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Analysis of the amplitude – phase parameters of time-dependent optical signals and transfer functions

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Abstract. **The possibilities of obtaining information on the time dependence of the amplitude and phase of an optical signal by the spectral modulation method are analysed. Three variants of the signal analysis are considered: a direct analysis of the signal whose parameters are changing in time; studies of the time-dependent optical characteristics of an object or a medium under study by probing them with radiation of the known structure; and measurements of the parameters of radiation propagated through the object and simultaneous determination and analysis of time-dependent structures of the signal and transfer function describing the effect of the transfer medium, the object, or an optical system on the propagated signal.**

Keywords: *phase problem in optics, measurement of ultrashort pulses.*

1. Introduction

The problems of obtaining information about the amplitude and phase structures of time-dependent optical signals and complex instrumental functions characterising the inner structure of media in which signals are propagating and the effect of these media on the signals, as well as about the parameters of optical inhomogeneities inherent in transfer media or objects under study attract considerable interest. This is explained by the fact that the solution of such problems is very important both for fundamental optical studies and technical applications [1 – 3].

The solution of problems on the amplitude – phase nature of signals and transfer functions for time-dependent optical signals involves the study of the signal transfer through distorting media, taking into account the corresponding distortions and the reduction of their influence. In addition, the structure of media or objects, which cause the time modulation of probing radiation due to a variety of physical, chemical or biological processes, should be analysed, as well as the effects observed upon the interaction of radiation with matter and resulting in the variation of the structure and optical characteristics of objects or media in time.

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Some important applied problems are also of interest. These include the data processing in optical systems by using time-dependent amplitude – phase interactions, the propagation of radiation and transfer of data on the signal structure in fibreoptic communication systems in the presence of time-dependent perturbing factors and optical inhomogeneities, as well as laser location in the optical spectral range, during which the time-dependent amplitude and phase actions of the structure of the objects under study or transfer media should be considered.

The methods for measuring the duration and shape of ultrashort laser pulses ($10^{-9} - 10^{-14}$ s) have been considered in the literature many times. First of all, these are direct methods for detecting signals with the use of fast electro-optical converters [4 – 6]. In these methods, the intensity distribution of a detected signal or its amplitude structure is measured. The time resolution of the method is limited by the dynamic resolution of electronic optics and the ultimate rate of the electronic scanning and is ~ 1 ps.

Among the indirect measurement methods available at present, various correlation methods provide the highest time resolution [2 – 4, 7, 8]. These methods allow one to obtain not only the shape of a correlation function but also information on the amplitude structure of a signal. In this case, additional information on a signal is required for an unambiguous interpretation of the results. This information can be obtained by the methods involving fast Kerr or Pockels switches or saturable-absorber switches [4].

The amplitude and phase structure of signals can be also obtained by holographic methods, which are capable of detecting the amplitudes and phases of individual spectral components of a signal being analysed. These include the methods using a nonstationary reference wave [8 – 10], which were later developed to the methods of spectral holography [11 – 14]. Note also an elegant method of hole burning in the absorption spectra of spectrally selective media, which have broad inhomogeneous absorption bands and narrow homogeneously broadened absorption lines [15 – 17].

However, both holographic and hole-burning methods require the use of a single pulse as a reference signal, whose duration should be shorter than the characteristic time scale of variation of a signal being analysed, because for detection the spectrum should be produced that covers the entire spectrum of a signal being studied. Some information on a signal can be obtained by spectral methods [18 – 20].

Therefore, all the above methods have certain limitations or their implementation is hindered – either only a part of information on the signal is detected (its duration, correlation function, spectrum) or a specially produced reference wave should be used or special detecting media should be used, which operate only at low temperatures.

The modern development of laser technology and methods of physical and coherent optics [21–23] makes it possible to solve efficiently the fundamental problems of detection, processing, and analysis of the amplitude and phase characteristics of time-dependent optical signals of arbitrary shapes, i.e., to solve the phase problem in optics as applied to time-dependent signals.

