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Interaction of light waves of arbitrary polarisation on a photorefractive grating in a cubic gyrotropic crystal in an external alternating electric field

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Abstract. **The symmetric two-wave interaction is considered, for small modulation indices of an interference light pattern, on a transmission photorefractive grating with stationary amplitude produced in a crystal of symmetry group 23 in an external alternating electric field. For an arbitrary off-duty ratio of the external field or arbitrary elliptic polarisation of incident light waves, the space-charge field is shown to contain both shifted and unshifted components, while the polarisation state and intensity of a weak light wave change upon the external field switching. The space-charge-field amplitude and the weak-wave intensity gain are analysed as functions of the interaction wavelength, the off-duty ratio of the external field, and the photorefractive-grating spacing for the longitudinal and transverse interaction geometry at a wavelength of 633 nm in a $Bi_{12}TiO_{20}$ crystal.**

Keywords: *photorefractive grating, gyrotropic crystal, alternating electric field.*

1. Introduction

Theoretical and experimental studies of the two-wave interaction in cubic photorefractive crystals extend our insight into a qualitative picture of the interaction of light waves on photorefractive nonlinearity [1 – 13].

The authors of paper [13] considered the dependence of the energy-exchange coefficient for the two-wave interaction in a $Bi_{12}TiO_{20}$ crystal in an external constant electric field $\boldsymbol{E}_0 = E_0\boldsymbol{z}^0$ (where $\boldsymbol{z}^0$ is the unit vector) on the sign of E_0 for the circular polarisation of light waves. For the same polarisation of light waves, the weak-wave gains for positive and negative values of an external square-wave field prove to be different [3]. The experiments [3] were performed for the external-field period T satisfying the condition $\tau_r \ll T \ll \tau_d$, where τ_r and τ_d are the recombination and dielectric relaxation times, respectively. In this case, the time modulation of the space-charge field $\boldsymbol{E}_{sc} = E_{sc}\boldsymbol{z}^0$ in the stationary regime is negligible [14 – 17]. However, when the sign of E_0 changes, the polarisation state of a light field in a crystal also changes due to a change in the sign of perturbations of the dielectric constant induced through a linear electrooptical effect. Such a change in the polarisation state of light waves in a crystal in the case of arbitrary elliptic polarisation of the incident waves results in a piece-constant periodic time dependence of the interaction efficiency and then light-intensity distribution over the interaction length. The two-wave interaction was analysed in papers [3 – 7] by neglecting the effect of the time dependence of the modulation index of the interference pattern on the field E_{sc}.

The aim of this paper is to analyse the symmetric two-wave interaction on a transmission photorefractive grating formed in a cubic gyrotropic crystal for an arbitrary polarisation of incident light waves and an arbitrary off-duty ratio of an external alternating electric field, taking into account the influence of the periodic time modulation of the interference pattern on the space-charge field.

2. The model

Consider the interaction of two plane monochromatic light waves in cubic gyrotropic photorefractive crystals of symmetry group 23, with an alternating electric field applied to the side faces of the crystals (Fig. 1a). Under the action of an interference pattern described by the expression $I = I_0(1 + m\cos Kz)$, a space-charge field is produced in the crystal, which induces, due to a linear electrooptical effect, the perturbation of the dielectric constant of the crystal $\Delta\hat{\varepsilon} = n^4\hat{r}\boldsymbol{z}^0E_{sc} = n^4r_{41}E_{sc}\hat{g}$ [18], where n is the refractive index, and $\hat{r}$ is the electrooptical tensor, which has nonzero components $r_{41} = r_{52} = r_{63}$ for crystals under study. The components of the introduced tensor $\hat{g}$ are determined by the relations $g_{ii} = 0$, $g_{12} = g_{21} = z_3^0$, $g_{13} = g_{31} = z_2^0$, $g_{23} = g_{32} = z_1^0$, where z_i^0 are the components of the vector $\boldsymbol{z}^0$ in a crystallophysic coordinate system. In a linear approximation in the modulation index m, the spatial distribution of the field $E_{sc} = (E_1/2)\exp(iKz) + \text{c.c.}$ [17] repeats the interference-pattern distribution, which is shifted in the general case. A similar perturbation $\Delta\hat{\varepsilon} = n^4r_{41}E_0\hat{g}$ is induced by the external electric field.

The light field in the crystal can be written as a superposition of the eigenwaves of the medium [4, 6, 7, 10, 13, 19]. The scalar amplitudes $S_{1,2}$ and $R_{1,2}$ of these waves are changed due to the interaction on the refractive grating with the vector $\boldsymbol{K} = K\boldsymbol{z}^0$. The vector diagram of the two-wave interaction is shown in Fig. 1b. For a small modulation index

