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Using waveguide scattering of laser radiation for determining the autocorrelation function of statistical surface roughness within a wide range of changes of the roughness correlation interval

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***Abstract*. An electrodynamic problem of laser radiation scattering in an integrated-optical waveguide containing small statistical irregularities (interface roughness and irregularities of the refractive indices of the waveguide-forming media) is considered. The possibility of using the waveguide scattering of laser radiation for extracting the information on the statistical properties of irregularities from noisy data of the scattering diagram in a far-field zone is shown. An algorithm for reconstructing the autocorrelation function of irregularities for the correlation interval changing within a wide range is described. The possibility of restoring a given Gaussian autocorrelation function that describes statistical irregularities of the waveguide substrate surface for a correlation interval changing between 10 nm and 10 μm and a high-level additive white real noise is shown by computer simulation.**

Keywords: *integrated-optical waveguide, statistical irregularities, inverse waveguide scattering problem, white noise.*

1. Introduction

The extensive development of integrated optics in the three recent decades has promoted the investigations of laser radiation scattering in irregular planar optical waveguides (PWs) in various aspects [1 – 10]. As a rule, the main attention of authors was focused on the solution of a so-called *direct scattering problem*, when the amplitude – phase distribution or the intensity distribution of laser radiation scattered in a waveguide are sought for. The scattering diagrams (indicatrices) thus found were employed to solve an *inverse scattering problem* (ISP) by the known comparison method, when, using one or another technique, the coincidence (e.g., within the rms accuracy) of the measured and theoretically calculated scattering diagrams is achieved.

Sometimes, in order to solve the ISP, simplified approaches based on a formal inversion of the expressions for scattering diagrams are exploited. As was correctly noted in [10], to acquire the statistical data for the spectral components of irregularities from the light scattering data in a sufficiently wide frequency band, more complex models than such a simplified model are necessary. Indeed, the use of a fairly simple expression for the scattering diagram when solving the ISP is justified, if a spatial resonance between the incident plane wave and continuous spectrum of irregularities [2, 11] is observed, i.e., when the observation point and the radiation source are located in the Fraunhofer zone with respect to the scattering region. In such a geometry of the problem, only a narrow beam of plane waves corresponding to a certain harmonic in the spectrum of irregularities reaches the point of interest. This type of scattering is sometimes called resonant or selective.

In this case, the scattered radiation is detected in a so-called zone of spectrum separation [11]. When irregularities are of statistical nature, diffraction spectra become spread and, instead of a sum of a finite number of plane waves, there is an integral field expansion in terms of plane waves at each point [2, 11, 12]. If the scattering conditions for the waveguide mode over the statistical ensemble of irregularities (this can be a statistical surface roughness or a statistical irregularity of the refractive index of the waveguide layer) satisfy the first approximation of the perturbation theory, then the scattered-radiation intensity, which is detected by a point (with a sufficiently small aperture) photodetector in a far-field zone or in a Fourier plane equivalent to it, is actually a mapping of the spectral density function (SDF) of the ensemble in the space of wave numbers.

As is known, in this case, the scattering coefficient depends only on the second-order statistical characteristics of the ensemble of irregularities: the SDF or its Fourier transform, which is the autocorrelation function (ACF). The ACF and SDF contain the complete information on the basic parameters of the ensemble under investigation. In the case considered, the first approximation of the perturbation theory is satisfied, if, for example, the rms height of statistical irregularities of the optical PW boundaries is small compared to both the laser radiation wavelength and waveguide layer thickness.

When direct and inverse scattering problems are solved, both differential (scattering diagrams) and integral scattering characteristics can be used. The following characteristics can be classified as integral ones: the relative loss of the waveguide mode power due to scattering (damping coefficient); the ratio of powers scattered backward and forward at a specified angle; and the ratio of upward- and downward-scattered light powers [2, 7, 8]. In this work, preference is given to the differential scattering characteristics

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due to the fact that they contain more information than the integral characteristics of waveguide scattering of laser radiation [12–14].

2. Theoretical analysis of the waveguide scattering problem

Consider the scattering of a waveguide mode excited by laser radiation in an integrated-optical waveguide containing random (statistical) irregularities (see Fig. 1). As a rule, a three-layer PW consists of the following layers: a framing medium, a wave-guiding layer, and a substrate with the refractive indices n_1, n_2, and n_3, respectively. The irregularities of the PW structure can be caused by irregularities (roughness) of the interfaces of the waveguide-forming media, subsurface defects (a so-called disturbed or cracked layer), and irregularities of the refractive index of the waveguide layer. When considering a scattering problem, the waveguide-layer irregularities and subsurface defects can be described in a similar way — as the refractive index irregularities in the corresponding waveguide medium. To simplify the analysis of the problem, we neglect these irregularities as well as the cross-correlation coupling between irregularities of interfaces.

