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Polarisation inhomogeneities of a ring resonator and nonreciprocity of counterpropagating waves

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***Abstract*. The nonreciprocity of the characteristics of counterpropagating waves in a ring resonator formed by totally reflecting prisms is analysed. The polarisation-inhomogeneous properties of a prism resonator and their effect on the nonreciprocity of the frequencies of counterpropagating waves are studied.**

Keywords: *ring laser, totally reflecting prism, polarisation-inhomogeneous wave, magnetic field*

1. Introduction

In optical ring resonators, whose anisotropy determines the linear polarisation in the ideal case, the eigenstates of polarisation of counterpropagating waves differ from linear ones in practice. In a magnetic field, such waves acquire a frequency shift even in a resting ring laser, which leads to errors in a laser gyroscope. As a rule, two main factors responsible for the appearance of ellipticity are considered: the fabricational and adjustment errors in the resonator, resulting in a nonplanar deformation of the axial contour, and the induced linear phase anisotropy (birefringence) of the material of reflectors.

This problem is of special significance for KM type laser gyroscopes whose resonator is formed by totally reflecting (TR) prisms [1]. In one of the first papers devoted to laser gyroscopes [2], the advantages of TR prisms as ideal reflectors were mentioned, however, it was also noted that the above-mentioned factors may lead to considerable difficulties in the use of prisms in ring resonators. In actual practice, the frequency shift in a magnetic field caused by the distortions of the prism resonator anisotropy can be reduced to the required minimum (no more than 0.01 Hz Oe^{-1}) through a combination of constructional and technological solutions based on the anisotropy studies for a prism resonator [3 – 6].

However, the resonator characteristics in the above papers were analysed without taking into consideration the polarisation inhomogeneity, which leads to the dependence of the polarisation state on transverse as well as longitudinal coordinates. The presence of such inhomogeneities is the third factor causing the nonreciprocity of counterpropagating waves. The aim of this paper is to make up this deficiency.

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2. Results of anisotropy studies in a prism resonator neglecting the polarisational inhomogeneity of prisms

It is convenient to calculate the ellipticity angle χ and the azimuth angle ψ for natural waves in a ring resonator by using the method of polarisation-induced perturbations [5, 7].

The frequency shift $\Delta\nu$ in a prism resonator placed in a magnetic field $\boldsymbol{H}$ depends on the geometry of the axial loop, the perturbation parameter, and the orientation of the magnetic field vector. For a *uniform* magnetic field, the indicatrix $\Delta\nu(\boldsymbol{H})$ lies in the resonator plane and has a figure-of-eight shape, i.e., there exists a zero sensitivity direction. An important feature of the combined action of anisotropic prisms is that the resultant frequency shift depends on the stress distribution in the prism and may even be equal to zero in principle.

In the ideal situation, the axial loop of a prismatic ring resonator is two-dimensional, and the amplitude – phase anisotropy of the prism is characterised by the coefficient $T \approx 0.8707\exp(\mathrm{i}0.15\pi)$. The counterpropagating waves have a linear polarisation, the loss in the p components being smaller. The nonplanar deformation of the axial loop of the resonator makes it sensitive to the magnetic field. Note that in this case, the local magnetic sensitivity nearly coincides with the sensitivity to a uniform magnetic field. This is due to the fact that the ellipticity of counterpropagating waves in such a resonator is almost the same in all resonator arms.

In a real resonator, it is necessary to take into consideration the mechanical stresses arising in prisms due to the fact that the prisms seal the vacuum channels of the monoblock and are subjected to mechanical strains under atmospheric pressure. If photoelasticity is taken into account, the polarisational properties of a prism are equivalent to the properties of a set of linear phase plates whose axes rotate gradually upon a displacement from the centre of symmetry to the periphery of the prism along the y axis [6]. Fig. 1 shows a system of lines along which the quasi-principal stresses are directed at each point in a TR prism (for two projections). Ellipticity does not arise if the vector $\boldsymbol{E}$ of linearly polarised light wave is directed along these lines.

