PACS numbers: 42.25.Gy; 42.65.Re
DOI: 10.1070/QE2002v032n04ABEH002189

Focusing femtosecond radiation with an axicon

A.A.Babin, D.V.Kartashov, D.I.Kulagin

***Abstract*.** **Focusing femtosecond radiation with a refracting axicon lens is stimulated within the framework of the second approximation of the dispersion theory, using the Kirchhoff – Fresnel method. It is shown that, within the angles considered (up to 0.2 rad), an increase in the pulse duration due to its dispersive spreading inside the axicon has the major effect on the field distribution under study, while the angular dispersion is rather small.**

Keywords: *axicon, focusing, material dispersion, femtosecond pulse.*

The use of an axicon (conical lens) for focusing terawatt femtosecond radiation into the bulk of transparent dielectrics makes it possible to form extended channels with an aspect ratio of $> 10^4$ [1]. However, despite the fact that such a lens has long been known, was often employed in laser experiments [2 – 5], and the focusing of monochromatic radiation with its help was studied experimentally in detail [5, 6], the formation of the caustic of an axicon onto which a femtosecod pulse is incident was not analysed. Note that the focusing of a short light pulse has certain specific features. In particular, a broad frequency spectrum of a femtosecond laser pulse leads to an increased influence of dispersion effects on the characteristics of the focal region, as in the case of an ordinary lens [7].

In this work, we have calculated the longitudinal and transverse field-intensity distributions on the axis of the refracting axicon with a rectilinear generatrix for a femtosecond laser pulse incident on its entrance. The calculations take into account the effect of the axicon material dispersion.

We simulated the femtosecond radiation focusing using the results obtained in [5] by the Kirchhoff – Fresnel method. In this study, the field distribution in a caustic was found for a monochromatic beam with a plane phase front incident onto an optically thin axicon. Since a linear problem is considered, the spatial distribution of the field $E(z,r,t)$ of a short pulse, can be written, using the results from [5], as the analytic expression for $E(z,r,t)$ formed by an axicon lens:

A.A.Babin, D.V.Kartashov, D.I.Kulagin Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, 603950 Nizhnii Novgorod, Russia

Received 16 October 2001
Kvantovaya Elektronika **32** (4) 308 – 310 (2002)
Translated by A.S.Seferov

$$E(z,r,t) = -\frac{\mathrm{i}\alpha}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}\omega (n_\omega - 1)(2\pi k z)^{1/2} E_\omega(0,r,\omega)$$
$$\times J_0[k\alpha(n_\omega - 1)r] \exp\left[\mathrm{i}k\left\{z + \frac{r^2}{2z} + \alpha(n_\omega - 1)\right.\right.$$
$$\left.\left.\times[R - \alpha(n_\omega - 1)z/2]\right\} - \mathrm{i}\omega t\right], \qquad (1)$$

where z and r are the longitudinal and transverse coordinates; t is the time; ω is the frequency; α is the angle at the axicon base; n_ω is the axicon refractive index; $k = 2\pi/\lambda$; λ is the radiation wavelength; R is the axicon radius; $E_\omega(0,r,\omega)$ is the spectrum of the field incident onto the axicon; and J_0 is the zero-order Bessel function. The integrand in (1) is, except for the factor $\exp(-\mathrm{i}\omega t)$, the field of a harmonic signal at an arbitrary point r, z behind the axicon [5].

Consider first the simplest case of $n(\omega) = \text{const}$. This approximation corresponds to a long pulse when the dispersion of the conical-lens material can be neglected. This approximation is obviously valid for a reflecting axicon and a femtosecond pulse, if the reflection coefficient of the working surface is frequency-independent. Analysing expression (1) shows that, in this approximation, the femtosecond pulse and continuous radiation are focused by the axicon almost identically, until the pulse duration reaches a few cycles of the laser field. In other words, the caustic parameters and the field intensity in the lens are virtually identical for a monochromatic signal and a not too short femtosecond pulse. This is a surprising fact. However, if we recall that the main contribution to the field at an arbitrary point z on the axicon axis is made by a ring with a radius $r = (n-1)\alpha z$ and a width $\Delta r \sim \lambda$ and also take into account that, for a femtosecond pulse, the caustic represents the result of the interference of the field radiated by an approximately the same ring, the result obtained above becomes clear.

