OPTICAL FIBRES AND FIBRE LASERS

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Electrodynamic approach to the estimate of the pumping efficiency of double cladding active optical fibres

I.E. Britov, A.S. Kurkov, A.S. Raevsky

***Abstract*. A method of increasing the pumping efficiency of active optical fibres doped with Er^{3+} ions by decreasing the diameter of the inner cladding is analysed. An active optical fibre is described by using the electrodynamic model of a three-layer dielectric waveguide. The dependence of the average pump power flux through the core on the ratio of the radii of the core and the inner cladding is obtained. It is shown that this dependence can also be interpreted as the dependence of the damping constant averaged over all the waves in the core and the cladding.**

Keywords: *active optical fibre, pumping efficiency, 'group damping coefficient'.*

1. Introduction

Intense studies have been aimed during recent years at the fabrication and analysis of fibre lasers and amplifiers based on active optical fibres (AOFs) with a double cladding, i.e., consisting of three layers: a single-mode core doped with an active rare-earth impurity and the impurities forming the refractive index profile, an inner quartz cladding, and an outer polymer or quartz cladding whose refractive index is smaller than that of the inner quartz cladding. The latter has a typical diameter of 20 – 500 μm, which makes it possible to couple the pump radiation from semiconductor sources having a power of tens of watts. This allowed the fabrication of high-power fibre lasers and amplifiers based on optical fibres with a core doped with Nd^{3+} [1], Yb^{3+} [2], Er^{3+} [3], and Tm^{3+} [4] ions.

To attain a high efficiency of the AOF devices, the pump radiation should be used to the maximum possible extent. However, a considerable part of the pump power in fibres with a circular geometry of the inner cladding propagates outside the active core region. Therefore, such AOFs have, as a rule, an inner shell with a noncircular cross section, which is achieved by grinding the fibre blank [5].

At present, the prevailing tendency is to fabricate semiconductor sources having a power of several watts and a width 25 – 50 μm of the light-emitting region, which makes it possible to improve considerably the characteristics of three-level fibre lasers and amplifiers, because the pump power density increases significantly in this case [6, 7]. However, it is not possible to use fibres in a polymer cladding in this case since the outer diameter of the fibre becomes too small.

It is proposed to solve this problem by using an AOF with an inner cladding made of quartz glass doped with germanium dioxide and the outer cladding made of pure quartz glass. Because the inner cladding has a circular cross section in this case, one has to consider the efficiency of absorption of pump radiation in such structures. This paper is devoted to a theoretical analysis of the efficiency of absorption of pump radiation in an AOF with a cylindrical inner cladding of small dimensions.

2. Formulation of the problem

We will analyse the dependence of the AOF characteristics on the ratio of the radii of the core and the cladding. We will describe an AOF using the three-layer model of an open dielectric waveguide (ODW). The first (inner) region of the ODW has a radius a, the second (corresponding to the inner cladding) has a radius b, and the third region corresponding to the outer cladding has an unlimited size. Such a model gives a correct picture of the physical processes occurring in a real guiding structure since the fields of the waves in the core and the inner cladding decrease quite rapidly along the radial coordinate in the outer cladding.

The analysis of the ODW is carried out by solving the Helmholtz equation written for the longitudinal components of the Hertz vectors with boundary conditions in each of the above-mentioned regions. Expressing the electric and magnetic field vectors in terms of the Hertz vectors and equating their components tangential to the interfaces between the regions, we obtain a system of linear homogeneous algebraic equations in the amplitude coefficients of the Hertz vectors. The nontriviality condition for solutions of this system of equations gives a dispersion equation for the ODW, which is solved together with the equations relating the wave numbers in the three regions. The active core is characterised by losses. The refractive index for the AOF core is a complex quantity. This is due to the fact that the pump radiation is absorbed by Er^{3+} ions.

The damping constant for a pump wave at 980 nm is 2.3 m^{-1}, which corresponds to an attenuation of 20 dB m^{-1}. The relative permittivity is also a complex

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quantity in this case: $\varepsilon = 2.158 - \mathrm{i}\,1.055 \times 10^{-6}$. Therefore, all the wave numbers will be complex. The dispersion equation should be solved on the complex plane of one of the wave numbers. For this purpose, we use the phase variation technique developed by one of the authors [8] and based on the familiar argument principle from the theory of functions of complex variable [9].

To estimate the extent of interaction between the pump radiation and the active core of the fibre, we should calculate the fraction η of the total pump power flux corresponding to the core. Note here that the pump power in the fibre under study is distributed in a certain manner between all the waves in the core and the cladding.

