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Phase locking of three lasers optically coupled with a spatial filter

A.F.Glova, A.Yu.Lysikov

Abstract. **An expression is obtained for approximate calculation of optical coupling coefficients for lasers in a 1D array phase-locked with a focal spatial filter. The phase-locking region for three lasers is found. The phase-locking efficiency is shown to be substantially dependent on the filter transparency. The results of calculations agree with the experimental data.**

Keywords: *optical coupling coefficients, laser-array phase locking, spatial filter*.

The aim of the laser-array phase locking is to enhance the axial radiation intensity. This is possible if the inphase distribution of the field is achieved over the combined aperture of the array. Upon laser-array phase locking with the help of optical coupling introduced between separate lasers, the inphase lasing is more stable in the case of a global optical coupling when each laser is coupled with many other lasers [1, 2].

One of the ways for achieving such a coupling in periodical laser arrays is the spatial filter method (see, for example, [3]). A spatial filter is usually placed in the focus of an intracavity optical system, which is matched with the cavity and consists of two focusing lenses [4 – 6], a focusing lens and a concave mirror [7, 8], or two mirrors [9]. The phase-locking stability and efficiency under these conditions depend on the filter parameters, which determine, in particular, the optical coupling coefficients between the lasers, which in turn impose restrictions on the admissible initial spread of the frequencies of the lasers [8].

It is most reasonable to use numerical methods for the theoretical study of phase locking of many lasers [10 – 13]. Nevertheless, the deriving of analytic expressions for basic parameters of lasers being phase-locked is undoubtedly of practical interest. Among these parameters are the coefficients of optical coupling between the lasers. In this paper, we obtained the expression for approximate calculation of these coefficients for a linear laser array with a focal spatial filter. The results of the calculation are used for determining the phase-locking region for three lasers depending on the filter parameters, the effect of its transparency on the phase-locking efficiency for two and three lasers, and are compared with experimental data.

A.F.Glova, A.Yu.Lysikov State Scientific Center of Russia, Troitsk Institute for Innovation and Fusion Research, 142190 Troitsk, Moscow oblast, Russia; e-mail: afglova@triniti.ru; lysikov@triniti.ru

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Consider a 1D laser array with a period d and common flat mirrors. One-dimensional arrays with a cavity of this type consist usually of semiconductor lasers [3, 4] or waveguide CO_2 lasers [6 – 8]. Let one the mirrors of the array be formed by a matched telescope [7, 8] containing a focusing lens with the focal distance F and a concave mirror. A spatial filter is placed in the telescope focus as a system of identical slits oriented perpendicular to the plane of the laser array and having a period $h = F\lambda/d$ (where λ is the radiation wavelength) and the width b of opaque regions.

We assume that the field of single-mode lasing on the aperture of the ith laser has the form

$$u_i(x_{1i}) = E_i U_i(x_{1i}), \tag{1}$$

where E_i is the amplitude; $U_i(x_{1i})$ is the normalised distribution of the transverse-mode field; and x_{1i} is the transverse coordinate in the aperture plane. The field of the ith laser on the aperture of the jth laser with the coordinate x_{1j} after passing of radiation through the telescope can be written in the form [14]

$$v_i(x_{1j}) = \left(\frac{k}{2\pi \mathrm{i} F}\right)^{1/2} \int q(x_2) \exp\left(\frac{\mathrm{i} k x_2 x_{1j}}{F}\right) \mathrm{d}x_2, \tag{2}$$

where $k = 2\pi/\lambda$; x_2 is the transverse coordinate in the filter plane; and

$$q(x_2) = f(-x_2 - \delta) f(x_2 - \delta) g(x_2). \tag{3}$$

Expression (3) contains the transmission function of the filter

$$f(x_2 - \delta) =$$

$$\begin{cases} 1 \text{ for } -a/2 + \delta + nh \leqslant x_2 \leqslant a/2 + \delta + nh, \\ 0 \text{ for } -a/2 + \delta + b + nh \leqslant x_2 \leqslant a/2 + \delta + b + nh, \end{cases} \tag{4}$$

where $n = 0, 1, 2, ..., a = h - b$ is the width of transparent parts of the filter; δ is the filter displacement perpendicular to the optical axis from the optimal position [7, 8], and the function

$$g(x_2) = \left(\frac{k}{2\pi \mathrm{i} F}\right)^{1/2} \int u_i(x_{1i}) \exp\left(\frac{-\mathrm{i} k x_2 x_{1i}}{F}\right) \mathrm{d}x_{1i}. \tag{5}$$

