-locking appearing in the case of a great difference between the lengths of subresonators was pointed out. In this paper, this case is analysed in detail for a MCF laser with a RW spatial filter.

2. Modes of a MCF laser with the Talbot filter in the limit of narrow microchannels

Consider the configuration of a MCF laser shown in Fig. 1a. The MCF is connected with the RW of length $z_{\mathrm{T}}/4$, which had a plane mirror at its end. We assume that the RW maintains only one radial mode. The resonator modes are usually found by studying a change in the radiation field observed during the round trip of radiation in the resonator. Although the radiation frequency is the same for all the microchannels, the field phase changes randomly over the microchannels. In the general case, taking into account that microfibres maintain one mode, the output field emerging from the MCF can be written in the form

$$u_{\mathrm{out}}(\boldsymbol{r}) = \sum_{n=0}^{N-1} a_n \exp(\mathrm{i}\varphi_n) f(\boldsymbol{r} - \boldsymbol{r}_{\mathrm{c}n}), \tag{1}$$

where a_n and φ_n are the amplitude and phase of the field emitted by the nth microchannel; $f(\boldsymbol{r})$ is a function describing the spatial distribution of the microfibre mode field centred at the point $\boldsymbol{r}_{\mathrm{c}n}$, which is the same for all the microchannels. The centres of microchannels are arranged equidistantly on a circle of radius R_{c} with the centre at the multimode fibre axis.

The propagation of radiation in the RW is described, in the approximation of paraxial and scalar optics, by the following equation for the field amplitude $u(x, y, z)$

$$2\mathrm{i}kn_0 \frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0\,, \tag{2}$$

where $k = 2\pi/\lambda$ is the wave number in vacuum; n_0 is the refractive index of a medium; z is the coordinate in the radiation propagation direction; and x and y are the transverse coordinates. The Talbot distance in the medium is $z_{\mathrm{T}} = 2n_0\Lambda^2/\lambda$, where Λ is the distance between the centres of adjacent microchannels.

Because we assume that the RW maintains one radial mode, the field distribution in the RW is fixed in the radial direction. We can show that the field propagation in this approximation is reduced to propagation in a planar waveguide with the periodicity condition corresponding to a cyclic angular pass around in the azimuthal direction with the period $2\pi R_{\mathrm{c}} = N\Lambda$. The Talbot distance $z_{\mathrm{T}N}$ for this period is N^2 times greater than the distance z_{T} determined by the period Λ. In Refs [10, 11], the field distribution was found for the fractional value of the Talbot distance for a periodic structure formed by step radiation sources with the aperture $2a$, which is less than Λ/N. It was shown in Ref. [12] that by using such a solution in the problem of propagation of the field (1) by the distance $z_{\mathrm{T}}/2 = z_{\mathrm{T}N}\times(2N^2)^{-1}$ and by projecting the result on the modes of microchannels

$$u_{\mathrm{in}}(\boldsymbol{r}) = \sum_{n=0}^{N-1} C_n f(\boldsymbol{r} - \boldsymbol{r}_{\mathrm{c}n}),$$

in the small filling factor limit

$$\frac{2a}{\Lambda} < \frac{1}{N} \tag{3}$$

we can obtain the following simple expression for the coefficients C_n

$$C_n = \frac{(-1)^n}{N\sqrt{\mathrm{i}}} \sum_{j=0}^{N-1} (-1)^j a_j \exp(\mathrm{i}\varphi_j). \tag{4}$$

This means that in the small filling factor limit for any initial set of a_j and φ_j, the distribution of the field incident on the MCF after the round trip in the RW proves to be antiphase, with equal amplitudes in all the microchannels. This field distribution is the only transverse mode of the MCF laser. The eigenvalue of this mode can be found by simple iterations

$$\gamma = \sum_{n=0}^{N-1} \frac{\exp(\mathrm{i}\varphi_n)}{N\sqrt{\mathrm{i}}}, \tag{5}$$

where φ_n are the phase incursions in microchannels. Expression (4) shows that the round trip of radiation in the RW for the limit of narrow microchannels results in the coupling of all the channels. The coupling matrix describes the so-called coupling of 'each channel with each channel' (or the global coupling), which was considered for the first time in paper [13].

It was shown in Ref. [4] that the global coupling results in the separation of an inphase mode in the case of the positive coupling coefficient, which is retained when the dispersion of laser frequencies is appreciable (when the coupling coefficient is negative, an antiphase supermode is separated). The methods for producing coupling that approaches in its exchange radius the global coupling, which were proposed earlier, result in substantial radiation losses [4].