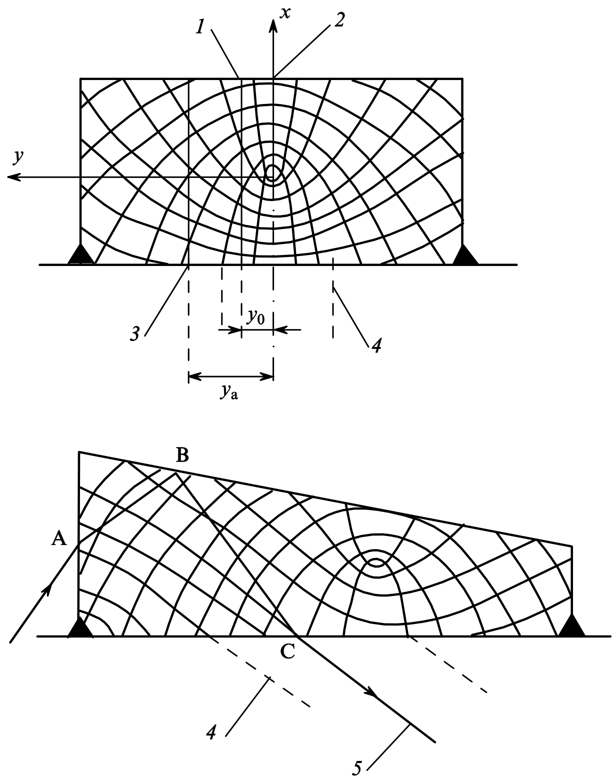

Figure 1. Induced linear phase anisotropy of a TR prism in two projections: (*1*) axial loop plane; (*2*) stress symmetry plane; (*3*) plane for anisotropy measurements on a polarimeter; (*4*) vacuum channel; (*5*) axial beam; ABC is the beam trajectory in the prism.

The ellipticity arises on the axis of a beam passing through a stressed prism if the plane formed by an axial beam in the prism does not coincide with the centre of symmetry of the stresses. In practice, this is caused by an inaccurate adjustment of the resonator or by a violation of stress symmetry due to defects in the soldered seam around the prism. The transformation of the components of the vector $\boldsymbol{E}$ upon the passage of radiation through the stressed region of the prism can be described in the first approximation by the Jones matrix

$$\mathbf{M} = \begin{pmatrix} 1 - \boldsymbol{a}^2(y - y_0)^2/2 & \mathrm{i}a(y - y_0) \\ \mathrm{i}a(y - y_0) & 1 - \boldsymbol{a}^2(y - y_0)^2/2 \end{pmatrix}. \quad (1)$$

Here, the coordinate axis y is connected with the axial loop of the resonator, which is displaced relative to the centre of symmetry of stresses by a small amount $y_0 = 0 - 1$ mm in the sagittal plane, and a is the linear phase anisotropy parameter, which depends on the conditions of fastening of the prism (in particular, on the state of the soldered or glued seam around the prism) and the fitting surface of the monoblock. This parameter characterises the variation in the anisotropy of the prism in the longitudinal and transverse directions. Since the prism rests on the monoblock all along the edge of the channel, the shear stresses determining the ellipticity for the incident p-component of the radiation increase in the longitudinal z-direction from zero to the maximum value near the centre of the prism in the total internal reflection (TIR) region. The value of a in an assembled resonator can be estimated from measurements with a polarimeter whose axes are oriented along the x and y axes (Fig. 1). Usually, the value of this parameter is $a = 1.4 \times 10^{-3} - 4.4 \times 10^{-3}$ rad mm^{-1}.

The results of investigation of the effect of nonplanarity and stresses on the polarisation parameters of the KM-11 prism ring resonator are presented in Table 1.