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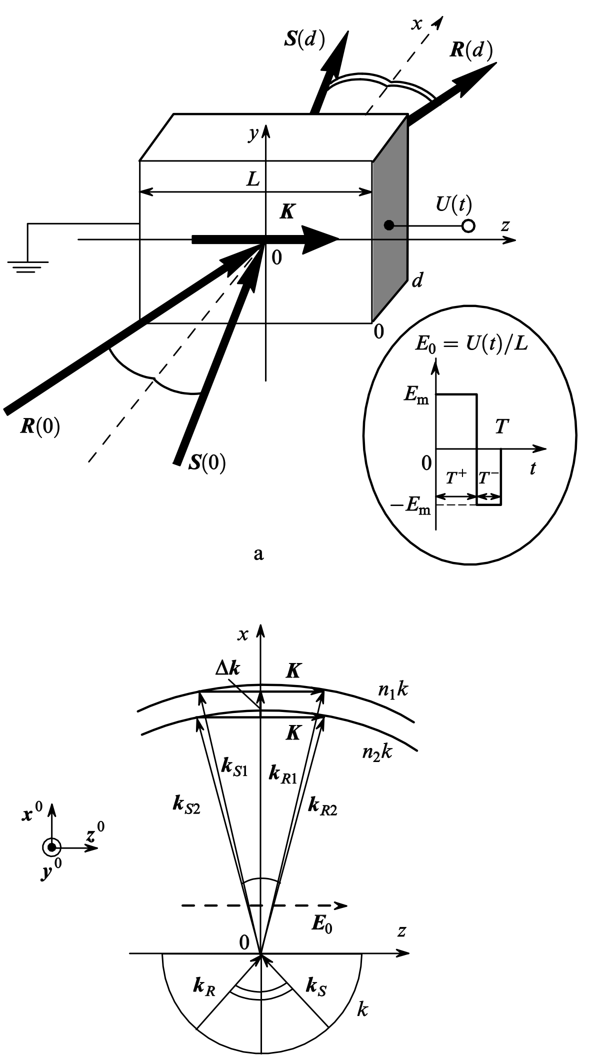

Figure 1. Scheme (a) and the vector diagram (b) of a symmetric two-wave interaction on the transmission photorefractive grating in a cubic gyrotropic crystal in an external alternating electric field changing in time as shown in the inset.

$$m = \frac{2(S_1 R_1^* + S_2 R_2^*)}{I_0} \ll 1, \tag{1}$$

the amplitudes $R_{1,2}$ can be considered specified $R_{1,2} \equiv R_{10,20}$. Then, $I_0 \approx |R_{10}|^2 + |R_{20}|^2$. In the approximation of slowly varying amplitudes S_1 and S_2, the equation for determining their spatial dependences can be obtained from the wave equation in the form

$$\frac{\mathrm{d}S_1}{\mathrm{d}x} = \mathrm{i}\frac{\pi n^3 r_{41}}{2\lambda} E_1 [g_{11} R_{10} + g_{12} \exp(\mathrm{i}\Delta nkx) R_{20}], \tag{2}$$

$$\frac{\mathrm{d}S_2}{\mathrm{d}x} = \mathrm{i}\frac{\pi n^3 r_{41}}{2\lambda} E_1 [g_{12}^* \exp(-\mathrm{i}\Delta nkx) R_{10} + g_{22} R_{20}], \tag{3}$$

where $k = 2\pi/\lambda$ is the wave number. The birefringence Δn, and convolutions $g_{11} = \boldsymbol{e}_1^* \hat{g} \boldsymbol{e}_1$, $g_{22} = \boldsymbol{e}_2^* \hat{g} \boldsymbol{e}_2$ and $g_{12} = \boldsymbol{e}_1^* \hat{g} \boldsymbol{e}_2$, of the tensor $\hat{g}$ with the polarisation vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ of the eigenwaves of the crystal are determined by expressions presented in papers [4, 6, 7, 13, 19]. The tensor convolutions g_{11} and g_{22} describe the influence of intramode processes on the interaction efficiency upon vector synchronism $\boldsymbol{k}_{S1} = \boldsymbol{k}_{R1} - \boldsymbol{K}$ и $\boldsymbol{k}_{S2} = \boldsymbol{k}_{R2} - \boldsymbol{K}$, respectively (Fig. 1b), while the convolution g_{12} describes this influence for synchronism $\boldsymbol{k}_{S1} = \boldsymbol{k}_{R2} - \boldsymbol{K} + \Delta\boldsymbol{k}$ and $\boldsymbol{k}_{S2} = \boldsymbol{k}_{R1} - \boldsymbol{K} - \Delta\boldsymbol{k}$.