One of the variants of coupling, which was also based on the Talbot effect applied to the laser array, was considered in Ref. [14]. The authors of paper [14] found that the filling factor $\sim 1/N$ provides the coupling that is close to the global one, which makes the inphase and antiphase modes insensitive to the fluctuations of parameters. However, the diffraction radiation losses are large in this case even for an ideal system. The configuration considered here demonstrates the possibility of obtaining the global coupling between the elements with zero losses for an ideal system. This unique property of the configuration is caused by the arrangement of the elements on the ring, so that no end effects are present in the system. Note that a laser array with the global coupling retains the coherence of the total field even in a nonstationary regime caused by the dispersion in the parameters of individual lasers and by the resonance of mode beats with relaxation oscillations [15].

3. Phase locking of a MCF laser with finite-size microchannels

The filling factor of the ring aperture by radiation from microchannels in experiments [7] did not satisfy the inequality (3). However, phase locking was observed when the estimated dispersion of the optical paths of microchannels was several tens of wavelengths. To explain this effect, recall that the phase incursion acquired after propagation in a microfibre depends on the radiation frequency. The difference of the phase incursions that is a multiple of 2π does not affect the summation of the fields. The residues from the division of the total phase incursion after the double passage of microchannels by 2π fill, generally speaking, the entire $0 - 2\pi$ interval with equal probabilities.

As the frequency is changed, a set of these phases also changes (below, we will consider only residual phases). The laser has a property of tuning to the frequency at which the difference between the gain and losses is maximal. The frequency quantisation caused by the presence of longitudinal modes proves to be insignificant in this configuration. This means that upon lasing, not the usual statistical distribution of phase incursions is established but the distribution which provides the better conditions for lasing, the radiation frequency playing the role of a parameter that is automatically matched with these conditions.

There is another parameter in our system, which characterises the radius of radiation transfer between microchannels, namely, the filling factor of the RW aperture by radiation from microchannels. Although this factor is small in experiments [7], it does not satisfies the criterion (3). We can expect that with increasing the microchannel aperture, the field observed after the round trip in the RW will not exactly correspond to an antiphase mode. This means that modes adjacent to the antiphase mode will appear whose structure does not strongly differ from that of the antiphase mode [12].

In particular, the filling factor is $\sim 3/N$ for the experimental conditions [7], and the distribution is excited in the MCF, which is a linear combination of distributions with $m = 8$, 9, and 10. In this case, the coefficients of expansion of the generated distribution over supermodes depend on the distribution of φ_n at the given frequency. Due to diffraction, the field emitted by one of the microchannels does not enter all the microchannels after a round trip in the RF because the condition (3) is not satisfied. In the MCF laser under study, the coupling between all the microchannels is established after a few round trips of radiation in the resonator. We will use, however, expression (5), which is rigorously valid for $2a/\Lambda < 1/N$, assuming that the conditions for the optimal reproduction of the field change weakly with increasing the filling factor.

The algorithm we propose for calculating a supermode in the laser under study consists in the following. For a given random set of the optical paths of microchannels, we find the phase shifts φ_n, which change when the radiation frequency is scanned within the gain band of the medium. For each given set of phases, the modulus of the eigenvalue of a mode of the MCF laser is calculated from expression (5), which is valid in the small filling factor limit (3). This procedure is repeated after changing the radiation frequency. The frequency dependence of the eigenvalue found in this way is used to select the lasing frequency for which a numerical calculation is performed with the help of the three-dimensional diffraction model described in paper [16].

Let us assume for definiteness that the dispersion of the radiation propagation constant in microchannels is caused by their different diameters. Consider a MCF of length L containing N microchannels with radii r_n. The radius of a microchannel is a random quantity, which depends on its number and has the average value a and the dispersion $\Delta \ll a$. We define the radiation wavelength as $\lambda = \lambda_0 + \delta\lambda$, where λ_0 is the wavelength corresponding to the centre of the gain band. Then, the phase incursion φ_n in the nth microchannel can be obtained excluding a term that is the same for all microchannels:

$$\varphi_n(\delta\lambda) \approx 2\delta\beta L \frac{\delta r_n}{a} + 4\delta\beta L \frac{\delta r_n}{a}\frac{\delta\lambda}{\lambda_0}, \tag{6}$$

where δr_n is the variation of the microchannel radius; $\delta\beta$ is the mode correction to the propagation constant averaged over all the microchannels, which can be approximately written in the form $\delta\beta \approx (2.405)^2\lambda_0/(4\pi n_0 a^2)$.