In this paper, the possibility of studying the time dependences of the amplitude and phase of an optical signal by the amplitude-spectral method is analysed.

The spectral, temporal, amplitude, and phase characteristics of laser pulses of an arbitrary shape are measured by the method based on the detection of the amplitude distributions of signals specially produced with the help of modulators, which was proposed earlier for analysis of the structure of stationary light fields [24]. This is the spectral modulation method for analysis and solving the phase problem in optics. The simplest variant of this method involves the formation of the signal spectrum and the spectrum of a signal transformed by an optical modulator with time-dependent characteristics [25].

Such a spectral modulation method is also used for studying the structure of time-dependent optical inhomogeneities in objects or media. An object under study is probed with an optical signal with the known shape. After propagation of radiation through the object, the signal becomes additionally modulated, and its amplitude and phase characteristics change. In addition, in solving some problems the situations appear when it is required to determine simultaneously the amplitude–phase structure of a time-dependent signal and the structure of a complex transfer or instrumental function, which characterises the action of the optical system or the medium on the signal. The problems of this type can be solved by the spectral modulation method, however, somewhat complicated [26]. Optical schemes considered in this paper are analysed and described in terms of physical optics [27–29].

2. Measurement of the amplitude–phase characteristics of time-dependent optical signals

The spectral modulation method for measuring the amplitude–phase structure of a time-dependent optical signal of an arbitrary shape involves the formation of the signal spectrum and the spectrum of the signal transformed due to an additional known time modulation of radiation being analysed.

Fig. 1 shows the scheme of illumination of the entrance slit of a spectral instrument used in the spectral modulation method involving two measurement channels, with a modulator (*1*) placed in one of the channels.

A laser radiation signal under study with the complex amplitude $\mathscr{E}_{\rm s}(t)$ and the average frequency ω_0

$$E_{\rm s}(t) = \mathscr{E}_{\rm s}(t)\,\exp(-{\rm i}\omega_0 t) \tag{1}$$

is characterised by the duration $T_{\rm s}$ and the complex amplitude $\mathscr{E}_{\rm s}(\omega)$ of the spectrum located in the vicinity of the frequency ω_0 and having the width $\Delta\omega_{\rm s} \ll \omega_0$. A part of the signal is transmitted through the modulator, whose action on radiation is described by the known function $M(t)$

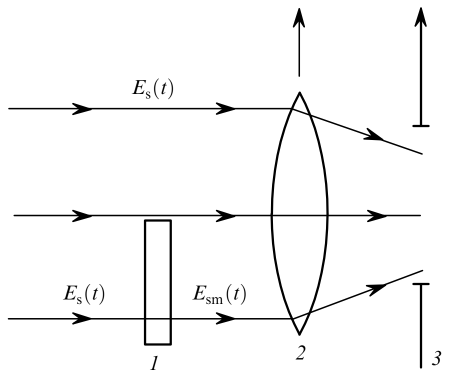

Figure 1. Principal optical scheme for analysis of the amplitude and phase structures of the time-dependent optical signal $E_{\rm s}(t)$ using a two-channel detection system with an additional modulator placed in one the channels: (*1*) optical modulator; (*2*) optical system; (*3*) entrance slit of a spectrometer.

or $M(\omega)$. The duration $T_{\rm m}$ of the modulator action on radiation should coincide with the signal duration or exceed it ($T_{\rm m} \geqslant T_{\rm s}$). As a result, the field

$$E_{\rm sm}(t) = \mathscr{E}_{\rm s}(t)\,M(t)\exp(-{\rm i}\omega_0 t) \tag{2}$$

is formed behind the modulator. The signals are directed with the help of the optical system to the entrance slit of a spectral instrument. A two-channel detection scheme is used. The radiation under study is directed to one part of the slit, and the radiation transmitted through the modulator is directed to another part of the slit.