When the period T of the external electric field is comparable with the time τ_{d} or $T > \tau_{\mathrm{d}}$, the amplitude E_1 of the space-charge field in the stationary regime exhibits a periodic time dependence [14–17]. If $T \ll \tau_{\mathrm{d}}$, the time modulation of E_1 is negligible. The amplitude E_1 weakly depends on T in the range $\tau_{\mathrm{r}} \ll T \ll \tau_{\mathrm{d}}$ and can be obtained in the form [14, 17]

$$E_1 = -\left\langle mE_{\mathrm{q}} \frac{E_0 + \mathrm{i}E_{\mathrm{d}}}{E_\mu + E_{\mathrm{d}} - \mathrm{i}E_0} \right\rangle \Big/ \left\langle \frac{E_{\mathrm{q}} + E_{\mathrm{d}} - \mathrm{i}E_0}{E_\mu + E_{\mathrm{d}} - \mathrm{i}E_0} \right\rangle$$

$$= -\frac{\langle mF_1 \rangle}{\langle \Gamma_1 \rangle}, \tag{4}$$

where $E_{\mathrm{d}} = 2\pi k_{\mathrm{B}} T'/(\Lambda e)$; $E_\mu = \Lambda/(2\pi\mu\tau_{\mathrm{r}})$; $E_{\mathrm{q}} = \Lambda e N_{\mathrm{a}}/(2\pi\varepsilon)$; $\Lambda = 2\pi/K$; N_{a} is the acceptor concentration; μ is the electron mobility; ε, k_{B}, e and T' are the static dielectric constant of the medium, the Boltzmann constant, the elementary charge, and the absolute temperature, respectively. The angle brackets denote averaging over the period T [10].

In the above approximations, equations (1)–(4) describe a symmetric two-wave interaction on a transmission photorefractive grating in a cubic gyrotropic crystal for an arbitrary orientation of the sample faces, an arbitrary polarisation of the incident wave, and an arbitrary period of the field $E_0(t)$.

3. Space-charge field in an external alternating electric field for an arbitrary polarisation of a pump wave

Taking relation (1) into account and using equations (2) and (3), we can express the derivative from the modulation index of the interference pattern with respect to the interaction length x in terms of the space-charge-field amplitude

$$\frac{\mathrm{d}m}{\mathrm{d}x} = \mathrm{i}\frac{\pi n^3}{\lambda} E_1 \frac{\mathrm{d}}{\mathrm{d}x}[r_{\mathrm{eff}}(x)x], \tag{5}$$

where the effective electrooptical coefficient

$$r_{\mathrm{eff}}(x) = r_{41}\left[\eta_{\mathrm{in}} - 2\mathrm{Im}\left\{\frac{\eta_{\mathrm{inter}}[1 - \exp(\mathrm{i}\Delta nkx)]}{\Delta nkx}\right\}\right], \tag{6}$$

is introduced, which is defined by the parameters $\eta_{\mathrm{in}} = (g_{11}|R_{10}|^2 + g_{22}|R_{20}|^2)/I_0$ and $\eta_{\mathrm{inter}} = g_{12} R_{10}^* R_{20}/I_0$ describing the contributions from intramode and intermode processes, respectively. By solving equations (4) and (5), we obtain E_1 in the form

$$E_1(x) = -m_0 \frac{\langle F_1 \rangle}{\langle \Gamma_1 \rangle} \exp\left[-\frac{\mathrm{i}\pi n^3 \langle r_{\mathrm{eff}}(x) F_1 \rangle x}{\lambda \langle \Gamma_1 \rangle}\right], \tag{7}$$

where m_0 is the modulation index at the boundary $x = 0$.

It follows from (7) that the distribution of the amplitude E_1 over x is determined by the value of $\langle r_{\mathrm{eff}}(x) F_1 \rangle / \langle \Gamma_1 \rangle$, which depends on the amplitude E_{m} and the time dependence $f(t)$ of the field $E_0 = E_m f(t)$. The time dependence $f(t)$, which is optimal for the amplification of E_1 with increasing x, can be determined from the solution of the extremal problem for the functional $\mathrm{Im}[\langle r_{\mathrm{eff}}(x)F_1\rangle/\langle\Gamma_1\rangle]$, which does not depend explicitly on $\mathrm{d}f/\mathrm{d}t$ and t in the approximations used. The analysis of this problem shows that the amplification of E_1 in an alternating external field with the amplitude E_m (Fig. 1), for example, in the square-wave field $E_0 = E_m \mathrm{sign}[\sin(2\pi t/T)]$, is greater than that in a continuous periodic field with the same amplitude, for example, in the sinusoidal field $E_0 = E_m \sin(2\pi t/T)$ [14–17]. For an alternating external field, the amplitude E_1 can be represented in the form

$$E_1(x) = -\mathrm{i}m_0 E_{\mathrm{eff}} \frac{1-\mathrm{i}\chi\delta_F}{1+\mathrm{i}\chi\delta_\Gamma}$$

$$\times \exp\left[\frac{\pi n^3 E_{\mathrm{eff}} x}{\lambda} \frac{r_\Sigma(x)(1-\mathrm{i}\chi\delta_F) + r_\Delta(x)(\chi - \mathrm{i}\delta_F)}{1+\mathrm{i}\chi\delta_\Gamma}\right], \quad (8)$$

where $\chi = (T^+ - T^-)/T$; $E_{\mathrm{eff}} = E_q[E_d(E_\mu + E_d) + E_m^2]/[(E_\mu + E_d)(E_q + E_d) + E_m^2]$; $\delta_F = E_m E_\mu/[E_d(E_\mu + E_d) + E_m^2]$; $\delta_\Gamma = E_m(E_q - E_\mu)/[(E_\mu + E_d)(E_q + E_d) + E_m^2]$; $r_\Sigma(x) = [r_{\mathrm{eff}}^+(x) + r_{\mathrm{eff}}^-(x)]/2$; $r_\Delta(x) = [r_{\mathrm{eff}}^+(x) - r_{\mathrm{eff}}^-(x)]/2$; and $r_{\mathrm{eff}}^\pm(x)$ is the coefficient r_{eff} for $E_0 = \pm E_m$.