The first term in the right-hand side of expression (6) corresponds to a random dispersion of the phases of the MCF output radiation upon lasing at the wavelength λ_0. This phase dispersion is much higher than 2π for the MCF parameters corresponding to the experiment [7], where 18 microchannels with the average radius $a = 4$ μm were arranged on a circle of radius $R_c = 131$ μm in a multimode fibre of radius 145 μm and $\Delta \sim 0.2$ μm and $L \sim 60$ cm. The second term in the right-hand side of equation (6) causes a change in the distribution of φ_n with changing the radiation frequency. It is obvious that it is reasonable to consider the change in the field frequency within the spectral gain band.

Taking into account (6), expression (5) for the eigenvalue in the small filling factor limit has the form

$$|\gamma(\delta\lambda)| \approx \left| N^{-1} \sum_{n=0}^{N-1} \exp\left[4i\delta\beta L \frac{\delta r_n}{a}\left(1 + \frac{\delta\lambda}{\lambda_0}\right)\right]\right|. \tag{7}$$

The modulus of the quantity γ calculated from expression (7) depending on the radiation-wavelength detuning normalised to the width of the gain band is presented in Fig. 2. The half-width $\delta\lambda_{max}$ of the gain band of a neodymium glass was taken equal to 50 Å in accordance with [17]. A set of radii r_n of microfibres was calculated using a random number generator. One can see from Fig. 2 that frequency variations of the eigenvalue modulus are significant for a given sampling of the optical paths of microfibres. There exist radiation wavelength at which the modulus of the eigenvalue of an antiphase mode exceeds 0.5.

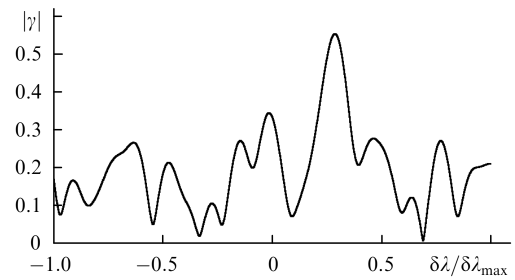

Figure 2. Dependence of the modulus of eigenvalue of an antiphase mode of the MCF laser on the normalised detuning of the radiation wavelength.

By introducing the standard concept of a locking band used in the theory of phase locking (see, for example, [4]), we can describe the observed effect in a simpler way. The number of microchannels for which the phase shift gets into the locking band also depends on the radiation frequency. We calculated the maximum number N_{max} of microchannels (within the spectral gain band) getting into the locking band as a function of the locking band half-width $\varDelta_c$. This dependence averaged over twelve random samplings of microchannel radii r_n is presented in Fig. 3. One can see that the possibility of tuning the radiation frequency within the spectral gain band in the case of a narrow locking band results in a substantial increase in the number of phase-locked elements.

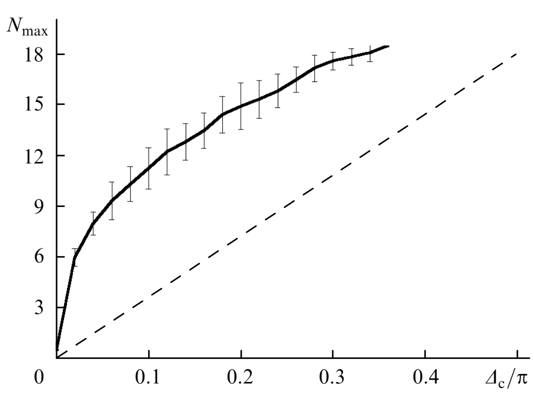

Figure 3. Maximum number of microchannels falling into the locking band averaged over twelve MCF realisations as a function of the half-width of the locking band. The dashed straight line corresponds to the average number of microchannels in the band.

