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Inversionless radiation amplification by ions having certain velocities in a magnetic field

A.I.Parkhomenko

***Abstract*.** **It is shown that the imposition of external magnetic field on a gas of ionised particles may give rise to inversionless radiation amplification by the ions having certain velocities, which is due to their Larmor rotation. In this case, the majority of ions may reside in the ground state. The integrated absorption coefficient remains positive due to an increase in the radiation absorption by the other half of ions.**

Keywords: *inversionless radiation amplification, active medium.*

1. Introduction

It is known that the Larmor rotation of ions in a magnetic field may result in a dramatic change of their absorption line shape [1, 2]: upon observation across the magnetic field when the condition $\omega_c \gtrsim \Gamma$ is fulfilled (ω_c is the cyclotron frequency of ion rotation and Γ is the homogeneous half-width of the absorption line of ions), the Doppler profile of the ion line splits into a series of equidistant peaks. The width of each peak is equal to the homogeneous width 2Γ of absorption line, and the separation between neighbouring peaks is ω_c. If the average Larmor radius of the ion orbit is much smaller than the wavelength, the absorption line is described by a Lorentzian with a homogeneous width 2Γ, which can be many times smaller than the Doppler width [1].

It is natural to assume that the imposition of an external magnetic field will change not only the shape of the absorption line of ions, but also the absorption by the group of ions having a certain velocity $\boldsymbol{v}$. The corresponding theoretical calculations lead to an entirely unexpected result: it turns out that the groups of ions moving in certain directions in the magnetic field amplify the radiation incident on the medium even when the majority of ions are in the ground state. This work is concerned with the theoretical study of this phenomenon.

Consider the gas of ionised particles in a permanent uniform magnetic field $\boldsymbol{B}$. We assume that the radiation in the form of a travelling monochromatic wave is resonantly absorbed at the $m-n$ transition between the ground (n) and first excited (m) ion levels. We restrict our consideration to the simplest case when the Zeeman splitting of the absorption line can be neglected. For instance, the line splitting does not occur in the case of a simple Zeeman effect (Landé g-factors for the states involved, m and n, are equal) when the radiation polarised linearly along the magnetic field $\boldsymbol{B}$ propagates transversely to $\boldsymbol{B}$.

The probability $P(\boldsymbol{v})$ of radiation absorption at the $m-n$ transition [$P(\boldsymbol{v})$ is the number of radiation absorption events per unit time by a particle with a given velocity $\boldsymbol{v}$ per unit velocity interval] is determined by the off-diagonal element $\rho_{mn}(\boldsymbol{v})$ of the density matrix [3]:

$$P(\boldsymbol{v}) = -\frac{2}{N}\,\mathrm{Re}[\mathrm{i}G^*\rho_{mn}(\boldsymbol{v})]\,,$$
$$|G|^2 = \frac{B_{mn}I}{2\pi}, \quad B_{mn} = \frac{\lambda^2\Gamma_m}{4\hbar\omega}, \tag{1}$$

where B_{mn} is the second Einstein coefficient for the $m-n$ transition; I is the radiation intensity; ω and λ are the radiation frequency and wavelength; and Γ_m is the rate of spontaneous radiative decay of the excited state m.

When the radiation intensity is low under stationary and spatially uniform conditions $\rho_{mn}(\boldsymbol{v})$ is determined from the equation [3]

$$\left[\frac{\Gamma_m}{2} - \mathrm{i}(\Omega - \boldsymbol{kv}) + \frac{\omega_c}{B}[\boldsymbol{vB}]\frac{\partial}{\partial\boldsymbol{v}}\right]\rho_{mn}(\boldsymbol{v})$$
$$= S_{mn}(\boldsymbol{v}) + \mathrm{i}GNW(\boldsymbol{v}), \tag{2}$$

where N is the ion concentration; $\omega_c = eB/Mc$ is the cyclotron frequency of the ion rotation; $\Omega = \omega - \omega_{mn}$ is the radiation frequency detuning; ω_{mn} is the $m-n$ transition frequency; $\boldsymbol{k}$ is the radiation wave vector; $W(\boldsymbol{v})$ is the function describing the Maxwell velocity distribution; $S_{mn}(\boldsymbol{v})$ is the 'off-diagonal' collision integral; e is the elementary electric charge; and M is the ion mass.