It follows from (8) that, when the period T of the field E_0 is constant, the local (proportional to $\mathrm{Re}E_1$) and nonlocal (proportional to $\mathrm{Im}E_1$) components of a photorefractive grating are determined both by the relation between the durations of the intervals of positive (T^+) and negative (T^-) values of the field E_0 (by the parameter χ) and the coefficients $r_{\mathrm{eff}}^+(x)$ and $r_{\mathrm{eff}}^-(x)$, which depend on the polarisation state of the pump wave. By using the expressions for the intrinsic refractive indices and tensor convolutions g_{11}, g_{22} and g_{12} presented in papers [4, 6, 7, 13, 19] and the expressions relating the scalar amplitudes of the eigenwaves of the medium with the polarisation parameters of the incident waves (which can be obtained from boundary conditions for the light field at the point $x = 0$), we can show that for a linearly polarised pump wave, the coefficient r_{eff} does not change with changing the sign of the field E_0. In this case, this coefficient can be conveniently expressed in terms of the angle θ between the polarisation vector of the pump wave and the y axis:

$$r_{\mathrm{eff}}^{\mathrm{lin}}(x) = r_{41}\Bigg\{H_\Sigma + \rho x \,\mathrm{sinc}^2(\Delta nkx/2)(H_{\mathrm{ME}}\cos 2\theta$$

$$+ H_\Delta \sin 2\theta) + \left[\left(\frac{2\delta n}{\Delta n}\right)^2 (H_\Delta^2 + H_{\mathrm{ME}}^2)\right.$$

$$\left. + \left(\frac{2\rho}{\Delta nk}\right)^2 \mathrm{sinc}(\Delta nkx)\right](H_{\mathrm{ME}}\sin 2\theta - H_\Delta \cos 2\theta)\Bigg\}, \quad (9)$$

where $\mathrm{sinc}\,\xi = (\sin\xi)/\xi$; $H_\Sigma = (H_{\mathrm{MM}} + H_{\mathrm{EE}})/2$; $H_\Delta = (H_{\mathrm{MM}} - H_{\mathrm{EE}})/2$; $H_{\mathrm{MM}} = \boldsymbol{z}^0\hat{\boldsymbol{g}}\boldsymbol{z}^0$; $H_{\mathrm{EE}} = \boldsymbol{y}^0\hat{\boldsymbol{g}}\boldsymbol{y}^0$; $H_{\mathrm{ME}} = \boldsymbol{z}^0\hat{\boldsymbol{g}}\boldsymbol{y}^0$; $\delta n = n^3 r_{41}E_0/2$; and ρ is the rotatory power. Note that the value of Δn, unlike δn, does not depend on time in an alternating field E_0. In the cases of right or left elliptic polarisation of pump waves, the coefficients r_{eff}^+ and r_{eff}^- on the input face of the crystal are related by the expressions

$$r_{\mathrm{eff}}^{+\mathrm{right}}(x) = r_{\mathrm{eff}}^{-\mathrm{left}}(x), \quad r_{\mathrm{eff}}^{+\mathrm{left}}(x) = r_{\mathrm{eff}}^{-\mathrm{right}}(x). \quad (10)$$

In the case of circular polarisation, the coefficient r_{eff} can be represented in the form

$$r_{\mathrm{eff}}^{\mathrm{circ}}(x) = r_{41}\Bigg\{H_\Sigma + [1 - \mathrm{sinc}(\Delta nkx)]$$

$$\times \frac{4\delta n\rho}{\Delta n^2 k}\left(H_{\mathrm{ME}}^2 + H_\Delta^2\right)\Bigg\}. \quad (11)$$

One can see from (8), taking (10) into account, that for an elliptically polarised pump wave ($r_\Delta \neq 0$), the spatial displacement of the field E_{sc} with respect to the initial interference pattern with the modulation index m_0 is not equal to the value $\Lambda/4$, which is typical for the photorefractive response of a crystal in an external square-wave ($\chi = 0$) field E_0 [2–7, 9, 10, 12, 14–17, 19]. As follows from (5), the distribution of the modulation index m of the interference pattern over x for $E_0 = E_m$ differs from that for $E_0 = -E_m$. This means that for $x \neq 0$, the interference pattern is sharply displaced with respect to the field E_{sc} after changing the sign of the field E_0.