Let us define the amplitude coefficient of optical coupling between the ith and jth lasers as

$$M_{ij} = \frac{\int v_i(x_{1j}) u_j(x_{1j}) dx_{1j}}{\int u_j^2(x_{1j}) dx_{1j}}. \quad (6)$$

After integration over an individual slit of the filter and replacing integration over all the slits and over x_{1i} by summation from $-\infty$ to $+\infty$, and taking into account the normalisation of the transverse modes and expressions (1)–(5) for the optical coupling coefficient, we obtain

$$M_{ij} = M_l = \frac{\sin(\pi a l/h)}{\pi l} \exp\left(2\pi i \delta \frac{l}{h}\right), \quad (7)$$

where $l = |i - j|$. One can see from (7) that the values of M_l become complex for displaced filters and determine not only the amplitude but also the phase of the coupling coefficients.

Table 1 presents coupling coefficients that were calculated from (7) and numerically calculated in paper [7] for an undisplaced filter with $a = 1.1$ mm and $h = 1.4$ mm, which was used for phase locking of five waveguide CO_2 lasers. One can see that the values of the corresponding coefficients that were calculated by these two methods are close.

Table 1.

M_0	M_1	M_2	M_3	M_4	Notes
0.789	0.198	−0.155	0.096	−0.035	Calculation by expression (7)
0.691	0.263	−0.181	0.084	0.005	Calculated in [7]

Let us use expression (7) for determining the phase-locking region for three CO_2 lasers in the region of detunings of the cavity lengths of adjacent lasers. For this purpose, we consider first the system of equations with dimensionless parameters and variables for an undisplaced filter, which can be obtained from [1] for lasers of this type:

$$\dot{A}_1 = {}^1/_2(g_1 - g_{th})A_1 + M\cos(\Delta\varphi_{21})A_2 + m\cos(\Delta\varphi_{31})A_3,$$

$$\dot{A}_2 = {}^1/_2(g_2 - g_{th})A_2 + M\cos(\Delta\varphi_{21})A_1 + M\cos(\Delta\varphi_{32})A_3,$$

$$\dot{A}_3 = {}^1/_2(g_3 - g_{th})A_3 + m\cos(\Delta\varphi_{31})A_1 + M\cos(\Delta\varphi_{32})A_2,$$

$$\dot{\varphi}_1 = \Delta_1 + M\frac{A_2}{A_1}\sin(\Delta\varphi_{21}) + m\frac{A_3}{A_1}\sin(\Delta\varphi_{31}), \quad (8)$$

$$\dot{\varphi}_2 = \Delta_2 - M\frac{A_1}{A_2}\sin(\Delta\varphi_{21}) + M\frac{A_3}{A_2}\sin(\Delta\varphi_{32}),$$

$$\dot{\varphi}_3 = \Delta_3 - m\frac{A_1}{A_3}\sin(\Delta\varphi_{31}) - M\frac{A_2}{A_3}\sin(\Delta\varphi_{32}),$$

$$\dot{g}_i = \frac{g_{0i} - g_i}{\tau} - g_i A_i^2,$$

where $i = 1, 2, 3$; A_i and φ_i are the amplitude and phase of the field in the ith laser; $\Delta\varphi_{ij} = \varphi_i - \varphi_j$ is the difference of field phases for the ith and jth lasers; g_i and g_{0i} are the gain and small-signal gain, respectively; g_{th} is the threshold gain, which is the same for all the lasers; Δ_i is the detuning of the emission frequency of the ith laser from the gain line centre, which coincides with the eigenfrequency of the laser cavity, neglecting the frequency pulling; τ is the inversion relaxation time; and $M = M_{12} = M_{21} = M_{32} = M_{23}$ and $m = M_{13} = M_{31}$ are the amplitudes of the optical coupling coefficients, which are independent of the permutation of indices and depend only on the absolute value of the difference between the indices [7]. The system (8) is a natural generalisation of the system of equations for two lasers [15] for the case of three lasers and is similar to the system of equations considered in paper [16]; however, unlike [16], it takes into account the coupling between two extreme lasers with $i = 1, 3$.