For the 'off-diagonal' collision integral $S_{mn}(\boldsymbol{v})$, we will employ the approximation [3] $S_{mn}(\boldsymbol{v}) = -(\Gamma - \Gamma_m/2)\rho_{mn}(\boldsymbol{v})$ commonly used in nonlinear spectroscopy, which implies that collisions cause complete dephasing of the oscillating dipole moment.

When the particle velocity distribution function is close to the Maxwell distribution, the kinetic equation (2) is solved by the Grad method (the method of moments) [4, 5]. We will solve the problem using the simplest approximation

A.I.Parkhomenko Institute of Automatics and Electrometry, Siberian Branch, Russian Academy of Sciences, prosp. akad. Koptyuga 1, 630090 Novosibirsk, Russia;
e-mail: par@iae.nsk.su

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of the Grad method in which the velocity dependence of the off-diagonal element of the density matrix in Eqn (2) can be represented as the sum of the equilibrium distribution $N_{mn}W(\boldsymbol{v})$ and the antisymmetric correction:

$$\rho_{mn}(\boldsymbol{v}) = \left[N_{mn} + \frac{2}{v_T^2}\,\boldsymbol{v}\boldsymbol{j}_{mn}\right]W(\boldsymbol{v}), \tag{3}$$

where $N_{mn} = \int \rho_{mn}(\boldsymbol{v})\mathrm{d}\boldsymbol{v}$; $\boldsymbol{j}_{mn} = \int \boldsymbol{v}\rho_{mn}(\boldsymbol{v})\mathrm{d}\boldsymbol{v}$; $v_T = (2k_\mathrm{B}T/M)^{1/2}$ is the most probable ion velocity; T is the temperature; and k_B is the Boltzmann constant. Expression (3) is applicable in the case of homogeneous broadening of the absorption line ($\Gamma \gg kv_T$) or Doppler broadening ($\Gamma \ll kv_T$) in strong magnetic fields ($\omega_\mathrm{c} \gg kv_T$) when the radiation wave vector $\boldsymbol{k}$ is perpendicular to the magnetic field $\boldsymbol{B}$.

For the zero and first momenta of the kinetic equation (2) we obtain, in view of expression (2):

$$(\Gamma - \mathrm{i}\Omega)N_{mn} + \mathrm{i}\boldsymbol{k}\boldsymbol{j}_{mn} = \mathrm{i}GN,$$
$$(\Gamma - \mathrm{i}\Omega)\boldsymbol{j}_{mn} + \mathrm{i}\boldsymbol{k}\,\frac{v_T^2}{2}\,N_{mn} = \omega_\mathrm{c}[\boldsymbol{j}_{mn}\boldsymbol{h}], \tag{4}$$

where $\boldsymbol{h} = \boldsymbol{B}/B$.

We solve the system of equations (4) to find the probability $P(\boldsymbol{v})$ of radiation absorption:

$$P(\boldsymbol{v}) = PW(v_\perp)W(v_\parallel)\left[1 + \frac{\Omega k v_\perp}{\Gamma}\,\tau(\varphi)\right], \tag{5}$$

where

$$P = \frac{2|G|^2\Gamma}{\Gamma^2 + \Omega^2};$$
$$\tau(\varphi) = \frac{2\Gamma(\Gamma^2 + \Omega^2)\cos\varphi - \omega_\mathrm{c}(3\Gamma^2 + \omega_\mathrm{c}^2 - \Omega^2)\sin\varphi}{\left[\Gamma^2 + (\Omega - \omega_\mathrm{c})^2\right]\left[\Gamma^2 + (\Omega + \omega_\mathrm{c})^2\right]}. \tag{6}$$