In order to calculate the power fluxes transferred by each of the waves through the ODW cross section, we substitute the values of the wave numbers obtained from the dispersion equation into the system of equations derived from the boundary conditions. By solving this system of equations, we obtain the amplitude coefficients of the Hertz vectors and calculate the components of the electric and magnetic field vectors which are used to determine the pump power fluxes through each region of the cross section.

3. Solution of the problem

The power flux for a wave passing through the cross section s of the structure averaged over the period is defined as

$$P = \frac{1}{2}\mathrm{Re}\int_s \left[\boldsymbol{EH}^*\right]\mathrm{d}s.$$

The quantity η for a wave can be defined by the formula

$$\eta = \frac{P_1}{P_1 + P_2 + P_3}, \tag{1}$$

where P_1, P_2, P_3 are the power fluxes through the core, the inner cladding and the outer cladding, respectively.

Calculations based on formula (1) have shown that the values of η for all the waves in the core and in the cladding are almost identical whether the losses are taken into consideration or not, i.e., losses do not affect the pump power distribution over the cross-sectional area. In this connection, we will treat an ODW in the following analysis with real values of the relative permittivity $\varepsilon_1 = 2.158$, $\varepsilon_2 = 2.1316$ and $\varepsilon_3 = 2.0736$, corresponding to the refractive indices $n_1 = 1.469, n_2 = 1.46$, and $n_3 = 1.44$. For $2a = 6$ μm, and $2b = 125$ μm, four waves can propagate in the AOF core. Calculations made by using formula (1) for these waves give $\eta(\mathrm{HE}_{11}) = 0.9072$, $\eta(\mathrm{H}_{01}) = 0.6964$, $\eta(\mathrm{E}_{01}) = 0.6939$, and $\eta(\mathrm{HE}_{21}) = 0.6952$. The number of waves in the cladding is much larger; there are about 5000 such waves for the same ODW parameters. Because it is quite difficult to take all these waves into account, we shall try to get an idea about the pumping efficiency by considering a limited number of waves.

It is known [10] that surface waves can be classified according to the azimuthal index n indicating the number of variations of the electromagnetic field over the angular coordinate. We calculated the dependences $\eta_n^{\mathrm{cl}}(m)$, i.e., the fractions of the power flux in the core region for waves with a fixed index n and different values of the index m. Calculations show that the dependences $\eta_n^{\mathrm{cl}}(m)$ are different for each value of n. In some cases (for $n = 1 - 5$), the dependences $\eta_n^{\mathrm{cl}}(m)$ exhibit peaks, while monotonic dependences are observed in some other cases (for $n = 6 - 9$). As an example, Figs 1 and 2 show the dependencesn $\eta_n^{\mathrm{cl}}(m)$ for asymmetric waves for $n = 2$ and 9.

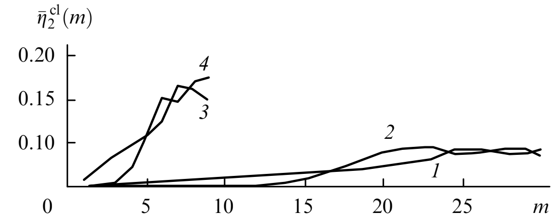

Figure 1. Dependences of the fraction of power flux through the core of an AOF for asymmetric waves with $n = 2$ on the index m for $2a = 6$ μm for HE_{nm} (*1*) and EH_{nm} waves (*2*) for $2b = 125$ μm and for HE_{nm} (*3*) and EH_{nm} waves (*4*) for $2b = 40$ μm.

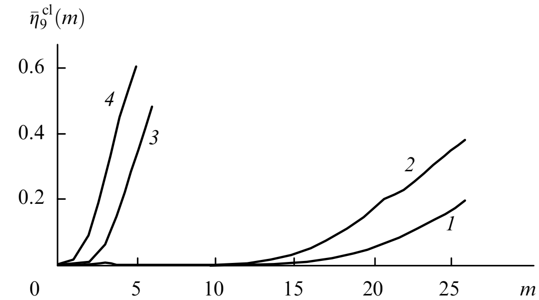

Figure 2. Dependences of the fraction of power flux through the core of an AOF for asymmetric waves with $n = 9$ on the index m for $2a = 6$ μm for HE_{nm} (*1*) and EH_{nm} waves (*2*) for $2b = 125$ μm and for HE_{nm} (*3*) and EH_{nm} waves (*4*) for $2b = 40$ μm.