At the output of the spectral instrument, the spectra of signals (1) and (2) are formed, which are described by the distribution of complex amplitudes

$$\begin{aligned} \mathscr{E}_{\rm s}(\omega) &= A_{\rm s}(\omega)\exp[{\rm i}\varPhi_{\rm s}(\omega)], \\ \mathscr{E}_{\rm sm}(\omega) &= \int \mathscr{E}_{\rm s}(\omega')M(\omega-\omega'){\rm d}\omega'. \end{aligned} \tag{3}$$

The parameter being detected is, as in other optical measurement systems, the intensity distribution. Taking into account the amplitude and phase structures of the spectrum of signal (3), the detected intensity distributions are described, with accuracy to factors that are insignificant for analysis, by the expressions

$$\begin{aligned} I_{\rm s}(\omega) &= A_{\rm s}^2(\omega), \\ I_{\rm sm}(\omega) &= \left[\int A_{\rm s}(\omega')\exp[{\rm i}\varPhi_{\rm s}(\omega')]M(\omega-\omega'){\rm d}\omega'\right] \times [\text{c.c.}] \end{aligned} \tag{4}$$

The intensity distributions (4) and the known function $M(\omega)$ describing the modulation of radiation by the filter allow one to find the amplitude and phase structures of the signal spectrum. The amplitude spectrum $A_{\rm s}(\omega)$ of the signal is determined from (4), and the phase structure $\varPhi_{\rm s}(\omega)$ of the signal spectrum is found by solving an integral equation. The amplitude–phase structure of the signal is reconstructed with the help of the inverse Fourier transform from the obtained values of spectral distributions.

As an example illustrating the operation of the scheme and showing the possibility of solving the problem, consider the use of a simple modulator that provides the amplitude modulation linear in time. The amplitude transmission of the modulator is described by the function $M(t) = t/T_{\rm m}$. In this case, the complex amplitudes of the spectral components of light fields produced at the output of the spectral instrument are described by expressions $\mathscr{E}_{\rm s}(\omega)$ and

$$\mathscr{E}_{\rm sm}(\omega) = \frac{\rm i}{T_{\rm m}} \int (-{\rm i}t)\mathscr{E}_{\rm s}(t)\exp(-{\rm i}\omega t){\rm d}t = \frac{\rm i}{T_{\rm m}} \frac{\rm d}{{\rm d}\omega}\mathscr{E}_{\rm s}(\omega). \qquad (5)$$

The detected spectral intensity distributions have the form [see (4)]

$$I_{\rm s}(\omega) = A_{\rm s}^2(\omega),$$
$$I_{\rm sm}(\omega) = \frac{1}{T_{\rm m}^2}\left[\left(\frac{\rm d}{{\rm d}\omega}A_{\rm s}(\omega)\right)^2 + A_{\rm s}^2(\omega)\left(\frac{\rm d}{{\rm d}\omega}\Phi_{\rm s}(\omega)\right)^2\right]. \qquad (6)$$

The amplitude distributions $A_{\rm s}(\omega)$ and ${\rm d}A_{\rm s}(\omega)/{\rm d}\omega$ are determined during the subsequent processing from the intensity distribution $I_{\rm s}(\omega)$. Then, taking into account the dependences thus obtained, the functions ${\rm d}\Phi_{\rm s}(\omega)/{\rm d}\omega$ and $\Phi_{\rm s}(\omega)$ are found from the intensity distribution $I_{\rm sm}(\omega)$. Finally, by using the amplitude and phase distributions $A_{\rm s}(\omega)$ and $\Phi_{\rm s}(\omega)$, respectively, the time dependence of the signal is calculated taking into account its amplitudes and phases:

$$\mathscr{E}_{\rm s}(t) = a_{\rm s}(t)\exp[{\rm i}\phi_{\rm s}(t)] = \frac{1}{2\pi}\int A_{\rm s}(\omega)\exp[{\rm i}\Phi_{\rm s}(\omega)]\exp({\rm i}\omega t){\rm d}\omega, \qquad (7)$$

which yields the solution of the problem.