The electrodynamic problem of the scattering of a waveguide mode propagating in an integrated-optical waveguide with random irregularities is solved by the method of coupled modes using the perturbation theory. In a general case, the electromagnetic field $\boldsymbol{E}$ in an irregular PW is described by an equation that has the following form in Cartesian coordinates [2]:

$$\nabla^2\boldsymbol{E}+\nabla\left(\boldsymbol{E}\frac{\nabla\varepsilon_i}{\varepsilon_i}\right)+\omega^2\mu\varepsilon_i\boldsymbol{E}=0, \qquad (1)$$

where ε_i is the permittivity of the ith PW layer ($i = 1, 2, 3$); $\omega = 2\pi f$; f is the frequency of the electromagnetic field $\boldsymbol{E}$; μ is the magnetic permeability of the layers; $\omega\sqrt{\mu\varepsilon_i} = n_i k$; n_i is the refractive index of the ith layer; $k = 2\pi/\lambda$; λ is the laser radiation wavelength in vacuum; and $\nabla^2 = \Delta$ is the Laplacian.

Consider the propagation of the fundamental TE mode with the components E_{0y}, H_x, and H_z in the waveguide along the z axis (the TM mode is analysed similarly). The total field in an irregular optical PW can be written as the sum of the fields of the incident waveguide mode $\boldsymbol{E}_{0y}$ and scattered wave $\boldsymbol{E}_{\rm s}$: $\boldsymbol{E} = \boldsymbol{E}_{0y}(x,z) + \boldsymbol{E}_{\rm s}(x,y,z)$. We assume that the permittivity can be represented in the form $\varepsilon_i(x,y,z) = \varepsilon_{0i}(x,z) + \Delta\varepsilon_i(x,y,z)$, where $\varepsilon_{0i}(x,z)$ describes the regular properties of the corresponding PW layer, and the additional term $\Delta\varepsilon_i(x,y,z)$ characterises three-dimensional irregularities of the waveguide structure (both the roughness of the interfaces of the PW media and irregularities of the refractive index of the PW ith layer).

In this case, Eqn (1) can be written as an approximate three-dimensional equation. Retaining only the first-order smallness terms in $\boldsymbol{E}_{\rm s}$ and $\Delta\varepsilon_i(\boldsymbol{r})$ in Eqn (1), we obtain an approximate inhomogeneous wave equation, which can be regarded as a homogeneous wave equation with a perturbation in the form of a source on the right-hand side:

$$\nabla^2\boldsymbol{E}_{\rm s}(x,y,z)+\omega^2\mu\varepsilon_{0i}\boldsymbol{E}_{\rm s}(x,y,z)$$

$$\approx -\omega^2\mu\varepsilon_{0i}\Delta\varepsilon_i(x,y,z)\boldsymbol{E}_{0y}(x,z), \qquad (2)$$

where $\boldsymbol{E}_{0y}$ is a solution of the homogeneous nonperturbed equation describing the propagation of the fundamental TE mode in the waveguide. From the energy viewpoint, the source on the right-hand side of Eqn (2) is the intensity of the mode incident on an irregular part of the waveguide and scattered in all directions (three-dimensional scattering). The solution to this inhomogeneous wave equation can be obtained in the form of a convolution of a certain Green function $G(x,y,z;x',y',z')$ with the expression for the source:

$$\boldsymbol{E}_{\rm s}(x,y,z) = -\omega^2\mu\varepsilon_{0i}\iiint\Delta\varepsilon_i(x',y',z')G(x,y,z;x',y',z')$$

$$\times\boldsymbol{E}_{0y}(x',z')\mathrm{d}x'\mathrm{d}y'\mathrm{d}z'. \qquad (3)$$

Analysis of Eqn (3) shows that, in this case, polarisation effects cannot be ignored, and the consideration of the problem of waveguide light scattering (which is multiple in essence) by three-dimensional irregularities becomes much more complex, because finding the analytical expression for the Green function $G(\boldsymbol{r},\boldsymbol{r}')$ is a serious problem. Indeed, in this approach, hybrid modes having six components (unlike the TE and TM modes with three components) may arise in the waveguide [2]. The condition $\partial/\partial y = 0$ is not satisfied for hybrid modes, so that field variations exist in this direction.