Fig. 2 shows the distributions of the local ($\mathrm{Re}\,E_1$) and nonlocal ($\mathrm{Im}\,E_1$) components of the field amplitude E_{sc} with a spatial period $\Lambda = 3.4$ μm over the interaction length x in a $Bi_{12}TiO_{20}$ crystal in an external square-wave electric field ($\chi = 0$) with the amplitude $E_m = 10$ kV cm^{-1}. We used in calculations the parameters that are typical for a light wave at 633 nm: $N_a = 10^{-22}$ m^{-3}, $\mu\tau_r = 10^{-12}$ V s m^{-2}, $\varepsilon = 416$ pF m^{-1}, $n = 2.58$, $r_{41} = -5$ pm V^{-1}, and $\rho = 6^\circ$ mm^{-1}. The distributions in Figs 2a, b correspond to the longitudinal interaction geometry, when light waves propagate in the $(\bar{1}10)$ crystal plane, and the positive direction of the coordinate axis z coincides with that of the [001] crystal axis. The distributions in Figs 2c, d correspond to the transverse

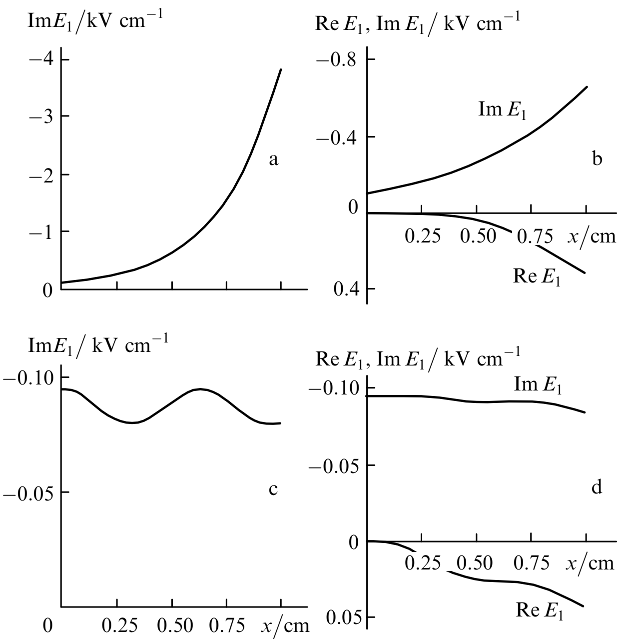

Figure 2. Distributions of the space-charge-field amplitude over the interaction length for the longitudinal (a, b) and transverse (c, d) geometry of the two-wave interaction in a $Bi_{12}TiO_{20}$ crystal in an external square-wave electric field for linear (a, c) and circular (b, d) polarisations of the incident pump wave.

interaction geometry, when light waves propagate in the (001) plane and the direction of the coordinate axis z coincides with that of the $[\bar{1}10]$ axis. The curves in Figs 2a, c correspond to the incident pump wave that is linearly polarised along the y axis, while the curves in Figs 2b, d correspond to the right circular polarisation of the pump wave.

Because in the case of a linearly polarised pump wave, the coefficient $r_{\rm eff}$ does not exhibit any jumps upon changing the sign of the field E_0, the photorefractive grating contains only a nonlocal component, which is proportional to $\operatorname{Im} E_1$ (Figs 2a, c). For a circularly polarised pump wave and $x \neq 0$, the coefficients $r^{+}_{\rm eff}(x)$ and $r^{-}_{\rm eff}(x)$ are different, resulting in the formation of a local component, which is proportional to $\operatorname{Re} E_1$ (Figs 2b, d).

In the case of a longitudinal geometry ($H_{\rm MM} = H_{\rm ME} = 0$, $H_{\rm EE} = -1$), the increase in the component $\operatorname{Im} E_1$ with increasing x from zero to $d = 1$ for a linearly polarised pump wave exceeds that for a circular polarisation by a factor of 5.7 (Figs 2a, b). This is explained by the fact that, under conditions considered, the polarisation ellipses of the eigenwaves, which are described by the vectors $\boldsymbol{e}^{+}_2 = \boldsymbol{e}^{-}_1$ and $\boldsymbol{e}^{+}_1 = \boldsymbol{e}^{-}_2$, are noticeably elongated (the axial ratio is $\delta \approx 0.2$) along axes y and z, and the tensor convolutions are $g^{+}_{11} = g^{-}_{22} \approx -0.04$, $g^{+}_{22} = g^{-}_{11} \approx -0.96$ and $g^{\pm}_{12} \approx -0.2$. For this reason, the pump wave with the polarisation vector close to the vector $\boldsymbol{e}^{+}_2 = \boldsymbol{e}^{-}_1$ is optimal for amplification of E_1. The contribution from intramode processes, which cause the exponential growth of $E_1(x)$, is greater for the pump wave that is linearly polarised along the y axis ($\eta_{\rm in} \approx -0.92$) than that for a circularly polarised pump wave ($\eta^{+}_{\rm in} \approx -0.32$, $\eta^{-}_{\rm in} \approx -0.68$). The intermode processes under these conditions weakly affect E_1 ($|\eta_{\rm inter}| < 0.1$ in both cases).