3. Measurement of the time-dependent amplitude – phase characteristics of the instrumental function or transfer medium

Fig. 2 shows the scheme for analysis of the time-dependent amplitude and phase characteristics of the medium or instrumental function under study by their probing with an optical signal with the known structure.

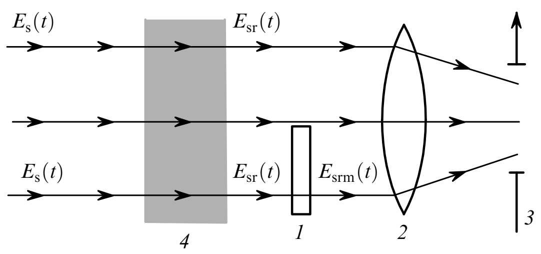

Figure 2. Principal two-channel optical scheme for analysis of the time dependence of optical characteristics of the medium in which a signal propagates. The notation as in Fig. 1; (*4*) medium under study.

The probe signal of type (1) propagates through the medium or object (*4*), whose action is described by the complex function $R(t)$ or $R(\omega)$. The duration $T_{\rm r}$ of the medium action is comparable with the signal duration ($T_{\rm r} \simeq T_{\rm s}$). An additional modulator (*1*) performs modulation of the known type, which is described by the function $M(t)$ or $M(\omega)$. The duration $T_{\rm m}$ of the modulator action on radiation coincides with the signal duration or somewhat exceeds it ($T_{\rm m} \geqslant T_{\rm s}$). Taking into account the actions of the medium and modulator, the fields are described, similarly to (2), by the expressions

$$E_{\rm sr}(t) = \mathscr{E}_{\rm sr}(t)\exp(-{\rm i}\omega_0 t),$$
$$E_{\rm srm}(t) = \mathscr{E}_{\rm sr}(t)M(t)\exp(-{\rm i}\omega_0 t). \qquad (8)$$

The radiation is directed with the help of optical system (*2*) to the entrance slit of a spectral instrument (*3*), which forms the signal spectra in the detection plane with the amplitude distributions, which, similarly to (3), have the form

$$\mathscr{E}_{\rm sr}(\omega) = A_{\rm sr}(\omega)\exp[{\rm i}\Phi_{\rm sr}(\omega)],$$
$$\mathscr{E}_{\rm srm}(\omega) = \int \mathscr{E}_{\rm sr}(\omega')M(\omega-\omega'){\rm d}\omega'. \qquad (9)$$

The intensity distributions $I_{\rm sr}(\omega)$ and $I_{\rm srm}(\omega)$ detected in the signal spectra, taking into account the known function $M(\omega)$ and (9), are described, similarly to (4), as

$$I_{\rm sr}(\omega) = A_{\rm sr}^2(\omega),$$
$$I_{\rm srm}(\omega) = \left[\int A_{\rm sr}(\omega')\exp[{\rm i}\Phi_{\rm sr}(\omega')]M(\omega-\omega'){\rm d}\omega'\right] \times [\text{c.c.}] \qquad (10)$$

and allow one to determine in principle the amplitude $[A_{\rm sr}(\omega)]$ and phase $[\Phi_{\rm sr}(\omega)]$ structures of the probe signal propagated through the medium or object and, hence, the frequency and amplitude characteristics of the medium or object, when the initial structure of the probe signal is known.

As above, consider as an example the modulation that provides the variation in the signal amplitude linear in time. The modulator transmission is described by the function $M(t) = t/T_{\rm m}$. The complex amplitudes in the spectrum of the probe signal, taking into account the modulator action, have the form $\mathscr{E}_{\rm sr}(\omega)$ [see (9)] and

$$\mathscr{E}_{\rm srm}(\omega) = \frac{\rm i}{T_{\rm m}}\int(-{\rm i}t)\mathscr{E}_{\rm sr}(t)\exp(-{\rm i}\omega t){\rm d}t = \frac{\rm i}{T_{\rm m}}\frac{\rm d}{{\rm d}\omega}\mathscr{E}_{\rm sr}(\omega). \qquad (11)$$