The authors of paper [9] studied a laser with a composite retroreflector and an angular selector and obtained the expression for the maximum brightness of radiation reflected from retroreflectors, which were placed at different distances from the initial plane of the incident-wave front. After averaging over random realisations of the positions of retroreflectors, the authors of paper [9] derived the asymptotic expression for the average maximal brightness using the theory of extreme values. In the limit of small filling factor, the radiation brightness in our system is described by the quantity $\langle|\gamma|^2\rangle N$, where the angle brackets mean averaging over the MCF realisations, so that we have

$$\langle|\gamma|^2\rangle N \approx C + \ln\left[NL\,\frac{\delta\lambda_{max}\varDelta}{a^3\pi}\left(\frac{2.405}{n_0}\right)^2\right], \qquad (8)$$

where $C \sim 0.577$. Fig. 4 shows the dependence of $\langle|\gamma|^2\rangle N$ on the MCF length, which was averaged over twenty five MCF realisations and calculated by expression (8). One can see that, within the error, the square of modulus of the eigenvalue does depend logarithmically on the MCF wavelength. For the parameters used, the efficiency of phase locking of the MCF laser virtually ceases to increase when the MCF length exceeds 1 − 2 m.

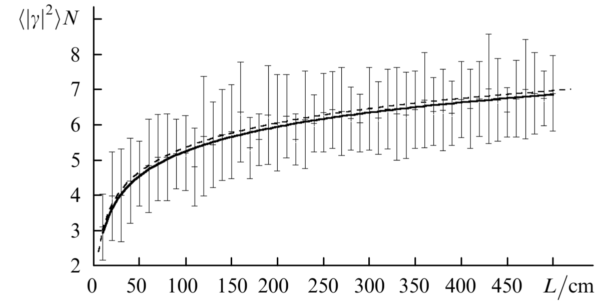

Figure 4. Quantity $\langle|\gamma|^2\rangle N$ averaged over twenty five MCF realisations on the MCF length as a function of the MCF length. The solid curve is the best approximation of the function $A + \ln L$, where $A = 0.67 \pm 0.03$; the dashed curve is calculated by expression (8).

4. Numerical simulation of a MCF laser with a broad spread of the optical paths of microchannels

An optical mode of the MCF laser was calculated numerically by the iteration method using the three-dimensional diffraction program described in Ref. [16]. Each iteration involved the calculation of the double passage of the field through the RW of length $z_T/4$, the projection of the field on the set of modes of microchannels and the addition of a set of phases determined for the given realisation of the MCF and the selected radiation wavelength. These iterations were performed until obtaining the convergence determined by the specified accuracy of the reproduction.

The output radiation of an ideal MCF has the antiphase field distribution between microchannels, which results in the appearance of many peaks in the far-field zone. The far-field zone structure can be substantially improved with the help of a phase plate, which adds the phase shift by π for the microchannels with odd numbers. The far-field distribution of output radiation of the MCF is presented in Fig. 5 for the same MCF realisation as in Fig. 2 for the wavelengths 1.0564 and 1.0554 μm. These wavelengths correspond to the maximum (0.67) and minimum (0.34) values of the modulus of the eigenvalue calculated from (7). In addition, we present for comparison the far-field distribution of output radiation of the MCF laser for the ideal case of the zero dispersion of the MCF parameters. Note that for the optimal frequency corresponding to the maximum eigenvalue, the field distribution exhibits a distinct regular structure (Fig. 5b), whereas when the antiphase mode is poorly reproduced, the distribution pattern is chaotic and no peak is observed on the axis.