Consider the dependence of the power flux fraction propagated across the core on the number of waves considered. We take all the waves in the core and will add to them all the waves in the cladding, first with $n = 0$, then with $n = 1$ in the next step, and so on. In each case, we will calculate the average power flux fraction through the core for such a group of waves with n varying from zero to N (assuming that their amplitudes are identical) using the formula

$$\bar{\eta}^{\mathrm{cl}}(N) = \frac{\sum_{n=0}^{N} N_n^{\mathrm{cl}}\bar{\eta}_n^{\mathrm{cl}}}{N^{\mathrm{cl}}(N)},$$

where $N^{\mathrm{cl}}(N) = \sum_{n=0}^{N} N_n^{\mathrm{cl}}$ is the number of waves in the cladding having indices from zero to N; N_n^{cl} is the number of waves with index n;

$$\bar{\eta}_n^{\mathrm{cl}} = \frac{1}{N_n^{\mathrm{cl}}}\sum_{i=1}^{N_n^{\mathrm{cl}}} \eta_{ni}^{\mathrm{cl}}$$

is the average fraction of the power flux for cladding waves with the azimuthal index n through the core; and η_{ni}^{cl} is the fraction of the power flux for the ith cladding wave with index n through the core.

We add all the waves in the core to the cladding waves. The average power flux fraction through the core for all the waves in the core and the cladding with indices from zero to N is:

$$\bar{\eta}(N) = \frac{N^{\mathrm{c}}\bar{\eta}_{\mathrm{c}} + N^{\mathrm{cl}}(N)\bar{\eta}^{\mathrm{cl}}(N)}{N^{\mathrm{c}} + N^{\mathrm{cl}}(N)}, \tag{2}$$

where $\bar{\eta}_c$ is the average power flux fraction through the core for all the waves in the core, and N^c is the number of waves in the core.

Fig. 3 shows the dependences $\bar{\eta}(N)$ calculated for the above-mentioned ODW parameters for $2a = 6$ μm and different values of $2b$. A common tendency is observed: as the value of n increases, the average power flux fractions through the core tend to limits that increase with the ratio a/b. It can be assumed that the steady-state value of $\bar{\eta}$ is attained for $n = 9$ for all the values of the ratio a/b.

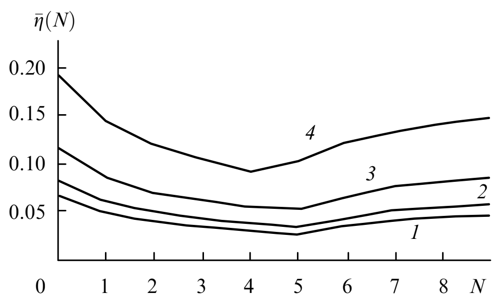

Figure 3. Dependences of the average fraction of power flux for all waves in the core and all waves in the cladding of an AOF with azimuthal index varying from zero to N for $2a = 6$ μm and $2b = 125$ (*1*), 100 (*2*), 70 (*3*) and 40 μm (*4*).

Fig. 4 shows the dependences of $\bar{\eta}$ on the ratio a/b. Points 1–4 correspond to the limits to which curves (*1–4*) in Fig. 3 tend. One can see that the dependence $\bar{\eta}(a/b)$ is almost linear. In order to verify the validity of the statement concerning the possibility of including waves with $n = 0, 1, ..., 9$, only for small values of the ratio a/b, we calculated $\bar{\eta}$ for $a/b = 0.4$ and 0.5 taking all the surface waves into account (42 and 29, respectively). The obtained values of $\bar{\eta}$ are shown by points 5 and 6 in Fig. 4. One can see that the points 1–6 lie on the same straight line with graphic accuracy.

As the value of a/b increases to 0.6 (point 7 in Fig. 4), all waves with $n = 9$ are cut off, so that the linear dependence $\bar{\eta}(a/b)$ is violated. Upon a further increase in the value of a/b (points 8–10 in Fig. 4), the dependence $\bar{\eta}(a/b)$ becomes linear once again. It should be noted that points 1, 2, 3, 4, 5, 6, 9, 10 lie on the same straight line passing through the origin (point $a/b = 0, \bar{\eta} = 0$) with graphic accuracy. As $a/b \to 1$, the value of $\bar{\eta}$ stops increasing (points 11, 12).

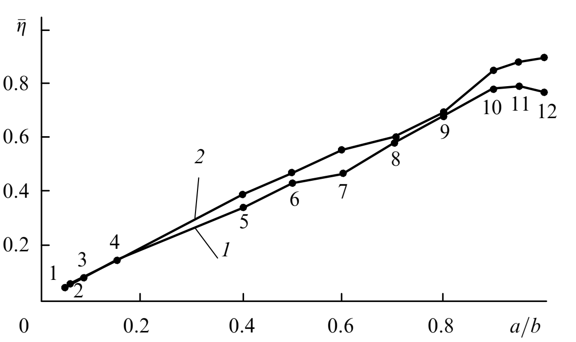

Figure 4. Dependences of the fraction of power flux corresponding to the core, averaged for all waves in the core and all waves in the cladding of an AOF, on the ratio of radii of the core and the cladding for $2a = 6$ (*1*) and 8 μm (*2*).