Here, $P \equiv \int P(\boldsymbol{v})\mathrm{d}\boldsymbol{v}$ is the velocity-integrated radiation absorption probability in the cases $\Gamma \gg kv_T$ or $\omega_\mathrm{c} \gg kv_T$; v_T is the projection of ion velocity $\boldsymbol{v}$ on the plane perpendicular to the magnetic field; φ is the angle between the radiation direction $\boldsymbol{k}$ and the velocity $\boldsymbol{v}_\perp$ in the coordinate system whose z axis is directed along the magnetic field $\boldsymbol{B}$ and the x axis is directed along the wave vector $\boldsymbol{k}$ (we assume that $\boldsymbol{k} \perp \boldsymbol{B}$); $W(v_\perp)$ and $W(v_\parallel)$ are the Maxwell distributions over the transverse and longitudinal (relative to the magnetic field $\boldsymbol{B}$) projections of the velocity $\boldsymbol{v}$. In the absence of a magnetic field (for $\omega_\mathrm{c} = 0$), the radiation absorption probability $P(\boldsymbol{v})$ is given by the known formula

$$P(\boldsymbol{v}) = \frac{2|G|^2\Gamma W(\boldsymbol{v})}{\Gamma^2 + (\Omega - \boldsymbol{k}\boldsymbol{v})^2}, \tag{7}$$

which coincides, of course, with expression (5) for $\Gamma \gg kv_T$.

The dependence of the absorption probability on the longitudinal projection of velocity $v_\parallel$ in (5) manifests itself only in the Maxwell factor $W(v_\parallel)$ (the magnetic field has no effect on the particle motion along the z axis). That is why it is of interest to consider the integral characteristics

$$P(v_\perp) \equiv P(\boldsymbol{v}_\perp, \varphi) = \int_{-\infty}^{\infty} P(\boldsymbol{v})\mathrm{d}v_\parallel,$$
$$P(\varphi) = \int_0^\infty P(v_\perp, \varphi)v_\perp \mathrm{d}v_\perp\,, \tag{8}$$

where $P(\varphi)$ is the number of radiation absorption events per unit time in a unit angular interval per ion for a given angle φ between the radiation direction and the projection of the ion velocity on the plane perpendicular to the magnetic field. It follows from (5) that

$$P(\boldsymbol{v}_\perp) \equiv P(v_\perp, \varphi) = PW(v_\perp)\left[1 + \frac{\Omega k v_\perp}{\Gamma}\,\tau(\varphi)\right],$$
$$P(\varphi) = \frac{P}{2\pi}\left[1 + \frac{\sqrt{\pi}}{2}\,\frac{\Omega k v_T}{\Gamma}\,\tau(\varphi)\right]. \tag{9}$$

The possibility of appearing negative values of absorption probabilities $P(\varphi)$, $P(v_\perp, \varphi)$, and $P(\boldsymbol{v})$ is determined by the behaviour of the function $\tau(\varphi)$. When the magnetic field is strong enough (for $\omega_\mathrm{c} \gg \Gamma$), the dependence of $\tau(\varphi)$ on Ω has the resonance character and has a maximum at $|\Omega| = \omega_\mathrm{c}$ equal to $\cos\varphi/2\Gamma$. In this case, the alternating (second) term in square brackets in expressions (5), (8), and (9) is, by the order of magnitude, equal to $\Omega k v_T/2\Gamma^2$ and can far exceed unity. Therefore, the ions having certain velocities may exhibit inversionless radiation amplification due to their rotation in the magnetic field.

Note that the alternating term in expressions (5), (8), and (9) responsible for the occurrence of inversionless radiation amplification in the cases $\Gamma \gg kv_T$ or $\omega_\mathrm{c} \gg kv_T$ under study does not make a contribution to the velocity-integrated absorption probability P, which determines the absorption line profile.

Fig. 1 shows the dependence $P(\varphi)$ calculated by expression (9) for different radiation frequency detunings in the case of Doppler broadening of the absorption line. As the unit of measurement, we adopted the probability $P_0 = 2\sqrt{\pi}|G|^2(kv_T)^{-1}$ of radiation absorption at the centre of the line with Doppler broadening in the absence of magnetic field.

Fig. 1a illustrates the behaviour of $P(\varphi)$ in the vicinity of the cyclotron resonance ($\Omega = \omega_\mathrm{c}$). The absorption probability $P(\varphi)$ takes negative values in the angular interval $\Delta\varphi = \pi$ near $\varphi \approx \pi$ (the radiation is amplified by the particles moving primarily oppositely to the radiation propagation). In the case shown in Fig. 1a, the amplitudes of negative and positive values of $P(\varphi)$ are identical and, despite the large radiation frequency detunings ($\Omega \approx 5kv_T$), are approximately equal to the radiation absorption probability P_0 at the line centre in the absence of a magnetic field.

