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On the possibility of compensating material dispersion in three-layer optical fibres in the wavelength range below 1.3 μm

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***Abstract*.** **Idealised three-layer model optical fibres consisting of an optically dense core with the refractive index n_1 surrounded by a 'holey' intermediate cladding having a much lower optical density and the refractive index n_2 and by a rather thick outer cladding with the refractive index n_3 are investigated. The idealised model assumes that the intermediate cladding is homogeneous. It is also assumed that $n_1 > n_3 > n_2$. It is shown that in such fibres with large differences $n_1 - n_2$ and $n_3 - n_2$, a single-mode regime can be realised in fact, when the nearest higher modes H_{01}, E_{01}, HE_{21} have additional radiative losses of several hundred dB km^{-1}, whereas the main operating mode HE_{11} has no such losses in the single-mode regime. It is important that the zero dispersion can be obtained in these fibres in the spectral region below 1.3 μm.**

Keywords: *optical fibre, dispersion compensation.*

Promising 'photonic-crystal' or 'holey' fibres have been extensively studied recently (see, for example, a detailed review [1]). The core of such fibres is surrounded by a cladding made of a set of densely packed hollow glass fibres drawn at high temperature. This or possibly some other technology allows the fabrication of three-layer optical fibres consisting of a core (with radius a and the refractive index n_1) surrounded by a 'holey' intermediate cladding with a lower optical density (with radius b and the effective refractive index n_2) and a rather thick outer cladding (with the refractive index n_3), the refractive indices satisfying the relations $n_1 > n_3 > n_2$.

The geometry and profile of the refractive index of such optical fibres correspond to those of the well-known three-layer fibres called the W-fibres, or fibres with a depressed intermediate cladding. The technologies existing earlier could not provide the fabrication of three-layer fibres with a large difference in the refractive indices of the cladding layers, hence they were used (and are still being used) mainly in the near IR range above 1.3 μm. For example, the material dispersion near 1.55 μm can be compensated in these fibres, whereas the material dispersion below 1.3 μm cannot be compensated in them.

The use of a holey intermediate cladding makes it possible to manufacture three-layered optical fibres with a large difference in refractive indices $n_1 - n_2$ and $n_3 - n_2$, allowing a compensation of the material dispersion in the short-wavelength spectral region below 1.3 μm, which is interesting for linear and nonlinear fibre optics.

An exact analysis of such structures with an azimuthal-periodic variations in the field distribution caused by the holey structure of the intermediate cladding is a quite complicated theoretical problem. Hence, we will study in this work model three-layer optical fibres of circular cross section, consisting of a germanium-doped quartz core, a holey (air-'doped') intermediate cladding, and an outer quartz cladding. The idealisation of the model is connected with the fact that the intermediate cladding is assumed to be homogeneous. The aim of this research is to find the parameters of optical fibres which allow a virtually single-mode operation and the compensation of the material dispersion at wavelengths below 1.3 μm.

We used the results presented in our earlier publications [2–4] in which three-layer fibres were analysed rigorously without resorting to the LP approximation, which is valid only for small differences between the refractive indices of the claddings. The problem of compensation of material dispersion in such fibres was also solved rigorously by taking into consideration the dependence of the refractive index of the cladding materials on the wavelength at the initial stage of solution of Maxwell's equations [5, 6].

The dispersion coefficient was defined by the rigorous expression

$$S = \frac{1}{c}\frac{\mathrm{d}n_{\mathrm{gr}}}{\mathrm{d}\lambda} = -\frac{\lambda}{c}\frac{\mathrm{d}^2 n_{\mathrm{m}}}{\mathrm{d}\lambda^2}, \quad (1)$$

where $n_{\mathrm{gr}} = c/\upsilon_{\mathrm{gr}} = n_{\mathrm{m}} - \lambda(\mathrm{d}n_{\mathrm{m}}/\mathrm{d}\lambda)$; $n_{\mathrm{m}} = c/\upsilon_{\mathrm{ph}}$; c is the velocity of light in vacuum; υ_{gr} and υ_{ph} are the group and phase velocities of the HE_{11} mode in the fibre.