Hence, in the case of three-dimensional irregularities, an arbitrary field distribution in an optical PW must be represented as an expansion in terms of all possible PW modes, including the summation and integration over the modes corresponding to the field variation along the second transverse coordinate, namely, along the y axis. A more detailed analysis of the direct problem of three-dimensional waveguide scattering and approaches to the solution of the corresponding inverse problem will be considered in our following works. Equation (3) can also be employed in the analysis of single (Rayleigh) scattering. Note that this scattering in irregular waveguides was analysed in several papers (e.g., [15]).

Equations (1)–(3) can be simplified, if the polarisation effects arising during scattering [2, 8] are neglected. This can be realised if we demand that the relative change of the permittivity on a distance of one wavelength is much smaller than unity [2]. This condition is often satisfied in optical media. In this case, the second (depolarisation) term in Eqn (1) is much smaller than the two other terms, because its ratio to any of them is of the order of $\Delta\varepsilon/\varepsilon_0$. Consequently, $\Delta\varepsilon/\varepsilon_0 \ll 1$, the exact equation (1)* can be replaced by an approximate wave equation

$$\Delta\boldsymbol{E}+n_i^2k^2\boldsymbol{E}=0, \qquad (4)$$

which is valid for each Cartesian component of the electric field vector. For the fundamental TE mode propagating along the z axis under the condition $\partial/\partial y = 0$, Eqn (4) takes the form

*A method for taking into account the three-dimensional character of random irregularities of the waveguide for the scattering observed at small angles of deviation from the plane of incidence xz was proposed in [8].

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + n_i^2 k^2 E_y = 0, \quad (5)$$

where $n_i^2(x, z) = n_{0i}^2(x, z) + \Delta n_i^2(x, z)$; $n_{0i}(x, z)$ describes the regular properties of the corresponding PW layer; and the addition $\Delta n_i^2(x, z)$ describes irregularities of the PW structure (roughness of the interfaces of the PW media and irregularities of the refractive index of the ith PW layer).

For the perturbation theory to be applied, it is not obligatory that $\Delta n_i^2(x, z)$ should be a small quantity. It is quite sufficient that the region within which this additional term is nonzero should be very narrow. A solution to Eqn (5) using the approximate method of cross sections (Marcuse method of ideal modes [2]) is sought for in the form of an expansion of an arbitrary radiation-field distribution in terms of PW orthogonal modes:

$$E_y = \int_0^{\infty} q(\rho, L) E_y(\rho, x, z) \mathrm{d}\rho, \quad (6)$$

where q is the effective scattering amplitude of the TE mode defined as the coefficient of field expansion in terms of all radiation modes; L is the length of the region with roughness; $\rho = (k^2 n_1^2 - \beta^2)^{1/2}$ is the transverse component of the scattering-mode propagation constant; β is the longitudinal component of the propagation constant of the scattering modes that form the diagram of scattering by irregularities (object's optical image). The expansion coefficients are found from the system of the derived integro-differential equations using the orthogonality relations and the perturbation theory. By using expression (6), we can derive the expressions for the fields in the near-, intermediate-, or far-field zone, as well as for the corresponding powers of the scattered radiation.

If the condition $\partial/\partial y = 0$ is satisfied, then any arbitrary field distribution in the PW can be written in the form of a superposition of orthogonal TE and TM modes of an ideal rectilinear waveguide [2]. After that, the total laser radiation power transferred in the waveguide or, for example, the radiation power determined by the scattering of the propagating mode by waveguide irregularities can be found. If the scattering occurs from random irregularities, the scattered radiation power is determined by averaging over the ensemble [2, 7–9, 12–14].

Thus, a solution to the direct waveguide-scattering problem can be obtained in the form of an amplitude–phase distribution or a solution that describes the intensity distribution of laser radiation scattered by irregularities. The applicability of the two-dimensional analysis of the scattering problem can be provided in experiments, for example, by placing a slit diaphragm parallel to the plane of incidence and a polariser in a far-field zone (or in the Fourier plane) [7, 8]. The two-dimensional scattering diagram measured in this way can be used for finding an approximate correct solution to the ISP for three-dimensional structural irregularities of an optical PW.