At the same time, the velocity-integrated radiation absorption probability P is low: $P/P_0 \approx \Gamma/25\sqrt{\pi}kv_T \approx 2.3 \times 10^{-4}$. In the case shown in Fig. 1a, the factor $\Omega k v_T/2\Gamma^2 \sim 2 \times 10^4$, and therefore the absorption probability $P(\varphi)$ in the magnetic field increases by more than a factor of 10^4. Therefore, the following dramatic situation takes place inside the medium. Approximately one half of the particles in the medium moving in one direction absorbs the radiation and the other half moving in the opposite direction amplifies it. However, the contributions of these two groups of particles to the integrated absorption proba-

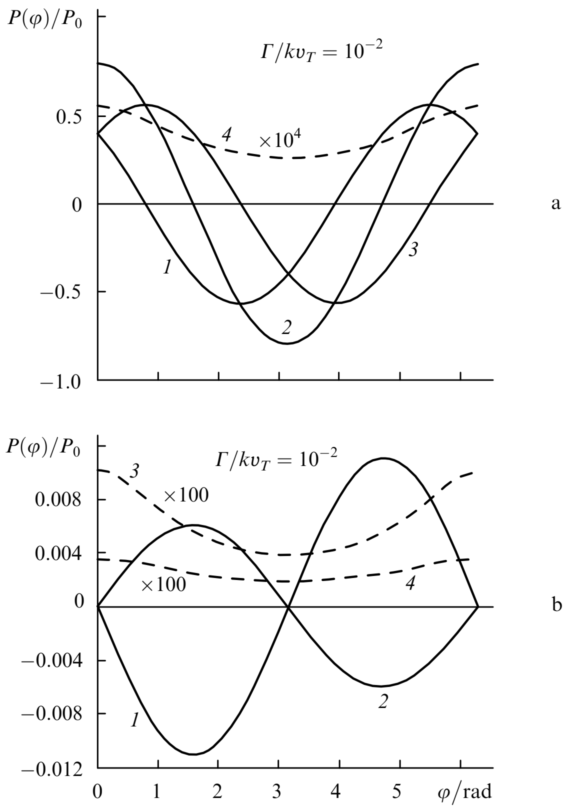

Figure 1. Dependences of $P(\varphi)$ on φ in the vicinity of cyclotron resonance for $\Omega/kv_T = 4.99$ (*1*), 5 (*2*, *4*), and 5.01 (*3*) (a) and away from the cyclotron resonance for $\Omega/kv_T = 4$ (*1*, *3*) and 6 (*2*, *4*) (b). The solid lines correspond to $\omega_c/kv_T = 5$ and the dashed lines, to $\omega_c = 0$.

bility compensate each other almost completely, and the medium as a whole absorbs the radiation weakly. The oscillation amplitude $P(\varphi)$ decreases with distance away from the cyclotron resonance (Fig. 1b), but can nevertheless be rather high compared to P_0.

The effect considered in this work is caused by the phase shift of the oscillations of the induced ion dipole moment, which depends on the ion velocity. This phase shift appears as ions are moving around their cyclotron orbits.

The inversionless radiation amplification by ions with certain velocities may occur in the case of both Doppler ($kv_T \gg \Gamma$) and homogeneous ($kv_T \ll \Gamma$) absorption line broadening. The Grad method, which was used to solve the problem, implies that the magnitude of magnetic field is bounded below ($\omega_c \gg kv_T$) in the case of Doppler broadening. This limitation is not fundamental from the physical standpoint. In the case of Doppler broadening, the effect of inversionless radiation amplification would therefore be expected to occur under the natural condition of the separation of cyclotron resonances in the absorption spectrum $\omega_c \gtrsim \Gamma$ [1, 2]. For $\Gamma \sim 10^7$ s^{-1} and the ion mass $M \sim 20$ au, the condition $\omega_c \gtrsim \Gamma$ is fulfilled in magnetic fields $B \gtrsim 2 \times 10^4$ G.