The detected intensity distributions in the signal spectra are described in this case by the expressions [see (6)]

$$I_{\rm sr}(\omega) = A_{\rm sr}(\omega)^2,$$
$$I_{\rm srm}(\omega) = \frac{1}{T_{\rm m}^2}\left\{\left[\frac{\rm d}{{\rm d}\omega}A_{\rm sr}(\omega)\right]^2 + A_{\rm sr}^2(\omega)\left[\frac{\rm d}{{\rm d}\omega}\Phi_{\rm sr}(\omega)\right]^2\right\}. \qquad (12)$$

The functions $A_{sr}(\omega)$ and $dA_{sr}(\omega)/d\omega$ can be found from the intensity distribution $I_{sr}(\omega)$, while the functions $d\Phi_{sr}(\omega)/d\omega$ and $\Phi_{sr}(\omega)$ can be found from the intensity distribution $I_{srm}(\omega)$. Then, the complex amplitudes $\mathscr{E}_{sr}(\omega)$ and $\mathscr{E}_{sr}(t)$ are determined.

In the following, one should take into account the type of the action of the medium or object on probe radiation. Two variants of such an action are considered in practice. The first one represents a modulation, and is described by the multiplication operation, which gives the function $R(t)$, which characterises the action of the medium or object on radiation, in the form

$$\mathscr{E}_{sr}(t) = \mathscr{E}_{s}(t)R(t), \quad R(t) = \frac{\mathscr{E}_{sr}(t)}{\mathscr{E}_{s}(t)}. \tag{13}$$

The second variant corresponds to the transformation of radiation, which is described by the convolution

$$\mathscr{E}_{sr}(t) = \int \mathscr{E}_{s}(t')R(t-t')dt' \quad \text{or} \quad \mathscr{E}_{sr}(\omega) = \mathscr{E}_{s}(\omega)R(\omega). \tag{14}$$

This allows one to describe the action of the medium or object in the form $R(\omega) = \mathscr{E}_{sr}(\omega)/\mathscr{E}_{s}(\omega)$ or $R(t)$ by using the inverse Fourier transform.

4. Simultaneous measurement of the amplitude–phase characteristics of time-dependent optical signals and instrumental and transfer functions

The scheme presented in Fig. 3 is intended for the simultaneous determination and analysis of the time-dependent amplitude–phase characteristics of a signal and the transfer or instrumental function describing the action of the transfer medium, an object, or the optical system on the propagating signal.

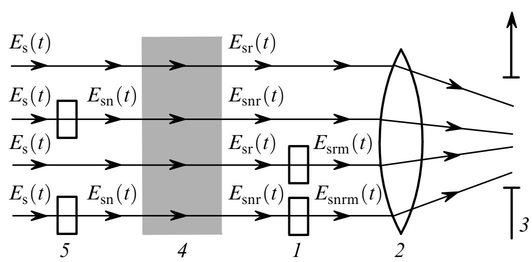

Figure 3. Principal four-channel optical scheme for a simultaneous analysis of the time dependences of the amplitude and phase structures of the light signal $E_s(t)$ and optical characteristics $R(t)$ of the medium. The notation as in Fig. 1; (*5*) additional modulator.

The probe signal of type (1) being analysed is characterised by the complex amplitude of spectral components $\mathscr{E}_s(\omega)$ located in the vicinity of the average frequency ω_0, the condition $\Delta\omega_s \ll \omega_0$ being satisfied for the frequency band $\Delta\omega_s$ of the signal spectrum. In the four-channel scheme, two signals propagate through an additional modulator (*5*), whose action on radiation is described by the function $N(t)$ or $N(\omega)$. The duration T_n of the modulator action on radiation should be equal to the duration of the signal being analysed or somewhat exceed it ($T_n \geqslant T_s$). Taking into account the modulator action and for a more compact presentation, the fields are written in the form