In order to take into account the axicon material dispersion, we expand its refractive index into a Taylor series in the vicinity of the central frequency ω_0 of the pulse up to the second-order terms with respect to $\Delta\omega$ inclusive. Such an approximation corresponds to the second approximation of the dispersion theory [7]

$$n(\omega) = n(\omega_0) + n'(\omega_0)(\omega - \omega_0) + \frac{n''(\omega_0)}{2}(\omega - \omega_0)^2, \quad (2)$$

where the derivatives $n'(\omega_0)$ and $n''(\omega_0)$ are related to the group velocity u and parameter k_2, which characterises the group velocity dispersion, via the expressions [7]

$$u = c[n + \omega_0 n'(\omega_0)]^{-1}, \quad k_2 = [n''(\omega_0)\omega_0 + 2n'(\omega_0)]/c. \quad (3)$$

In this approximation, integral (1) was calculated by the method of stationary phase and the input signal was assumed to be transform-limited with a Gaussian shape $E(r, \omega)$:

$$E(0, r, \omega) = E_0 \exp\left[-\frac{r^2}{r_0^2} - (\omega - \omega_0)^2 \tau^2 / 4\right].$$

Here, τ is the input pulse duration at the $1/e^2$ intensity level. The refractive index, from which the quantities (3) were determined, was calculated from the Sellmayer formulas presented in [8] for the K8 optical glass and KU1 fused silica. The values of other parameters in (1) used in calculations were as follows: $\alpha = 0.2$ rad, $R = 1$ cm, the wavelength $\lambda = 0.8$ μm, and a typical radius of the incident beam $r_0 = 0.5$ cm.

The longitudinal and transverse distributions of the light intensity at the axicon axis normalised to the input intensity I_0 are shown in Figs 1 and 2. As follows from the analysis of these dependences, the dispersion of the axicon material becomes substantial beginning with the pulse duration approximately 30 fs for the K8 and KU1 glasses. The maximum radiation intensity at the axis decreased by a factor of ~ 2, as the pulse duration decerased down to 10 fs.

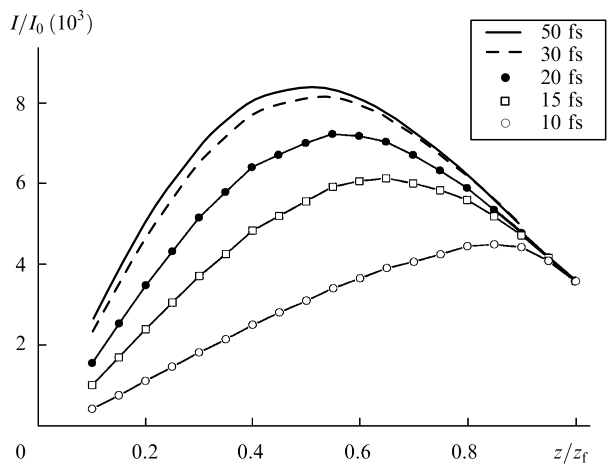

Figure 1. Dependences of the peak intensity on the longitudinal coordinate z/z_f, where $z_f = R/[\alpha(n_0 - 1)]$, for various pulse durations τ. The curve for $\tau = 50$ fs almost coincides with the curve for a continuous signal.

Note that the dispersion-induced distortions of the longitudinal and transverse field distributions are substantially different. The peak of the longitudinal distribution $I(z)$ is shifted to larger z. Physically, this fact is explained by a more significant effect of dispersion on the near-axial part of the incident beam, which passes through the thickest layer of the axicon substance. As follows from Fig. 1, the angular dispersion during refraction is insignificant, because the field strength at the axis at $z/z_f \approx 1$ for input pulse durations τ in a range of 10–50 fs almost coincides with the field strength for a continuous signal. The transverse structure of the