The dispersion coefficient is usually calculated from the expression:

$$S = -\frac{1}{\lambda c}\left[\lambda^2\frac{\mathrm{d}^2 n}{\mathrm{d}\lambda^2} + \Delta n V\frac{\mathrm{d}^2(VB)}{\mathrm{d}V^2}\right], \quad (2)$$

where $\Delta n = n_1 - n_2$; $V = \kappa a(n_1^2 - n_2^2)^{1/2}$; $B = (n_{\mathrm{m}}^2 - n_2^2)\times(n_1^2 - n_2^2)^{-1}$; $\kappa = 2\pi/\lambda$. The first term in expression (2) des-

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cribes the material dispersion (it is usually assumed that $d^2n_1/d\lambda^2 \approx d^2n_2/d\lambda^2 \approx d^2n_3/d\lambda^2$ for quartz fibres), while the second term describes the waveguide dispersion. This expression is approximate and is valid for fibres with a small difference in the refractive indices ($\Delta n < 0.01$). It was shown in Refs [5, 6] that the use of the approximate expression for $\Delta n > 0.01$ leads to significant errors, and the dispersion coefficient should be calculated by using the rigorous expression (1) that takes into account the dependence of n_m not only on the type of mode and the geometrical dimensions a and b but also on the refractive indices $n_{1,2,3}$ of the claddings which, in turn, depend on the wavelength λ.

The refractive indices of the materials of the core, intermediate and outer claddings were presented in the calculations in the form of Sellmeyer series. We analysed fibres in which the core was made of quartz doped with GeO_2, the intermediate cladding was 'doped' with air, and the outer cladding was assumed to be made of fused quartz. The molecular concentration C_1 of GeO_2 in the core was 5 %, 10 % and 20 %, while the molecular concentration C_2 of air in the intermediate cladding was 36 %, 54 % and 72 %. The three values for concentrations C_1 and C_2 correspond to the relatively weak, intermediate, and high concentrations of GeO_2 and air. The outer cladding was assumed to be made of pure quartz, for which $C_3 = 0$.

The refractive indices of the fibre layers were calculated from the expression

$$n_i^2(\lambda) = 1 + \sum_{k=1}^{3} \frac{[(1-0.01C_i)A_k^0 + 0.01C_iA_k^i]\lambda^2}{\lambda^2 - [(1-0.01C_i)\lambda_k^0 + 0.01C_i\lambda_k^i]^2}, \quad i = 1, 2, 3, \tag{3}$$

where A_k^i, λ_k^i are the coefficients in the Sellmeyer series for pure SiO_2 ($j = 0$) and GeO_2 ($j = 1$) [7]; $k = 1, 2, 3$. Air was treated as a dielectric for which all the coefficients in the Sellmeyer series are equal to zero ($A_k^2 = \lambda_k^2 = 0$).

Thus, the intermediate cladding is made of fused quartz 'doped' with air, and its refractive index n_2 (like n_1 and n_3) was calculated by formula (3) using the linear concentration dependence of the refractive index [7]. Therefore, the core, the intermediate and outer claddings of the fibre were assumed to be homogeneous with refractive indices n_1, n_2, and n_3, respectively. Calculations were performed by using the concentrations C_1 and C_2.

Fig. 1 shows the calculated dependences of the dispersion coefficient S on the wavelength λ for fibres Nos 1–9, whose parameters are presented in Table 1. Here, V_0 and λ_0 are the characteristic parameter of the fibre and the wavelength corresponding to zero dispersion, i.e., to the case $S = 0$; V_c and λ_c are the characteristic parameter and the wavelength corresponding to the cut-off of the nearest higher modes H_{01}, E_{01}, and HE_{21}. Table 1 also presents the values of the refractive indices $n_{1,2,3}$, the phase parameter B, and the parameter V_0 for each obtained value of λ_0.