$$E_j(t) = \mathscr{E}_j(t)\exp(-i\omega_0 t).$$

Hereafter, the subscript $j =$ s, sn. The action of the transfer medium or instrumental function of the object is described by the function $R(t)$ or $R(\omega)$, the duration T_r of the medium or object action being comparable with the signal duration: $T_r \simeq T_s$. Another modulation of two channels is performed with modulator (*1*), whose action on radiation is described by the function $M(t)$ or $M(\omega)$. The duration T_m of the modulator action, as above, coincides with the signal duration or somewhat exceeds it ($T_m \geqslant T_s$). The fields that are produced taking into account the action of the medium and two modulators have the form [see (2) or (8)]

$$E_{jr}(t) = \mathscr{E}_{jr}(t)\exp(-i\omega_0 t),$$

$$E_{jrm}(t) = \mathscr{E}_{jr}(t)M(t)\exp(-i\omega_0 t).$$

At the spectral instrument output, four spectra are formed, which are described, similarly to (3) or (9), by the complex amplitudes

$$\begin{gathered}\mathscr{E}_{jr}(\omega) = A_{jr}(\omega)\exp[i\Phi_{jr}(\omega)], \\ \mathscr{E}_{jrm}(\omega) = \int \mathscr{E}_{jr}(\omega')M(\omega-\omega')d\omega'.\end{gathered} \tag{15}$$

The detected spectral intensity distributions are described, similarly to (4) or (10), by the expressions

$$\begin{gathered}I_{jr}(\omega) = A_{jr}^2(\omega), \\ I_{jrm}(\omega) = \left[\int A_{jr}(\omega')\exp[i\Phi_{jr}(\omega')]M(\omega-\omega')d\omega'\right] \times [\text{c.c.}]\end{gathered} \tag{16}$$

To illustrate the implementation of this method, consider, as above, a linear amplitude modulation.

The modulator action is described by the function $M(t) = t/T_m$. In this case, the complex amplitudes in the spectra of the four fields at the output of the spectral instrument are described by the functions $\mathscr{E}_{jr}(\omega)$ [see (15)] and

$$\mathscr{E}_{jrm}(\omega) = \frac{i}{T_m}\int(-it)\mathscr{E}_{jr}(t)\exp(-i\omega t)dt = \frac{i}{T_m}\frac{d}{d\omega}\mathscr{E}_{jr}(\omega).$$

The spectral intensity distributions detected in the output plane have, similarly to (6) or (12), taking into account (16) the form

$$I_{jr}(\omega) = A_{jr}^2(\omega),$$

$$I_{jrm}(\omega) = \frac{1}{T_m^2}\left[\left(\frac{d}{d\omega}A_{jr}(\omega)\right)^2 + A_{jr}^2(\omega)\left(\frac{d}{d\omega}\Phi_{jr}(\omega)\right)^2\right].$$

The amplitude structures $A_{jr}(\omega)$ and $dA_{jr}(\omega)/d\omega$ are found from the intensity distributions $I_{jr}(\omega)$. Then, the phase distributions $d\Phi_{jr}(\omega)/d\omega$ and $\Phi_{jr}(\omega)$ are found from the intensity distribution $I_{jrm}(\omega)$. The calculated amplitude $[A_{jr}(\omega)]$ and phase $[\Phi_{jr}(\omega)]$ distributions determine the complex amplitudes of the fields in the frequency $[\mathscr{E}_{jr}(\omega)]$ and time $[\mathscr{E}_{jr}(t)]$ representations.

In the following, it is necessary to consider the type of action of the medium or object on probe radiation. In the case of the action of the medium described by a convolution, the complex amplitudes of the fields have the form [see (14)]

$$\mathscr{E}_{jr}(t) = \int \mathscr{E}_j(t')R(t-t')\mathrm{d}t', \ \mathscr{E}_{jr}(\omega) = \mathscr{E}_j(\omega)R(\omega). \quad (17)$$