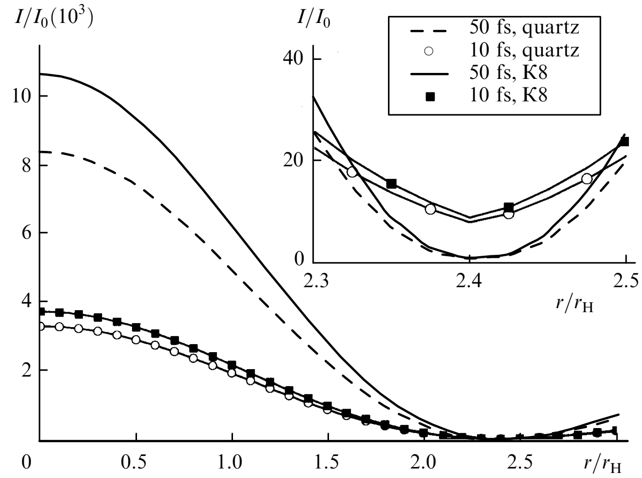

Figure 2. Calculated transverse distributions of the field intensity at the axicon caustic for $z/z_f = 0.5$ and $\tau = 10$ and 50 fs for silica and K8 glass axicons. The inset shows the function $I(r/r_H)/I_0$ near $r/r_H = 2.4$, $r_H = [k\alpha(n_0 - 1)]^{-1}$.

beam in the caustic of the axicon (Fig. 2) is virtually independent of τ for pulse durations under study.

A comment concerning the precision of the stationary phase method used in calculations should be made. Because a significant difference between the parameters of focusing of continuous and pulse radiation by the axicon begins with rather short (< 30 fs) pulses for lenses used in experiments, we will perform the estimate for simplicity for a harmonic signal. In this case, the axicon-caustic formation can be described by a parabolic equation

$$2ik \frac{dA(z, r_\perp)}{dz} + \Delta_\perp A(z, \boldsymbol{r}_\perp) = 0, \quad (4)$$

where $A(z, \boldsymbol{r}_\perp)$ is the field amplitude. We will not consider the beam transformation inside the axicon, and will take into account only the distortion of the initially plane phase front by the axicon, which is valid for an optically thin element. Thus, the field distribution

$$A(0, \boldsymbol{r}_\perp) = A_0 \exp\left[-\frac{r_\perp^2}{r_0^2} - ik\alpha(n-1)(R - r_\perp)\right], \quad (5)$$

will play the role of the initial condition for Eqn (4), where r_0 is the beam radius.

Since Eqn (4) is linear, it can be solved using the Fourier transform. When carrying out numerical calculations, the parameter r_0 and the grid pitch along the transverse coordinates, in which the Fourier transform is performed, should be selected.

It is known that when the beam is focused by an axicon, a typical transverse beam size in the caustic is determined only by the radiation wavelength and axicon parameters, is independent of the beam radius r_0 at the entrance aperture [5], and is rather small (~ 2.7 μm in our case). This leads to a maximum beam radius $r_0 \approx 0.1$ mm that can be obtained using the available computational capabilities.

The results of the numerical solution of Eqn (4) show that the stationary phase method used for calculating the Kirchhoff–Fresnel integral well describes the longitudinal intensity distribution at the axicon axis, and the transverse distribution of the light intensity is described well within a region of the order of the caustic diameter.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant No. 01-02-17512).

References

1. Babin A.A., Kiselev A.M., Pravdenko K.I., Sergeev A.M., Stepanov A.N., Khazanov E.A. *Usp. Fiz. Nauk*, **169**, 80 (1999).
2. Mc.Cleod J.H. *J. Opt. Soc. Am.*, **44**, 1871 (1954).
3. Korobkin V.V., Margolin L.Ya., Polonskii L.Ya., Pyatnitskii L.N. *Kvantovaya Elektron.*, **16**, 9 (1989) [*Sov. J. Quantum Electron.*, **19**, 4 (1989)].
4. Durfee III C.G., Milchberg H.M. *Phys. Rev. Lett.*, **71**, 2461 (1993).
5. Korobkin V.V., Polonskii L.Ya., Poponin V.P., Pyatnitskii L.N. *Kvantovaya Elektron.*, **13**, 265 (1986) [*Sov. J. Quantum Electron.*, **16**, 178 (1986)].
6. Andreev N.E., Bychkov S.S., Kotlyar V.V., Margolin L.Ya., Pyatnitskii L.N., Serafimovich P.G. *Kvantovaya Elektron.*, **23**, 130 (1996) [*Quantum Electron.*, **26**, 126 (1996)].
7. Akhmanov S.A., Vysloukh V.A., Chirkin A.S. *Optika femtosekundnykh lazernykh impul'sov* (Optics of Femtosecond Laser Pulses) (Moscow: Nauka, 1988).
8. USSR–DDR 'Optical Glass' Catalogue.