Because it is very difficult to analyse the stability of stationary solutions of system (8), we will restrict ourselves to a search for the relation between the stationary values of $\Delta\varphi_{ij}$ and $\Delta_{ij} = \Delta_i - \Delta_j$ by assuming that $M, m \ll g_{th}$. This assumption means that the amplitudes of fields in optically coupled lasers and uncoupled lasers only slightly differ from each other and leads to the approximate equality $g_i \approx g_{th}$. Taking this equality into account, it follows from the last three equations from system (8) that $A_i^2 \approx A_{0i}^2 = (1/\tau)(g_{0i}/g_{th} - 1)$. Therefore, the relation between the output powers of individual lasers is determined by the distribution of the pump power over them. Let this distribution be symmetrical with respect to the central laser, i.e., $A_2/A_1 = A_2/A_3 = c$, which allows us to represent the fourth, fifth, and sixth equations of system under stationary conditions in the form

$$\Delta_1 + Mc\sin(\Delta\varphi_{21}) + m\sin(\Delta\varphi_{31}) = 0,$$

$$\Delta_2 - \frac{M}{c}\sin(\Delta\varphi_{21}) + \frac{M}{c}\sin(\Delta\varphi_{32}) = 0, \quad (9)$$

$$\Delta_3 - m\sin(\Delta\varphi_{31}) - Mc\sin(\Delta\varphi_{32}) = 0.$$

By subtracting the second equation of system (9) from the first and third equations and subtracting the third equation from the first one, we pass to the system of equations containing the relative detunings Δ_{ij}. Because $\Delta\varphi_{31} = \Delta\varphi_{21} + \Delta\varphi_{32}$ и $\Delta_{31} = \Delta_{21} - \Delta_{23}$, we obtain the following system of two equations (since the equation containing Δ_{31} is a linear combination of these equations and can be neglected)

$$\Delta_{21} + M\left(c + \frac{1}{c}\right)\sin(\Delta\varphi_{21}) - \frac{M}{c}\sin(\Delta\varphi_{32})$$
$$+ m\sin(\Delta\varphi_{21} + \Delta\varphi_{32}) = 0, \quad (10)$$
$$\Delta_{23} + \frac{M}{c}\sin(\Delta\varphi_{21}) - M\left(c + \frac{1}{c}\right)\sin(\Delta\varphi_{32})$$
$$- m\sin(\Delta\varphi_{21} + \Delta\varphi_{32}) = 0.$$

A system that is more convenient for analysis can be obtained by adding and subtracting equations (10):

$$\Delta_+ + M\left(c + \frac{2}{c}\right)\sin(\Delta\varphi_{21}) - M\left(c + \frac{2}{c}\right)\sin(\Delta\varphi_{32}) = 0,$$
$$\Delta_- + Mc\sin(\Delta\varphi_{21}) + Mc\sin(\Delta\varphi_{32}) \quad (11)$$
$$+ 2m\sin(\Delta\varphi_{21} + \Delta\varphi_{32}) = 0,$$

where $\Delta_+ = \Delta_{21} + \Delta_{23}$ and $\Delta_- = \Delta_{21} - \Delta_{23}$.

In the case of an undisplaced filter, which selects an inphase mode, phase locking can be achieved when $|\Delta\varphi_{21}| \leqslant \pi/2$ and $|\Delta\varphi_{32}| \leqslant \pi/2$, i.e., the phase-locking region lies within a square on the $\Delta\varphi_{21}, \Delta\varphi_{32}$ plane, which is limited by straight lines $\Delta\varphi_{21} = \pm\pi/2$ and $\Delta\varphi_{32} = \pm\pi/2$. The boundaries of the phase-locking region on the $\varDelta_+, \varDelta_-$ plane can be found from system (11) by substituting to it by turn the phase differences $\Delta\varphi_{21} = \pm\pi/2$ for $-\pi/2 \leqslant \Delta\varphi_{32} \leqslant \pi/2$ and $\Delta\varphi_{32} = \pm\pi/2$ for $-\pi/2 \leqslant \Delta\varphi_{21} \leqslant \pi/2$. For example, a part of the boundary on the given plane for $\Delta\varphi_{21} = -\pi/2$ and $-\pi/2 \leqslant \Delta\varphi_{32} \leqslant \pi/2$ is described by the second-order curve