3. Consideration of the polarisation inhomogeneity of a prism resonator

Theoretical and experimental studies have revealed that laser radiation is polarisation-inhomogeneous in most optical instruments, including laser gyroscopes. The state of polarisation of polarisation-inhomogeneous waves (PIWs), specified in a unified polarisation basis at all points in space, varies regularly in both transverse and longitudinal directions. A plane wave is a polarisation-homogeneous wave in a Cartesian basis, while the dipole radiation in the wave zone is polarisation-homogeneous in a spherical basis.

The entire computational formalism existing at present [7] has been developed for plane waves, and the interrelation between polarisation and wave characteristics (the magnitude and direction of the wave vector, the beam diameter, and the wavefront curvature) is completely neglected.

The analysis of waves with a complicated polarisation structure [8–11] is mainly based on the ray approach according to which the radiation is treated as a set of rays with different states of polarisation. In such an approach, one has to consider the evolution of the polar-

Table 1.

Polarisation characteristics of the resonator	Nonplanar axial loop		Stress (birefringence)
	Misalignment of the reflecting spherical face of the prism	Tilt of the refracting face of the prism in sagittal plane	
Error parameter	$\sigma_g = 3.7'$	$\sigma = 10'$	$\varepsilon_a = 2^\circ$, $y_0 = 0.5$ mm
Maximum ellipticity angle $/'$	2	3	6
Polarisation loss (%)	5×10^{-5}	1.8×10^{-4}	0.9×10^{-4}
Maximum sensitivity to local magnetic field/Hz Oe^{-1}	~ 0.38	~ 0.57	~ 0.8
Maximum sensitivity to uniform magnetic field/Hz Oe^{-1}	~ 0.45	~ 0.62	~ 2.3

isation state of many rays, but the phase relations arising between them because of a difference in their path lengths that remains constant over the cross section of the rays are neglected. It is important to note that the ray approach is not applicable for solving the polarisational problems in optical resonators since the methods used in this approach are based, as a rule, on separate calculations of polarisation and optical wave characteristics. In actual practice, such characteristics for a resonator with TIR are not separable in principle.

The main property of PIWs is that the components of the vector $\boldsymbol{E}$ have different amplitude – phase distribution in an arbitrary polarisation basis that is common for all points in space. In other words, the presence of a PIW can be described with the help of a superposition of three completely or partially coherent waves with orthogonal orientations of the vector $\boldsymbol{E}$. The ray vectors of these waves as well as their phase velocities may not coincide at an arbitrary point in space.

The prism ring resonator considered here has a considerable polarisation inhomogeneity. It was mentioned in Ref. [1] that even in a perfectly adjusted resonator, the disagreement between the curvature of the wave front and the optical surface leads to polarisation inhomogeneity of the radiation, i.e., to the spatial dependence of the ellipsometric parameters in transverse and longitudinal directions. A similar situation takes place for a stressed prism. If the axial beam passes in the stress symmetry plane in which the shear stresses are equal to zero ($y_0 = 0$), the ellipticity angle at the periphery of the beam ($y = w_y \approx 0.35$ mm) attains a value of $6'$. In this case, the ellipticity angle at the other end of the beam will have the same value but the opposite sign.

Consider now the effect of such a polarisation inhomogeneity on the nonreciprocity of counterpropagating waves. For this purpose, we use the method of polarisation-wave matrix [13 – 15] applicable for calculations of distortions of the polarisation structure, losses, and radiation frequency in an optical resonator.

The basic idea behind the method is as follows. The polarisation-inhomogeneous laser radiation is presented as a coherent vector superposition of the transverse Hermitian – Gaussian modes with different polarisation states, amplitudes, and their own phase shifts. The intensity of modes decreases with increasing mode order, and the analysis can be restricted only by zeroth, first and second-order modes in the paraxial region.