For the transverse geometry ($H_{\rm MM} = H_{\rm EE} = 0$, $H_{\rm ME} = 1$) and linear polarisation of the pump wave, the distribution of the component $\operatorname{Im} E_1$ over x is formed only due to intermode processes ($\eta_{\rm inter} \approx -0.11$, $\eta_{\rm in} = 0$). In this case, the dependence $\operatorname{Im} E_1(x)$ is periodic, with period $\Lambda_x = 2\pi/(\Delta n k) = 0.63$ cm (Fig. 2c). For a circularly polarised pump wave, the distribution of E_1 over x is formed due to intramode ($\eta^{+}_{\rm in} = -\eta^{-}_{\rm in} \approx 0.21$) and intermode ($\eta^{+}_{\rm inter} = -\eta^{-}_{\rm inter} \approx -0.1$) processes. In this case, the component $\operatorname{Re} E_1$ increases with increasing x, whereas the component $\operatorname{Im} E_1$ decreases. (Fig. 2d).

It follows from expression (8) that the local component can be formed not only due to the elliptic polarisation of the pump wave but also due to a different duration ($\chi \neq 0$) of the intervals of positive and negative values of the field E_0. Fig. 3 shows the dependence of the local and nonlocal components of the amplitude of the field $E_{\rm sc}$ produced in the cross section $x = 1$ cm of a $Bi_{12}TiO_{20}$ crystal on the parameter χ. The remaining conditions of the calculations of curves in Figs 3a–d are the same as for curves in Figs 2a–d, respectively.

For a linearly polarised pump wave (Figs 3a, c), the local component is an asymmetric function of χ, while the nonlocal component is a symmetric function of χ. The component $\operatorname{Im} E_1$ reaches its maximum for $\chi = 0$ (a square-wave external field). The modulus $|\operatorname{Re} E_1|$ has a maximum at $|\chi| = 0.25$ for a longitudinal geometry (Fig. 3a) and at $|\chi| \to 1$ for a transverse geometry (Fig. 3c). In the case of the right circular polarisation of the pump wave (Figs 3b, c), the dependences of $\operatorname{Re} I_1$ and $\operatorname{Im} E_1$ on χ do not have symmetry properties, and their maxima and minima are achieved for values of the parameter χ that differ from those in the case of a linearly polarised pump wave.

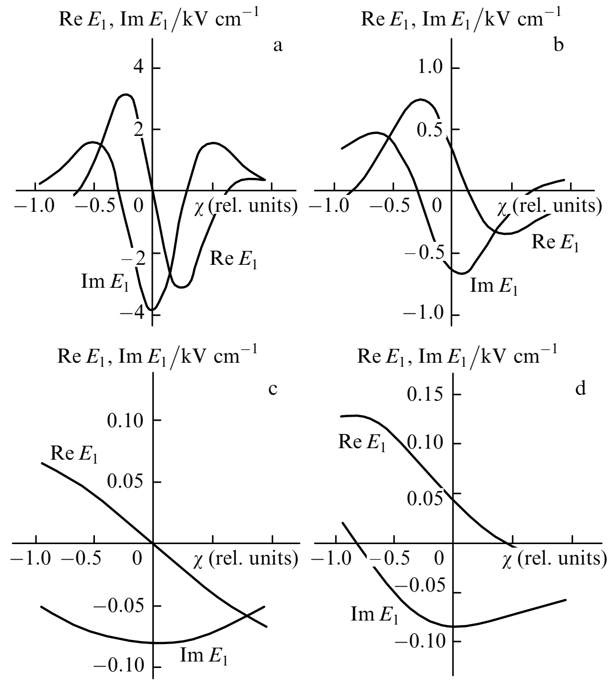

Figure 3. Dependences of the space-charge-field amplitude at a distance $x = 1$ cm from the input face of a $Bi_{12}TiO_{20}$ crystal in an external alternating electric field on the parameter χ for the longitudinal (a, b) and transverse (c, d) geometry of the two-wave interaction for linear (a, c) and circular (b, d) polarisations of the incident pump wave.

4. Polarisation state and intensity of a weak light wave

Relation (7) for the space-charge-field amplitude allows one to integrate equations for coupled waves (1) and (2) and represent the vector amplitude $\boldsymbol{S}(x)$ of the total light field of a weak wave in the form

$$\boldsymbol{S}(x) = \boldsymbol{S}_{\parallel}(x) - \mathrm{i} m_0 \frac{\pi n^3 \langle F_1 \rangle}{2\lambda \langle \Gamma_1 \rangle} \times \left\{ \boldsymbol{R}_{\parallel}(x) \int_0^x \exp\left[-\mathrm{i}\frac{\pi n^3 \langle r_{\rm eff}(\zeta) F_1 \rangle \zeta}{\lambda \langle \Gamma_1 \rangle}\right] \times \mathrm{d}[r_{\rm eff}(\zeta)\zeta] + r_{41} X(x) \boldsymbol{R}_{\perp}(x) \right\}, \quad (12)$$

where

$$X(x) = \int_0^x g(\zeta) \exp\left[-\mathrm{i}\frac{\pi n^3 \langle r_{\rm eff}(\zeta) F_1 \rangle \zeta}{\lambda \langle \Gamma_1 \rangle}\right] \mathrm{d}\zeta; \quad (13)$$