The inversionless radiation amplification under study is a 'hidden' effect in the sense that it makes no contribution to the integrated absorption probability $P \equiv \int P(\boldsymbol{v})\mathrm{d}\boldsymbol{v}$, which defines the absorption line profile. The question arises of whether the alternation of sign of radiation absorption probability $P(\boldsymbol{v})$ can have any effect on observable physical effects at all. The answer to this question is positive. Indeed, one of the effects sensitive to the features of radiation absorption probability $P(\boldsymbol{v})$ is the effect of light-induced drift (LID) [6, 7].

The LID velocity is known [7] to be proportional to the first-order moment $\boldsymbol{Q}$ of the radiation absorption probability $P(\boldsymbol{v})$ [$\boldsymbol{Q} \equiv \int \boldsymbol{v}P(\boldsymbol{v})\mathrm{d}\boldsymbol{v}$]. For the LID of ions in a weakly ionised gas in an external magnetic field perpendicular to the radiation propagation direction ($\boldsymbol{k} \perp \boldsymbol{B}$), the moment $\boldsymbol{Q}$ is defined by the function $P(\boldsymbol{v})$ (5). The contribution to the moment $\boldsymbol{Q}$ is made only by the second term in square brackets, which completely determines the alternating behaviour of $P(\boldsymbol{v})$. The LID of ions in the magnetic field was theoretically investigated in Ref. [8]. The formulas for the projections $u_\parallel$ and $u_\perp$ of the ion drift velocity $\boldsymbol{u}$ on the directions $\boldsymbol{k}$ and $\boldsymbol{n}$ derived in Ref. [8] can be written as:

$$u_\parallel \equiv \frac{\boldsymbol{k}}{k}\boldsymbol{u} = \tau_\sigma \left[A\frac{\boldsymbol{k}}{k}\boldsymbol{Q} - C\boldsymbol{n}\boldsymbol{Q} \right],$$
$$u_\perp \equiv \boldsymbol{n}\boldsymbol{u} = \tau_\sigma \left[C\frac{\boldsymbol{k}}{k}\boldsymbol{Q} + A\boldsymbol{n}\boldsymbol{Q} \right], \tag{10}$$

where

$$A = 1 - \frac{\omega_c^2}{\nu_n(\Gamma_m + \nu_m)};$$
$$C = \frac{\omega_c(\Gamma_m + \nu_m + \nu_n)}{\nu_n(\Gamma_m + \nu_m)}; \quad \boldsymbol{n} = \frac{[\boldsymbol{k}\boldsymbol{B}]}{kB}; \tag{11}$$

and ν_i is the average transport frequency for the collisions of ions in the state $i = m, n$ with buffer particles; the formula for τ_σ is given in Ref. [8]. Under laboratory conditions, the light-induced ion drift can manifest itself in the form of an electric current (light-induced current [9]). Therefore, the electric signal, which arises from the light-induced ion drift and is proportional to the first-order moment $\boldsymbol{Q}$ of radiation absorption probability $P(\boldsymbol{v})$, will be produced due to both positive and negative values of $P(\boldsymbol{v})$. If the dependence of light-induced ion drift signal on different parameters (on the radiation frequency detuning Ω, the cyclotron frequency of the ion rotation ω_c, etc.) is adequately described by expressions (10), this will confirm that the light-induced ion drift is entirely due to the alternating part of $P(\boldsymbol{v})$ under the conditions involved.

One can also expect that the alternation of sign of $P(\boldsymbol{v})$ will affect the absorption of a probe (relatively weak) field. Indeed, the off-diagonal element $\rho_{mn}(\boldsymbol{v})$ of the density matrix, which defines the alternating radiation absorption probability $P(\boldsymbol{v})$ according to expression (1), will appear in the kinetic equations for the density matrix describing the interaction of the probe field with ions. For this reason, the final expressions for the velocity-integrated absorption probability for the probe field will include the $\boldsymbol{v}$-averaged products of $P(\boldsymbol{v})$ by other functions of $\boldsymbol{v}$, which also depend on the radiation frequency detunings of the strong and probe fields. Eventually, this will determine the sensitivity of the probe field to the alternating part of $P(\boldsymbol{v})$.

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