Calculations are performed in the following order:

(1) The parameters of the fundamental mode of the optical system are calculated by neglecting the polarisation inhomogeneity.

(2) The polarisation-wave vector $\boldsymbol{D}$ whose components are complex Hermitian – Gaussian modes is written. The radiation of an optical resonator without polarisation inhomogeneity contains only one nonzero component, which is the Jones vector of the fundamental Hermitian–Gaussian mode. 'Parasitic' modes appear in a real resonator, and the vector $\boldsymbol{D}$ can be written as $\boldsymbol{D}_0 = (\boldsymbol{D}_{00}, \boldsymbol{D}_{10}, \boldsymbol{D}_{01}, \boldsymbol{D}_{20}, \boldsymbol{D}_{02}, \boldsymbol{D}_{11})^{-1}$.

(3) The block polarisation-wave matrix is constructed for each polarisation-inhomogeneous element (PIE) taking into account the wave parameters of radiation that were determined earlier. For this purpose, the Jones matrix is recorded for the PIE, its elements being functions of transverse coordinates. Each such function is expanded into a descending series in the Hermitian polynomials, the quantities $\sqrt{2}x/w_x$ and $\sqrt{2}x/w_y$ being chosen as their arguments, where w_x and w_y are the beam radii in the meridional (xz) and sagittal (yz) planes. The polarization-wave matrix describes the interaction between modes forming the PIW.

(4) The block matrix of isotropic optical gaps between PIEs is written. This is a diagonal matrix, each matrix element being the product of a unit matrix and the coefficient G_{mn}, which has the form $G_{mn} = [(1 + d_1 Q^*)/(1 + d_1 Q)]^{1+(m+n)/2}$ in the particular case of an optical system without astigmatism. Here, $Q = \rho - \mathrm{i}\omega$; $\omega = \lambda/\pi w^2$; ρ is the curvature of the wave front; w is the beam radius; and d_1 is the length of the optical gap. This matrix describes the intermode dispersion.

In the general case, a laser system may also contain polarisation devices, which can be treated as polarisation-homogeneous. Such devices are described by a diagonal polarisation-wave matrix whose elements are Jones matrices.

Let us apply this method for analysing the nonreciprocity of counterpropagating waves in a prism resonator caused by polarisation inhomogeneity of the TR prisms. Consider a simplified resonator scheme: four reflectors have the amplitude – phase anisotropy of the TR prisms with a coefficient $T = |T| \exp(\mathrm{i}\theta)$; the focal length of the reflecting surface in the sagittal plane is $f_y = 2100$ mm, the length of the resonator arm is $l = 110$ mm, and the PIE which is a nonreciprocating element in this case, is placed in the middle of the resonator arm (in the waist as shown in Fig. 2). The elements of the Jones matrix (1) of the given PIE depend on the coordinate y.

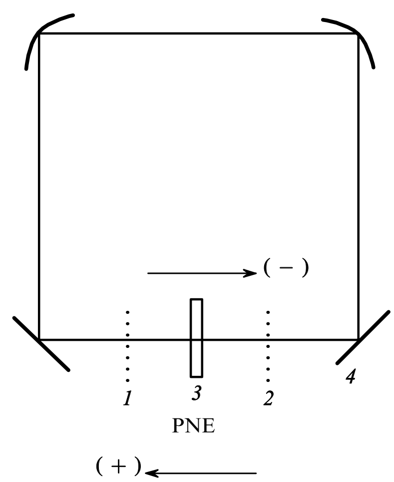

Figure 2. Schematic diagram of a ring resonator: (*1*) resonator cross section behind polarisation-nonreciprocal element along the forward wave (+); (*2*) resonator cross section behind polarisation-nonreciprocal element in the direction of the backward wave (–); (*3*) polarisation-nonreciprocal element; (*4*) reflector with an amplitude-phase transmission coefficient T.