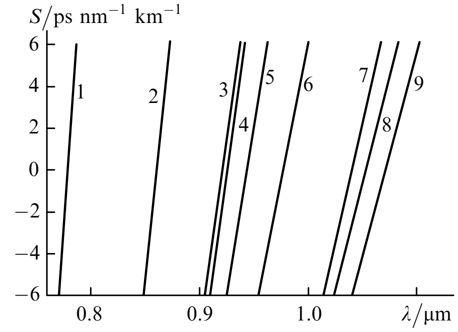

Figure 1. Wavelength dependences of the dispersion coefficient for three-layer fibres Nos 1–9, whose parameters are presented in Table 1.

It was shown in Refs [2, 4] that these modes are characterised by the relation

$$V_c \to 3.832\left[\frac{(n_1^2 - n_2^2)}{(n_1^2 - n_3^2)}\right]^{1/2}. \tag{4}$$

Note that the mode HE_{11} also has a cut-off in such fibres, and for this mode we have [2, 4]

$$V_c \to 2.405\left[\frac{(n_1^2 - n_2^2)}{(n_1^2 - n_3^2)}\right]^{1/2}. \tag{5}$$

An analysis of Fig. 1 and Table 1 shows that three-layer fibres with large differences in refractive indices $n_1 - n_2$ and $n_3 - n_2$ may provide a compensation of the material dispersion in the spectral region $\lambda < 1.3$ μm. The compensation of the material dispersion for much shorter wavelengths can be attained by increasing the differences in refractive indices $n_1 - n_2$ and $n_3 - n_2$, or by reducing the core diameter $2a$.

An increase in the diameter $2b$ of the intermediate cladding also leads to a decrease in the wavelength λ_0 corresponding to zero dispersion. However, the following circumstance should be taken into account in this case. It is known that at frequencies lower than the cut-off frequency, the directed modes of the optical fibres become the leak modes in a broad frequency range. The properties of the leak modes have been studied quite thoroughly for two-layer open dielectric waveguides [8–10]. The wave numbers of the leak modes are complex even for real values of the

Table 1.

Fibre No.	$2a$/μm	$2b$/μm	C_1 (GeO_2) (%)	n_1	C_2 (air) (%)	n_2	n_3 (fused quartz)	B	λ_0/μm	V_0	V_c	λ_c/μm
1	2	4	20	1.483	72	1.145	1.453	0.9176	0.780	7.586	10.983	0.540
2	2.4	4.8	20	1.482	54	1.228	1.452	0.9121	0.860	7.265	9.643	0.649
3	2.5	5	20	1.481	36	1.306	1.451	0.8769	0.921	5.952	7.979	0.688
4	3.5	7	10	1.466	72	1.143	1.451	0.9580	0.923	10.936	15.714	0.643
5	3.5	7	10	1.466	54	1.227	1.451	0.9444	0.944	9.347	13.601	0.649
6	3.5	7	10	1.465	36	1.305	1.450	0.9179	0.978	7.488	11.122	0.659
7	5.5	11	5	1.457	72	1.141	1.449	0.9769	1.041	15.041	22.410	0.699
8	5.5	11	5	1.457	54	1.225	1.449	0.9696	1.053	12.914	19.357	0.704
9	5.5	11	5	1.456	36	1.304	1.449	0.9553	1.072	10.466	15.787	0.711

refractive indices of the fibre layers due to radiation losses (hereafter, additional losses). Therefore, the longitudinal mode propagation constant is also complex, i.e., $\gamma = \beta_1 - i\beta_2$, where β_1 is the phase constant of the mode, and β_2 is the damping constant of mode related to its emission to the outer cladding of the fibre.