3. Direct and inverse waveguide scattering problems

Consider the solutions to the direct and inverse problems of waveguide scattering of laser radiation in the presence of random additive noise specified over a limited range of wave numbers (in the region of existence of the observed scattering modes). Note that papers devoted to the study of the correctness of the inverse problem of waveguide scattering of laser radiation are actually inavailable. Works [12–14] are evidently the only papers in which the direct and inverse waveguide scattering problems were formulated and analysed for the case of a scattering diagram measured in a far-field zone. It should be also noted that no publications on the theory and model analysis of waveguide scattering, in which the influence of additive white noise on the solution of the direct and inverse problems has been investigated, are available.

3.1 Direct waveguide scattering problem

Using the solution to the problem of scattering of a propagating waveguide mode in an irregular optical PW obtained earlier for a high signal-to-noise ratio [12], we proceed to solving a direct waveguide scattering problem in the presence of a random additive noise. The solution to this problem consists in finding the laser-radiation scattering diagram due to the PW irregularities under study (see Figs 1–4). The scattering diagram is measured predominantly in the near-field or far-field zone. When a point photodetector is used (the photodetector filtration function is the delta function), the scattering diagram with noise $P(\beta, \gamma)$ can be represented in the far-field zone in the form

$$\langle P(\beta, \gamma) \rangle = C_0 \langle \Phi(\beta, \gamma) F(\beta, \gamma) \rangle + \langle N_{\mathrm{w}}(\beta, \gamma) \rangle, \quad (7)$$

where γ is the effective refractive index (phase slowing-down coefficient); C_0 is a normalisation factor; $\Phi(\beta, \gamma)$ is the optical transfer function of the PW*; $F(\beta, \gamma)$ is the SDF of the statistical ensemble of irregularities; $N_{\mathrm{w}}(\beta, \gamma)$ is the intensity of the white additive real noise specified over the measurement range of the scattering diagram; angle brackets denote the averaging over the ergodic ensemble of statistically identically systems.

The first term on the right side of Eqn (7) is the scattering diagram at a signal-to-noise ratio $S/N \geqslant 10^2$, where S and N are the signal and noise spectral densities, respectively. The scattering diagram of laser radiation is written in the form of a discrete digital set of response intensities at $\sim 500 - 3000$ experimental points. The noise nature in the experiment is the subject of a special study and is not treated in this paper.

3.2 Inverse waveguide scattering problem

The solution to the inverse problem of laser-radiation scattering by statistical irregularities in a waveguide consists in the reconstruction of the ACF (and/or SDF) and determination of the appropriate parameters of irregularities from the data of waveguide-mode scattering obtained in the near-field or far-field (Fourier plane) zone [12–14].

If the scattering mode intensity is recorded in the far-field zone by a point photodetector, the reconstructed smoothed ACF $R_{\mathrm{sm}}(u, \gamma)$ can be determined from the measured scattering diagram (7) using the formula [12]

$$R_{\mathrm{sm}}(u, \gamma) = C_0^{-1} \int \frac{\langle P(\beta, \gamma) \rangle \Phi^* E(u, \beta)}{|\Phi|^2 + \mu_{\mathrm{r}} M} \exp[\mathrm{i}(\beta_0 - \beta)u] \mathrm{d}\beta, \quad (8)$$

*The expression for the optical transfer function of a symmetrical PW is similar to that of an asymmetrical PW (see the integrand in the second term of (1) in [9]).

where μ_r is the regularisation parameter; the simplest pth-order stabilisers are taken in the form $M = \beta^{2p}$ or $M = (\beta_0 - \beta)^{2p}$; $p \geqslant 0$ is the regularisation order; $\beta_0 = k\gamma$ is the propagation constant of the waveguide mode; $E(u, \beta)$ is the filtering (smoothing) function selected from the condition of the minimum rms error of the ACF reconstruction [12]; $u = z - z'$; z and z' are the coordinates in the waveguide substrate plane. In principle, the function M may have an arbitrary order of rise at $\beta \to \infty$. Formula (8) makes it possible to obtain an approximate correct solution to the ISP using the quasi-optimal filtration procedure [12, 14]. For $M = \mu_r^{-1}(S/N)^{-1}$, formula (8) yields an optimal regularised solution to the ISP, which coincides with the result of applying the Wiener optimal filtration [16].