$$\varDelta_- + \frac{c^2}{2+c}\varDelta_+ - 2Mc = 2m\left\{1 - \left[1 - \frac{c}{M(2+c^2)}\varDelta_+\right]^2\right\}^{1/2}, \quad (12)$$

where $0 \leqslant \varDelta_+ \leqslant 2M(2+c^2)/c$. Analysis of expression (12) and similar expressions for other phase differences shows that all these expressions are equations of ellipse. Therefore, the phase-locking region on the $\varDelta_+, \varDelta_-$ plane is located between segments of ellipses. Fig. 1 shows these regions for three filters with the period $h = 0.5$ mm and different b for $c = 1.5$.

Note that the form of the phase-locking region in the coordinates $\varDelta_+, \varDelta_-$ substantially depends on the value and sign of the coefficient m. For $m > 0$, there is no self-intersections in the phase-locking region (Fig. 1a); for $m < 0$, this region can be divided into three subregions (Fig. 1c); while for $m = 0$, this region represents a rhomb (Fig. 1b). The optical coupling between 'nearest neighbours' corresponds to the case $m = 0$ when the form of the region coincides that found in paper [16].

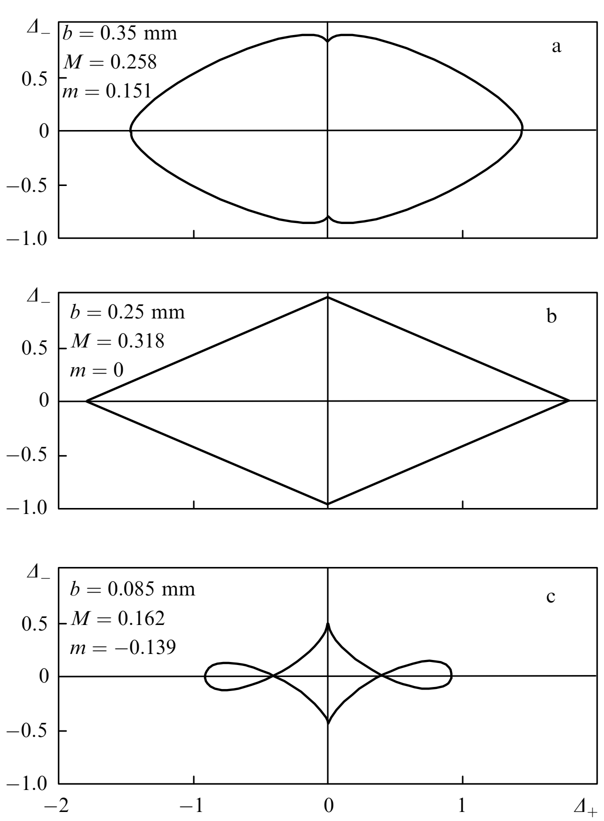

Figure 1. Phase-locking regions calculated for three lasers for $c = 1.5$, $h = 0.5$ mm, $b = 0.35$ (a), 0.25 (b) and 0.085 mm (c), $M = 0.258$ (a), 0.318 (b), and 0.162 (c), m= 0.151 (a), 0 (b), and −0.139 (c).

The parameters of a spatial filter in Fig. 1c correspond to the conditions of the experimental study [8] of the efficiency and stability of phase locking of three waveguide CO_2 lasers. It was shown in paper [8] that in the case of stable phase locking of extreme pairs of lasers with the given filter when the third laser is uncoupled and the dimensional detunings of laser cavity lengths are $\varDelta_{12}^{\mathrm{d}} = \lambda/30$ and $\varDelta_{23}^{\mathrm{d}} = \lambda/20$, the coupling of the third laser may result in quenching of coherent lasing in all the three lasers. To explain this effect, we represent the region in Fig. 1c in the coordinates of real dimensionless detunings $\varDelta_{21}$ and $\varDelta_{23}$ by using the transformations

$$\varDelta_{23} = (\varDelta_+ - \varDelta_-)/2, \varDelta_{21} = (\varDelta_+ + \varDelta_-)/2\,. \quad (13)$$