The use of a modulator (*5*), which provides a linear amplitude modulation $N(t) = t/T_{\mathrm{n}}$, gives the complex amplitudes in the spectral representation, which have the form $\mathscr{E}_{\mathrm{s}}(\omega)$ [see (3)] and

$$\mathscr{E}_{\mathrm{sn}}(\omega) = \frac{\mathrm{i}}{T_{\mathrm{n}}}\int(-\mathrm{i}t)\mathscr{E}_{\mathrm{s}}(t)\exp(-\mathrm{i}\omega t)\mathrm{d}t = \frac{\mathrm{i}}{T_{\mathrm{n}}}\frac{\mathrm{d}}{\mathrm{d}\omega}\mathscr{E}_{\mathrm{s}}(\omega).$$

By separating the function

$$R(\omega) = \frac{\mathscr{E}_{\mathrm{sr}}(\omega)}{\mathscr{E}_{\mathrm{s}}(\omega)} = \frac{\mathscr{E}_{\mathrm{snr}}(\omega)}{\mathscr{E}_{\mathrm{sn}}(\omega)} \quad (18)$$

from expression (17) for the found functions $\mathscr{E}_{jr}(\omega)$, taking into account the amplitude structures $A_{jr}(\omega)$, $\mathrm{d}A_{jr}(\omega)/\mathrm{d}\omega$, and the phase distributions $\mathrm{d}\Phi_{jr}(\omega)/\mathrm{d}\omega$ and $\Phi_{jr}(\omega)$, we obtain the differential equation

$$\frac{\mathrm{i}}{T_{\mathrm{n}}}\mathscr{E}_{\mathrm{sr}}(\omega)\frac{\mathrm{d}}{\mathrm{d}\omega}\mathscr{E}_{\mathrm{s}}(\omega) = \mathscr{E}_{\mathrm{snr}}(\omega)\mathscr{E}_{\mathrm{s}}(\omega).$$

for determining $\mathscr{E}_{\mathrm{s}}(\omega)$.

The solution of this equation with the boundary conditions

$$\mathscr{E}_{\mathrm{s}}(\omega) = 0 \ \text{for} \ \omega = \omega_0 \pm \frac{\Delta\omega_{\mathrm{s}}}{2}$$

yields the function $\mathscr{E}_{\mathrm{s}}(\omega)$ and $R(\omega)$ from (18). The inverse Fourier transform gives the required functions $\mathscr{E}_{\mathrm{s}}(t)$ and $R(t)$.

When the action of the medium or object on radiation is described by the multiplication operation [see (13)]

$$\mathscr{E}_{jr}(t) = \mathscr{E}_j(t)R(t), \quad (19)$$

a modulator (*5*) performs a linear amplitude modulation over the spectrum in the vicinity of the frequency ω_0 in the range $\Delta\omega_{\mathrm{n}}$, which has the form $N(\omega) = \omega/\Delta\omega_{\mathrm{n}}$. The distributions of complex amplitudes in the spectral and time representations are described in the form $\mathscr{E}_{\mathrm{s}}(\omega)$ [see (3)] and

$$\mathscr{E}_{\mathrm{sn}}(\omega) = \mathscr{E}_{\mathrm{s}}(\omega)N(\omega),$$

as well as in the form $\mathscr{E}_{\mathrm{s}}(t)$ [see (1)] and

$$\mathscr{E}_{\mathrm{sn}}(t) = -\frac{\mathrm{i}}{\Delta\omega_{\mathrm{n}}}\int(\mathrm{i}\omega)\mathscr{E}_{\mathrm{s}}(\omega)\exp(\mathrm{i}\omega t)\mathrm{d}\omega$$

$$= -\frac{\mathrm{i}}{\Delta\omega_{\mathrm{n}}}\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{E}_{\mathrm{s}}(t).$$