4. Computer simulation. Results of calculations

The irregular integrated-optical waveguide considered and schematic diagram of the system for detecting the scattered laser radiation are shown in Fig. 1. The point photodetector scans the scattering diagram in the far-field zone in the domain of existence of the observed scattering modes. The laser that excites the TE modes and certain devices (for signal processing, etc.) are not shown. A symmetrical PW is formed by two quartz plates and a thin waveguide layer of optically transparent liquid located between them with refractive indices $n_1 = n_3 = 1.46$ and $n_2 = 1.59$, respectively (for the wavelength $\lambda = 0.63$ μm of a He – Ne laser radiation). The scattering from the surface irregularities of identically treated quartz plates (taken from a common ergodic ensemble) is investigated. This allows us to consider that the plate surfaces under study are described by one and the same SDF with identical statistical surface-roughness parameters. Therefore, we further analyse one of two surfaces. During computer simulation, the effective refractive index γ of the fundamental TE mode was taken equal to 1.479, 1.525, 1.556, and 1.571 for the waveguide thickness $h = \lambda/5$, $\lambda/2$, λ and $3\lambda/2$, respectively.

The computer simulation was performed for a Gaussian SDF of statistical stationary irregularities of the PW substrate surface

$$\langle F(\beta, \gamma)\rangle = 2\sigma^2 r L^{-1} \exp\left[-(\beta_0 - \beta)^2 r^2/2\right], \qquad (9)$$

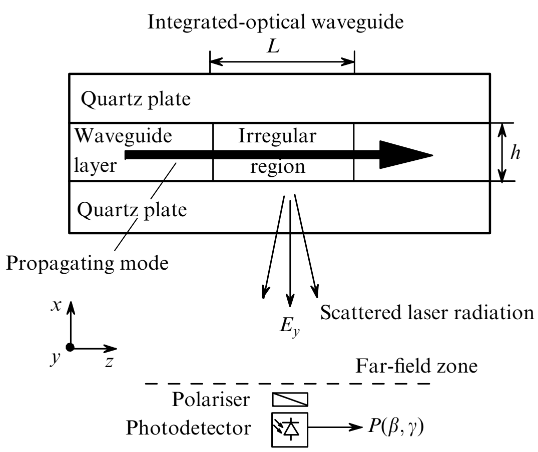

Figure 1. Irregular integrated-optical waveguide.

where σ is the rms roughness height and r is the correlation interval. The ACF corresponding to it has the form $B(u) = \sigma^2 \exp[-(u/r)^2]$. Note the surface irregularities of flat samples with a high degree of surface smoothness is determined predominantly by a random roughness. Such irregularities are usually studied using a Gaussian (normal) distribution law [11, 17]. Therefore, the use of a Gaussian function in model calculations seems to be justified.

Figs 2 – 4 show the most typical curves of the experimental scattering diagrams with noises and the ACFs reconstructed from these diagrams for subwavelength correlation ranges of surface irregularities. The signal-to-noise ratio in Figs 2 – 4 is given for those parts of the scattering diagrams where the scattered laser radiation intensity is maximum. The dynamics of the scattering indicatrices with changing r is obvious. At $r = 10$ nm, the rms error in the reconstruction of the specified ACF for the phase slowing factor $\gamma = 1.571$ (curve *4* in Fig. 2) is $\sim 35\,\%$ and can be lowered by selecting the ISP and $E(u, \beta)$ parameters. The rms reconstruction error for $r = 30$ nm and $\gamma = 1.571$ (curve *4* in Fig. 3) is $\sim 22\,\%$; for $S/N \geqslant 10^2$, selecting the ISP and $E(u, \beta)$ parameters allows us to determine the ACF to within an accuracy of $< 20\,\%$. The reconstruction error at $r = 0.3$ μm for optimal $\gamma = 1.525$ [12, 13] (curve *2* in Fig. 4) amounts to $\sim 70\,\%$ and can be reduced to 60 % by selecting the ISP parameters. For $\gamma = 1.525$, the scattering intensity in the PW reaches its maximum and, correspondingly, the ratio S/N is also maximum.