A formal inclusion of this matrix in the matrix equation according to the Jones method is not possible because the PIE distorts the wave parameters of the beam and, conversely, the variation of the polarisation state in the cross section depends on the curvature of reflecting surfaces and the perimeter of the resonator. We shall use the following notation: $\hat{\mathbf{M}}_d$ is the block matrix; $\mathbf{M}_{mn}$ is the internal matrix in which the indices m and n correspond to the Hermitian–Gaussian mode indices like as for $\boldsymbol{D}_{mn}$. The natural waves are obtained from a solution of the equation for the polarisation-wave matrices:

$$\Lambda\begin{pmatrix}\boldsymbol{D}_{00}\\ \boldsymbol{D}_{01}\\ \boldsymbol{D}_{02}\end{pmatrix}=\hat{\mathbf{M}}_d\begin{pmatrix}\mathbf{I} & 0 & 0\\ 0 & \mathbf{I}\exp(\mathrm{i}\phi_{0y}) & 0\\ 0 & 0 & \mathbf{I}\exp(\mathrm{i}2\phi_{0y})\end{pmatrix}$$

$$\times\begin{pmatrix}\mathbf{M}_{00} & \mathbf{M}_{01} & 0\\ \mathbf{M}_{01} & \mathbf{I} & 0\\ \mathbf{M}_{02} & \mathbf{M}_{01} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\boldsymbol{D}_{00}\\ \boldsymbol{D}_{01}\\ \boldsymbol{D}_{02}\end{pmatrix}, \qquad (2)$$

where

$$\mathbf{M}_{00}=\begin{pmatrix}A_{00} & 0\\ 0 & D_{00}\end{pmatrix};\ \ \mathbf{M}_{01}=\begin{pmatrix}0 & B_{01}\\ C_{01} & 0\end{pmatrix};$$

$$\mathbf{M}_{02}=\begin{pmatrix}A_{02} & 0\\ 0 & D_{02}\end{pmatrix};$$

$A_{02}=D_{02}=-a^2w_{0y}^2/16$; $B_{01}=C_{01}=\mathrm{i}aw_{0y}/2\sqrt{2}$; $A_{00}=D_{00}=1+2A_{02}$; $w_{0y}\approx 0.35$ mm is the beam radius in the cross section in front of the PIE (at the beam waist) for the sagittal plane; ϕ_{0y} is the phase incursion for the TEM_{01} mode as compared to the TEM_{01} mode after the round trip in the resonator, which is determined by the trace of the cyclic ray matrix $\mathbf{M}_{1y}$ for the sagittal section: $\phi_{0y}=\arccos[(A_y+D_y)/2]$ (A_y, D_y are the diagonal elements of $\mathbf{M}_{1y}$; in our case, $\phi_{0y}\approx 140°$).

By solving Eqn (2), we obtain the correction to the eigenvalues of the unperturbed resonator, the correction to the polarisation variable on the beam axis, and the distribution of the ellipsometric parameters of the wave in transverse as well as longitudinal direction. From Eqn (2) we obtain the polarisation variable for a forward wave passing through the resonator clockwise (in the cross section (*1*) behind the PIE along the path of the forward wave):

$$\Gamma_1^{(+)}=\frac{\mathrm{i}ay}{1-T^4\exp(\mathrm{i}\phi_{0y})}.$$

Thus, the presence of a PIE leads to the appearance of the orthogonal s-component of the vector $\boldsymbol{E}$ in the resonator, having the same distribution as the TEM_{01} mode. The beam is stretched along the y axis. In addition, the radiation also contains the TEM_{02} mode having the p-polarisation, as the fundamental mode, but its effect on the polarisation state can be neglected. Because of the phase incursion between the modes TEM_{00} and TEM_{01} forming a PIW, the optical gap is equivalent to the linear phase anisotropy distributed in the longitudinal direction. Measuring the distance z from the waist containing the PIE, we can write

$$\Gamma^{(+)}=\frac{\mathrm{i}ay\exp[\mathrm{i}\phi_y(z)]}{1-T^4\exp(\mathrm{i}\phi_{0y})}.$$