Similar calculations of $\bar{\eta}$ were made for a core diameter $2a = 8$ μm. The results are shown in Fig. 4 (curve *2*). One can see that the dependence $\bar{\eta}(a/b)$ behaves almost in the same way as for $2a = 6$ μm, but is somewhat steeper and is slightly higher in the limit. This may be explained by the fact that at the pump wavelength, the number of waves in the core with large values of η is seven, and not four.

After calculating $\bar{\eta}$ for a/b close to unity for a fixed value of a and extrapolating the dependence $\bar{\eta}(a/b)$ linearly to the origin, we can determine the value of $\bar{\eta}$ for a given a and for any radius of the inner cladding with graphic accuracy.

An analysis of the results of calculation of the propagation constants β for waves in the core and the cladding taking into account the loss in the active core shows that the damping constant β_2 for all waves is directly proportional to η for a given wave. Calculations show that the core wave HE_{11} with the maximum value of η, for which $\beta_2 = 2.1$ m^{-1}, is absorbed most rapidly. The slowest absorption is observed for the cladding wave HE_{41}. For this wave, $\beta_2 = 0.000338$ m^{-1} for $\eta = 0.0001463$. Thus, averaging over η actually gives averaging over the damping constant. The dependence $\bar{\eta}(a/b)$ can be treated as the dependence of the damping constant β_2 averaged over all waves, if we introduce the scaling factor $\chi = \beta_2(\text{wave})/\eta(\text{wave}) = 2.31$ m^{-1}. Using for analysis the concept of group velocity used for the wave packet in the theory of wave processes, we can introduce the concept of the group damping index β_2^{gr} for the entire set of waves in the core and cladding of an AOF at the pump frequency:

$$\beta_2^{\text{gr}} = \chi\bar{\eta}.$$

We assume that the pump radiation power varies along the longitudinal coordinate z of the fibre according to the law

$$P_{\text{p}}(z) = P_{\text{p0}} \exp(-2\beta_2^{\text{gr}} z).$$

The obtained dependence $\beta_2^{\text{gr}}(a/b)$ can be used to calculate the dependence $l(a/b)$ of the length of the AOF, over which 90% of pump power is absorbed, on the ratio of the core radius to the cladding radius (Fig. 5). The solid curve in this figure shows the dependencee $l(a/b)$ for a core diameter $2a = 6$ μm, while the dashed curve shows the same dependence for $2a = 8$ μm. The dark square in the same figure shows the results of measurements of the fibre length ensuring a pump power absorption of 10 dB. The sample under investigation had a core of diameter 8 μm and a

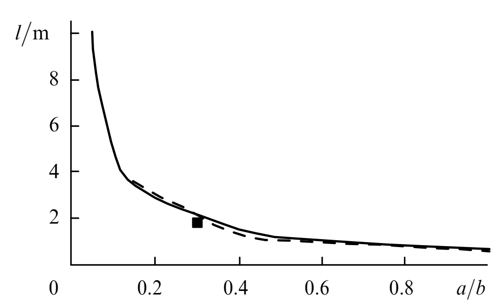

Figure 5. Dependences of the AOF length over which 90 % of the pumping power is absorbed on the ratio of radii of the core and inner cladding for $2a = 6$ (solid curve) and 8 μm (dashed curve).

cladding of diameter 25 μm. A comparison of the experimental and theoretical data reveals a good matching of the results.

4. Conclusions

Thus, an electrodynamic analysis of the processes proceeding during the pumping of the AOF to the inner cladding shows that the pumping efficiency can be estimated from the fraction of the power flux corresponding to the core. A one-to-one correspondence between η (pump power fraction corresponding to the AOF core) and the damping constant β_2 exists for each wave in the core and the cladding, i.e., for the entire set of waves transporting the pump power. This allows us to introduce the concept of group damping index for the pump radiation and to estimate its value from the value of $\bar{\eta}$ averaged over all the surface waves in the core and the inner cladding of the AOF. In order to estimate the value of $\bar{\eta}$, it is sufficient to calculate it for a value of a/b close to unity. The model described here is applicable for the case when the pump power is not high enough to cause a saturation of the metastable level $I_{13/2}$ of Er^{3+} ions.

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