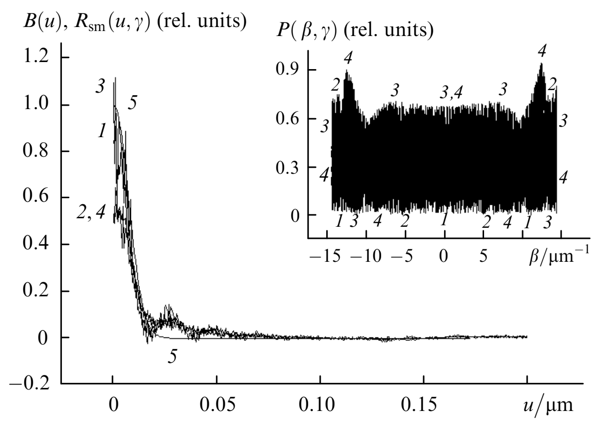

Figure 2. Specified [$B(u)$, *5*] and reconstructed [$R_{sm}(u, \gamma)$, *1 – 4*] autocorrelation functions at $M = (\beta_0 - \beta)^{2p}$, regularisation parameter $\mu_r = 1.3$, regularisation order $p = 1.0$, parameter $m = 25$ of the smoothing function $\sin(mx)/(mx)$, geometrical parameters of the surface roughness $\sigma = 5$ nm and $r = 10$ nm, and an effective refractive index of the waveguide $\gamma = 1.479$ (*1*), 1.525 (*2*), 1.556 (*3*), and 1.571 (*4*). The inset shows the scattering diagrams for laser radiation in the far-field zone at a signal-to-noise ratio $S/N \approx 1$.

The computations performed have shown that, at $S/N > 10$, the algorithm developed enables one to reconstruct the specified Gaussian ACF of irregularities with a maximum error of at most 60 % for the correlation interval changing from $\lambda/60$ to 15λ. The specified ACF can be determined with a maximum error below 30 %, if the correlation interval changes from $\sim \lambda/30$ to 3λ. In this case, the parameters of irregularities are determined with super-resolution [12 – 14]. When developing planar waveguides and integrated-optical devices of new generation, this method can undoubtedly serve as an efficient tool for

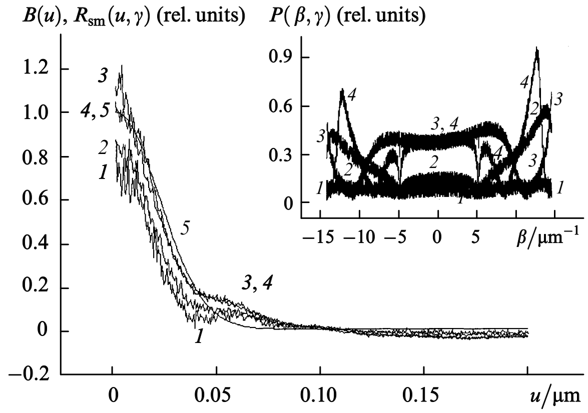

Figure 3. Functions similar to those in Fig. 2 for $r = 30$ nm, $\mu_r = 1.0$, $p = 0.8$, $m = 12$, and $S/N \approx 10$.

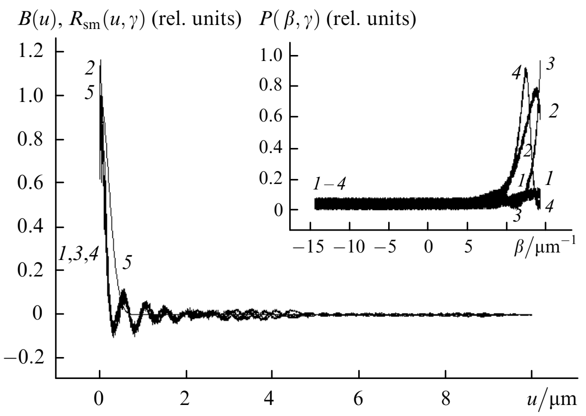

Figure 4. Functions similar to those in Fig. 3 for $r = 0.3$ μm, $\mu_r = 4.0$, $p = 0.9$, and $m = 0.8$.

monitoring their metrological characteristics and parameters. This is an absolute method, because σ and r are determined only from the waveguide scattering data [12, 13]. The problem of increasing the precision of the solution of the inverse waveguide scattering problem was analysed in detail in [12–14].

5. Conclusions

The method developed for finding an approximate correct solution to the ISP allows the reconstruction of the ACF of statistical irregularities from the data on laser radiation scattering in a PW obtained in a far-field zone in the presence of significant additive white noise. This technique has an acceptable experimental accuracy. The solution is based on the application of a quasi-optimal regularisation operator employing the least-squares method. An important advantage of the waveguide scattering technique is the in-phase scattering of laser radiation by the surfaces under study, which increases the measurement sensitivity by two–three orders of magnitude compared to single scattering. Another advantage, similar to that of the Mie theory, is the possibility of studying the light scattering within a wide range of the lateral sizes of irregularities.

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