$\boldsymbol{S}_{\parallel}(x) = S_{10}\boldsymbol{e}_1 + S_{20}\boldsymbol{e}_2 \exp(\mathrm{i}\Delta n k x)$ and $\boldsymbol{R}_{\parallel}(x) = R_{10}\boldsymbol{e}_1 + R_{20}\boldsymbol{e}_2 \times \exp(\mathrm{i}\Delta n k x)$ are the vector amplitudes of the signal wave in the absence of the interaction and of the pump wave, respectively; $\boldsymbol{R}_{\perp}(x) = R^{*}_{20}\boldsymbol{e}_1 - R^{*}_{10}\boldsymbol{e}_2 \exp(\mathrm{i}\Delta n k x)$ is the vector orthogonal to the vector $\boldsymbol{R}_{\parallel}(x)$ ($\boldsymbol{R}_{\parallel}(x)\boldsymbol{R}^{*}_{\perp}(x) = 0$); $\boldsymbol{R}$ is the vector amplitude of the light wave; and the function $g(\zeta)$ is defined by expression (14) in paper [13].

When the incident light waves have the same polarisation, the amplitude $\boldsymbol{S}(x)$ can be represented as a superposition of two components, which are proportional to the vectors $\boldsymbol{S}_{\parallel}(x)$ and $\boldsymbol{S}_{\perp}(x) = S_{20}^{*}\boldsymbol{e}_1 - S_{10}^{*}\boldsymbol{e}_2 \exp(\mathrm{i}\Delta nkx)$ [6, 7, 11, 13]. Note that the distribution of the modulation index $m = 2\boldsymbol{S}_{\parallel}(x)\boldsymbol{R}_{\parallel}^{*}(x)/I_0$ over x is formed upon the interference of the pump wave and the signal-wave component, which is proportional to $\boldsymbol{S}_{\parallel}(x)$ and is self-consistent with the amplitude E_1 of the field E_{sc} due to self-diffraction [see (5)]. For $m \ll 1$, the contribution from the signal-wave component [which is proportional to $\boldsymbol{S}_{\perp}(x)$] to the interference pattern is negligible [$\boldsymbol{S}_{\perp}(x)\boldsymbol{R}_{\parallel}^{*}(x) = 0$ and $I_S = |\boldsymbol{S}(x)|^2 \ll I_0$], so that it does not affect the field E_{sc}. This component appears due to anisotropic diffraction of the pump wave from an inhomogeneous photorefractive grating, with the transformation of the initial polarisation state to the orthogonal state [17].

When the incident waves have the same linear polarisation in an alternating external electric field, the two-wave gain $\Gamma = [\ln(I_S/I_{S0})]/x$ of a weak signal-wave intensity I_{S0} (where $I_{S0} = |\boldsymbol{S}_{\parallel}(x)|^2$) can be represented in the form

$$\Gamma(x) = \frac{2\pi n^3 r_{\mathrm{eff}}^{\mathrm{lin}}(x) E_{\mathrm{eff}}}{\lambda} \frac{1 - \chi^2 \delta_F \delta_\Gamma}{1 + \chi^2 \delta_\Gamma^2} + \Gamma_{\perp}(x)$$

$$= \Gamma_{\parallel}(x) + \Gamma_{\perp}(x), \qquad (14)$$

where

$$\Gamma_{\perp}(x) = \frac{1}{x} \ln \left\{ 1 + \left(\frac{\pi n^3 r_{41} E_{\mathrm{eff}}}{\lambda} \right)^2 |X^{\mathrm{lin}}(x)|^2 \frac{1 + \chi^2 \delta_F^2}{1 + \chi^2 \delta_\Gamma^2} \right.$$

$$\left. \times \exp[-\Gamma_{\parallel}(x) x] \right\}; \qquad (15)$$

$$X^{\mathrm{lin}}(x) = \int_0^x \exp\left[\frac{\pi n^3 r_{\mathrm{eff}}^{\mathrm{lin}}(\zeta) E_{\mathrm{eff}}}{\lambda} \frac{1 - \mathrm{i}\chi\delta_F}{1 + \mathrm{i}\chi\delta_\Gamma} \zeta \right]$$

$$\times \left\{ \frac{4\delta n \rho}{k \Delta n^2} (H_\Delta^2 + H_{\mathrm{ME}}^2)[\cos(\Delta nk\zeta) - 1] \right.$$

$$+ \frac{\mathrm{i}2\rho}{k\Delta n} (H_{\mathrm{ME}} \sin 2\theta - H_\Delta \cos 2\theta) \sin(\Delta nk\zeta) \qquad (16)$$

$$- \mathrm{i} \left[\frac{4\delta n^2}{\Delta n^2} (H_\Delta^2 + H_{\mathrm{ME}}^2) + \frac{4\rho^2}{k^2 \Delta n^2} \cos(\Delta nk\zeta) \right]$$

$$\left. \times (H_{\mathrm{ME}} \cos 2\theta + H_\Delta \sin 2\theta) \right\} \mathrm{d}\zeta.$$

The component $\Gamma_{\parallel}$ determines the contribution of the component to the intensity I_S, which is proportional to $\boldsymbol{S}_{\parallel}$, and describes usual unidirectional energy transfer due to which a weak light wave can be amplified or attenuated [1–17]. The second component $\Gamma_{\perp}$ is always positive and determines the non-unidirectional contribution to I_S from the component proportional to $\boldsymbol{S}_{\perp}$ [6–8, 11, 13]. The coefficient $\Gamma_{\perp}$ at the positive $\Gamma_{\parallel}$ is greater than that at the negative.