By separating the function

$$R(t) = \frac{\mathscr{E}_{\mathrm{sr}}(t)}{\mathscr{E}_{\mathrm{s}}(t)} = \frac{\mathscr{E}_{\mathrm{snr}}(t)}{\mathscr{E}_{\mathrm{sn}}(t)} \quad (20)$$

in (19), we obtain the differential equation

$$-\frac{\mathrm{i}}{\Delta\omega_{\mathrm{n}}}\mathscr{E}_{\mathrm{sr}}(t)\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{E}_{\mathrm{s}}(t) = \mathscr{E}_{\mathrm{snr}}(t)\mathscr{E}_{\mathrm{s}}(t)$$

for finding the function $\mathscr{E}_{\mathrm{s}}(t)$ taking into account the found functions $\mathscr{E}_{\mathrm{s}}(t)$ and $\mathscr{E}_{\mathrm{snr}}(t)$ and the boundary conditions

$$\mathscr{E}_{\mathrm{s}}(t) = 0 \ \text{for} \ t = 0 \ \text{and} \ t = T_{\mathrm{s}}.$$

The solution of this equation yields $\mathscr{E}_{\mathrm{s}}(t)$ and, according to (20), allows one to find $R(t)$, i.e., to obtain the required functions.

5. Estimates of the characteristics of spectral instruments

Let us make some numerical estimates of the characteristics of spectral instruments. The total detected spectrum $\Delta\nu$ or $\Delta\lambda$ determines the time resolution

$$\delta t = \frac{1}{\Delta\nu} = \frac{\lambda^2}{c\Delta\lambda}.$$

The spectral resolution $\delta\nu$ or $\delta\lambda$ provided by the instrument is characterised by the width of the instrumental function and determines the total detection time

$$T_{\mathrm{s}} = \frac{1}{\delta\nu} = \frac{\lambda^2}{c\delta\lambda}.$$

For a spectrometer with a reflection diffraction grating with the triangle line profile, the ratio

$$\frac{\Delta\nu}{\delta\nu} = \frac{\Delta\lambda}{\delta\lambda}$$

is sufficiently high, amounting to $10^4 - 10^5$ and more. Therefore, the ratio of the total detection time T_{s} to the time resolution δt is equally high.

For the spectrum of a signal at $\lambda \sim 1000$ nm and a 800-lines mm^{-1} diffraction grating of length 150 mm, the resolving power is $R = \lambda/\delta\lambda = 120\times10^3$, which provides the spectral resolution $\delta\lambda \sim 0.008$ nm and, therefore, the total detection time is $T_{\mathrm{s}} \sim 4\times10^{-10}$ s. The time resolution $\delta t \sim 10^{-14}$ s is provided upon detection of the spectral range $\Delta\lambda \sim 300$ nm.

Similarly, for the spectrum of a signal at $\lambda \sim 800$ nm and a 1000-lines mm^{-1} diffraction grating of length 150 mm, the resolving power is $R = \lambda/\delta\lambda = 150\times10^3$, $\delta\lambda \sim 0.005$ nm, and $T_{\mathrm{s}} \sim 4\times10^{-10}$ s. The time resolution $\delta t \sim 10^{-14}$ is achieved upon detection of the spectral range $\Delta\lambda \sim 200$ nm.

When the narrow face of the diffraction grating line is used, it is possible to work in the second diffraction order, which results in an increase in the total detection time up to $\sim 10^{-9}$ s [30, 31].

Thus, spectrometers with typical spectral characteristics can provide a rather high time resolution.

The possibility of obtaining the total detection time $T_{\mathrm{s}} \sim 10^{-9}$ s shows a promise of using the available modulators for realisation of the required operating regime. These can be mechanical modulators on gas bearings with a turbine gear, similar to those applied in photochronographs [6], or electrooptical crystal modulators [4].

6. Conclusions

It has been shown that the spectral, amplitude, and phase characteristics of the time-dependent optical signals of an arbitrary shape can be in principle measured and analysed with a high time resolution using the two-channel spectral analysis and an additional modulation of the signal. This approach can be also used for studying the time-dependent amplitude and phase characteristics of optical media or objects by probing them with radiation with the known time-dependent structure. The use of a four-channel spectral instrument with two additional modulators provided information on the signal structure and on the action of the medium or object on probe radiation.

No additional information or special conditions are required for solving the problems. The information is processed without any iteration procedures, which yields the results in quasi-real time.

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