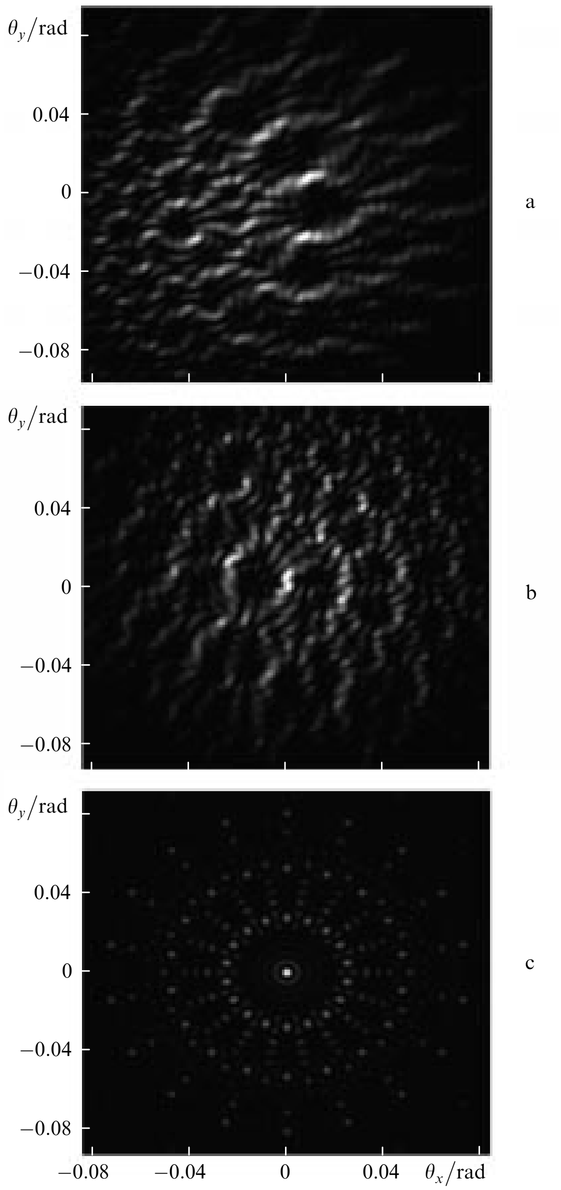

Figure 5. Distribution of the output radiation field of the MCF laser in the far-field zone after propagation through a phase screen at the wavelengths 1.0554 (a) and 1.0564 µm (b), as well as for the zero dispersion of the MCF parameters (c).

Fig. 6 shows the corresponding near-field zone distributions presented in polar coordinates, the radiation amplitude from each microchannel corresponding to a vector length and the radiation phase corresponding to an angle. One can see that the minimum of the eigenvalue (7) (Fig. 6a) corresponds to generation with large field amplitudes in several microchannels and strongly different phases of the fields, so that the destructive interference of the fields results in large losses. These results show that phase locking of the MCF laser is possible at the experimental values of parameters in paper [7]. The MCF laser can be also phase-locked at the wavelengths corresponding to a poor reproduction of the antiphase distribution; however, in this case the losses will be greater and the quality of the output radiation will be worse.

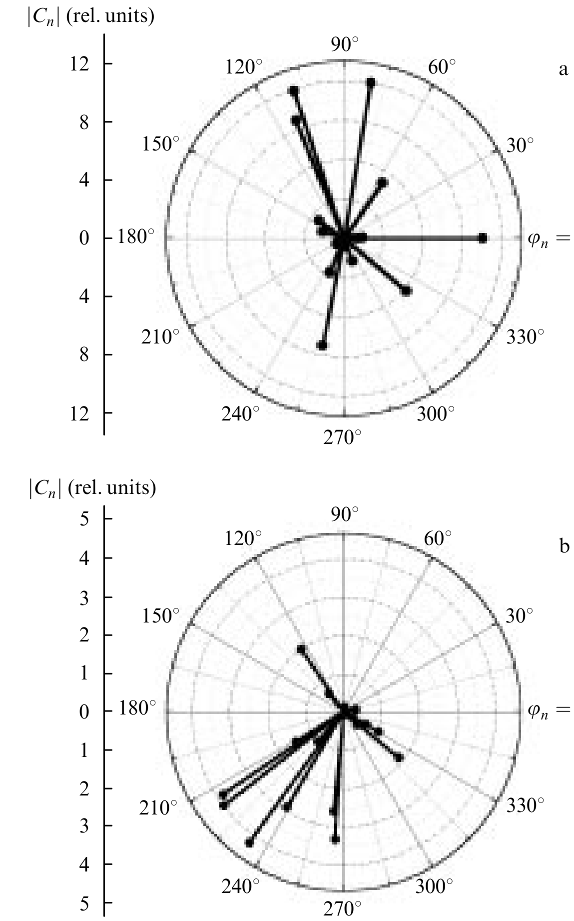

Figure 6. Diagrams of phase φ_n and amplitude C_n of the output radiation for the 1.0554-µm (a) and 1.0564-µm (b) modes of the MCF laser in the near-field zone.

5. Conclusions

We have considered the problem of phase locking of a MCF laser using an intracavity RW filter in the case of a large spread of the optical paths of microchannels. In the limit of small filling factor for the emitting aperture, we found the explicit expression for the eigenvalue of an antiphase mode in the case of an arbitrary spread of the optical paths of microchannels. The use of the RW of length $z_T/4$ as a spatial filter provides a global optical coupling of identical channels without losses. The algorithm has been proposed for calculating the lasing frequency of the system with a large spread of channel lengths, which provides minimal losses for the antiphase mode with an acceptable optical quality of the output radiation.

The numerical simulation has shown that radiation from microchannels is also phase-locked in the case of a finite filling factor, which explains phase locking observed in a MCF laser with a large dispersion of radiation propagation constants in different microchannels. A similar lasing at the frequency providing a minimum of losses of the antiphase mode results in a partial compensation of the effect of optical misalignment of phase locking.

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