Here, $\phi_y(z)=\arctan(\lambda z/\pi w_{0y}^2)$. Therefore, the ellipticity and the azimuth vary continuously not only in transverse, but also in longitudinal direction. At each reflector, the value of $\Gamma^{(+)}(z)$ is multiplied by the amplitude–phase anisotropy parameter T. As a result, we obtain in the cross section (*2*) after the round trip of radiation in the resonator

$$\Gamma_2^{(+)}=\frac{B_{01}T^4\exp[\mathrm{i}\phi_{0y}(z)]}{1-T^4\exp(\mathrm{i}\phi_{0y})}.$$

For the backward wave (see Fig. 2), the distribution of the polarisation state in the resonator is symmetric relative to the PIE: $\Gamma_1^{(-)}=-\Gamma_2^{(+)}$ and $\Gamma_2^{(-)}=-\Gamma_1^{(+)}$.

The above analysis shows that, in contrast to the conventional polarisation nonreciprocity of counterpropagating waves [4, 5, 16], the polarisation inhomogeneity leads to the dependence of the difference in the intensities of counterpropagating waves in a given cross section on the transverse coordinates:

$$I^{(+)}-I^{(-)}=Ka^2y^2\exp\left(-\frac{2x^2}{w_x^2}\right)\exp\left(-\frac{2y^2}{w_y^2}\right), \qquad (3)$$

where K is a coefficient that depends on the amplitude–phase anisotropy of the reflectors, curvatures of the reflecting surfaces, and the optical length of the resonator.

The corrections to the eigenvalue of the Jones operator caused by the polarisation inhomogeneity are identical for counterpropagating waves. This means that there is no nonreciprocal frequency shift in a resonator without an active medium. The losses are proportional to the square of the product of the anisotropy parameter a and the beam diameter w_y.

If the prism resonator is placed in a magnetic field H but there are no distortions of linear polarisation on the beam axis (the polarisation variable Γ_0 is equal to zero [7]), there is no nonreciprocity of frequencies and losses of counterpropagating waves either. Thus, the reciprocity condition of counterpropagating waves is the symmetry of polarisation inhomogeneity relative to the meridional and sagittal cross sections of Gaussian beams in the ring resonator.

If the polarisation inhomogeneity is asymmetric, the frequency shift $\Delta\nu_n$ and the losses of natural waves in the prism resonator placed in a magnetic field H are proportional to the product $B_{02}\Gamma_0VHd$, where d is the optical path length in all prisms, V is the Verdet constant, and $B_{02}=a^2w_y^2\approx 10^{-7}$. In this case, the polarisation inhomogeneity gives a small correction to the existing nonreciprocity of counterpropagating waves, and hence it can be neglected while calculating $\Delta n/H$ for a prism resonator.

It should be noted that in the case of frequency degeneracy of transverse modes ($\phi=0$), the above method is not applicable (the corrections increase sharply) and further investigations must be carried out.

4. Conclusions

Thus, an analysis of the nonreciprocity of counterpropagating waves in a prism resonator shows that the optical elements of the prism resonator generate polarisation-inhomogeneous waves with an ellipticity angle varying over a wide range (from $-6'$ to $+6'$) in the cross section.

There is no polarisation-inhomogeneity-induced frequency shift of counterpropagating waves in a magnetic field. Upon a distortion of the resonator anisotropy caused, for example, by tension, the contribution to the magnetic sensitivity due to polarisation inhomogeneity is negligibly small.

The beam diameter, and the curvature of the wave front of counterpropagating waves in the resonator in the general case, which are caused by the polarisation inhomogeneity, do not coincide in a given cross section. In the presence of an active medium, this may lead to a nonlinear nonreciprocity of counterpropagating waves.

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