Fig. 2 shows the form of the region in new coordinates. The straight lines AA, BB, CC, and DD drawn through the coordinate values ±0.4 restrict the phase-locking band of each of the pairs of extreme lasers, the third laser being uncoupled, which has, according to [8], the dimensional boundaries $\varDelta^{\mathrm{d}} = \pm\lambda/16$. The point R corresponds to detunings $\varDelta_{12}^{\mathrm{d}} = \lambda/30$ and $\varDelta_{23}^{\mathrm{d}} = \lambda/20$. The dimensionless detunings are obtained by multiplying dimensional detunings by $2\pi/\lambda$. One can see from Fig. 2 that stable phase locking of the extreme pairs of lasers, when the third laser is uncoupled, is not sufficient for phase locking of all the three lasers because all the three lasers can be found outside the phase-locking region due to the narrowing of this region upon coupling with the third laser. The phase-locking region is narrowed due to the destructive interference of the intrinsic and injected fields in lasers when m is negative.

An increase in the number of lasers in an array from two to three, a filter and detunings being the same, not always enhances the maximum efficiency of phase locking. For example, for small values of b, the loss of radiation on the filter for three lasers becomes greater than that for two lasers

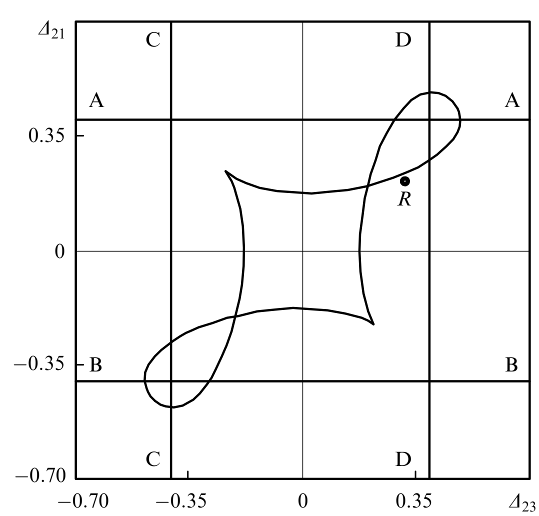

Figure 2. Phase-locking region for three lasers calculated in coordinates $\varDelta_{23}, \varDelta_{21}$ for a filter with $h = 0.5$ mm and $b = 0.085$ mm. Coordinates of the point R are taken from paper [8].

because the intermediate maxima of the intensity distribution are overlapped by the filter. The corresponding boundary values of b can be estimated from the stationary system (8) for $g_{0i} = g_0 =$ const, if, unlike the previous analysis, we take into account small variations of the field amplitudes caused by optical coupling and neglect the initial frequency detunings by assuming that $\Delta_i = 0$:

$$\delta A_i = -\delta g_i(1 + A_0^2\tau)/(2A_0 g_{\mathrm{th}}\tau), \tag{14}$$

where $A_0 = [(1/\tau)(g_0/g_{\mathrm{th}} - 1)]^{1/2}$; $\delta g_1 = \delta g_3 = -2(M + m)$ and $\delta g_2 = -4M$ for three lasers, and $\delta g_1 = \delta g_2 = -2M$ for two lasers.

Let us represent the phase-locking efficiency in the form

$$P = \frac{\sum A_i^2}{\sum A_{0\mathrm{f}}^2}, \tag{15}$$

where $A_i^2 = (A_0 + \delta A_i)^2$; $A_{0\mathrm{f}} = [(1/\tau)(g_0/g_{\mathrm{t}} - 1)]^{1/2}$ and $g_{\mathrm{t}} = g_{\mathrm{th}} + 2\ln M_0$ [14] are the field amplitude and threshold gain in the absence of a filter, respectively. If P_3 and P_2 are the phase-locking efficiencies for three and two lasers, respectively, then the requirement $P_3/P_2 > 1$, taking into account (14) and (15), reduces to the fulfilment of the condition

$$M > -2m\,. \tag{16}$$

For a filter with $h = 0.5$ mm, condition (16) means that P_3 exceeds P_2 when $b > 0.17$, in qualitative agreement with experiments [8] performed at small detunings of the order of $\lambda/40$.

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