The value of β_2 for leak modes was determined by using the phase variation technique [11] based on the argument principle from the theory of functions of complex variable [12]. We treat the equation

$$f(\gamma) = 0 \tag{6}$$

as the mode dispersion equation for a three-layer fibre defined on the complex plane γ. It is assumed that the function $f(\gamma)$ is analytic everywhere in the closed domain D with the exception of a finite number of isolated singularities. We also assume that $f(\gamma)$ does not vanish at any point of the contour $\mathcal{L}$ confining the domain D. In this case, the difference between the total number of zeros and poles of the function $f(\gamma)$ in domain D is determined, according to the argument principle, as the number of total revolutions performed by the point $w = f(\gamma)$ around the point $w = 0$ during a positive circumvention of the contour $\mathcal{L}$.

The dispersion equation is written in such a way that the function $f(\gamma)$ has no poles. For the leak modes, the domain D is located in the fourth quadrant of the complex plane γ. The contour $\mathcal{L}$ confining the search region should be chosen in such a way that it does not intersect the cut line of the Hankel function describing the radial dependence of the electric and magnetic fields of the mode in the outer cladding of the fibre. Having determined the domain of existence of the root of the dispersion equation, we localise it with the help of a certain algorithm until the desired accuracy of the solution is attained.

Thus, the damping constant β_2 and the dependence $P(L) = P_0 \exp(-2\beta_2 L)$ of the power of the mode on the fibre length L, where P_0 is the power at the fibre input, are determined for the specified values of the fibre parameters. Additional losses for the leak mode expressed in dB km^{-1} are determined from the expression

$$\alpha = \frac{10}{L} \log \frac{P_0}{P(L)}, \tag{7}$$

where L is the fibre length in kilometres, or

$$\alpha = 8.686 \times 10^9 \beta_2, \tag{8}$$

where β is in inverse micrometers.

These losses depend strongly on the thickness of the intermediate cladding. For relatively large values of the difference $b - a$, the nearest higher modes may have small radiation losses, and the optical fibre will not be a single-mode fibre. Fig. 2 shows the additional losses in the nearest higher modes H_{01}, E_{01}, and HE_{21} as functions of λ for fibre No. 1, whose parameters are presented in Table 1. For the working mode HE_{11} in the single-mode regime, there are no additional losses because this mode is not an outgoing mode and does not have radiation losses, i.e., $\beta_2 = 0$ for this mode.

An analysis of the additional losses for fibre No. 1 shows that among the nearest higher modes H_{01}, E_{01}, and HE_{21},

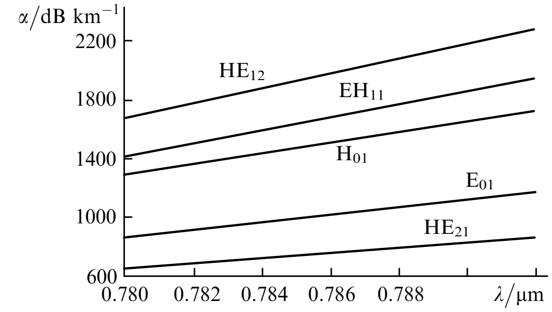

Figure 2. Additional radiation losses in the nearest higher leak modes for the three-layer fibre No. 1, whose parameters are presented in Table 1.

the mode HE_{21} has the smallest radiation losses, 645.8 dB km^{-1} for $\lambda_0 = 0.780$ μm, i.e., its power attenuates by a factor of 4.4 over the length $L = 10$ m of the fibre. Therefore, this fibre can be treated as a single-mode fibre. A decrease in the thickness of the intermediate cladding causes a further attenuation of the leak higher modes, but in this case the value of λ_0 corresponding to zeroth dispersion increases slightly. The dispersion characteristics of the fibre (for example, the slope of the dependence of S on λ, i.e., $dS/d\lambda$ for the HE_{11} mode) can be improved by varying its parameters.

Thus, three-layer optical fibres with large differences $n_1 - n_2$ and $n_3 - n_2$ in the refractive indices of the layers can provide a single-mode operation, allowing zero dispersion in the spectral region below 1.3 μm.

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