The coefficients $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ depend on the parameter χ. The coefficient $\Gamma_{\parallel}$ does not change when the sign of E_0 changes and is a symmetric function of χ, which has a maximum at $\chi = 0$ (square-wave field E_0) and a minimum at $|\chi| \to 1$. The coefficient $\Gamma_{\perp}^{+}$ for $E_0 = E_{\mathrm{m}}$ [$\delta n > 0$, see (16)] differs from the coefficient $\Gamma_{\perp}^{-}$ for $E_0 = -E_{\mathrm{m}}$ ($\delta n < 0$) $\chi \neq 1$. However, the relation $\Gamma_{\perp}^{+}(\chi) = \Gamma_{\perp}^{-}(-\chi)$ is valid. The type of the dependence $\Gamma^{+}(\chi)$ is determined by the orientation of the interaction with respect to the crystallophysic axes and by the ratio δ_F/δ_Γ, which strongly depends of the grating spacing Λ.

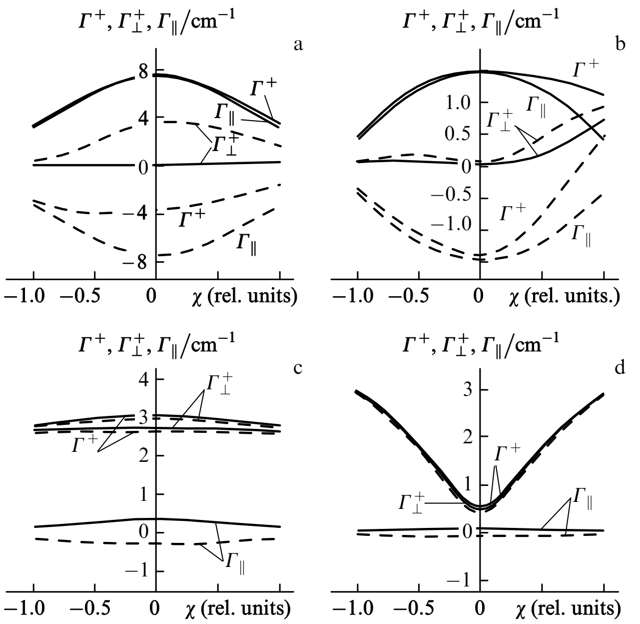

Figure 4. Dependences of the total two-wave gain Γ^{+} and its unidirectional ($\Gamma_{\parallel}$) and non-unidirectional ($\Gamma_{\perp}$) components on the parameter χ for the longitudinal (a, b) and transverse (c, d) geometry of interaction in a $Bi_{12}TiO_{12}$ crystal of thickness $d = 1$ cm in an external alternating electric field for linearly polarised incident light waves and the photorefractive grating spacing $\Lambda = 3.4$ (a, c) and 34 μm (b, d). The solid and dashed curves correspond to different orientations of a sample rotated by 180° around the x axis.

Fig. 4 shows the dependences of the total two-wave gain Γ^{+} and its unidirectional ($\Gamma_{\parallel}$) and non-unidirectional ($\Gamma_{\perp}^{+}$) components on the parameter χ for the two-wave interaction in a $Bi_{12}TiO_{20}$ crystal of thickness $x = d = 1$ cm in an external alternating field E_0 with the amplitude $E_{\mathrm{m}} = 10$ kV cm^{-1}. The dependences in Figs 4a, b correspond to the longitudinal geometry, and those in Figs 4c, d – to the transverse geometry. The curves in Figs 4a, c are calculated for $\Lambda = 3.4$ μm, and the curves in Figs 4b, d are calculated for $\Lambda = 34$ μm. The solid and dashed lines correspond to different orientations of a sample obtained by its rotation around the x axis by 180°.

In the case of the longitudinal geometry, the component $\Gamma_{\perp}^{+}$ can be either smaller or greater than $\Gamma_{\parallel}$. The gain Γ^{+} has a maximum at $\chi = 0$ (square-wave field E_0). For $\Lambda = 3.4$ μm ($\delta_F/\delta_\Gamma = 0.77$), as for $\Lambda < 12.7$ μm ($\delta_F/\delta_\Gamma < 5.77$), energy transfer is unidirectional for all values of χ (Fig. 4a). For $\Lambda = 12.7$ μm, $\chi \to 1$, and the crystal orientation at which the z axis is directed along the $[00\bar{1}]$ crystal axis, the attenuation of the signal wave caused by unidirectional energy transfer is compensated by its amplification due to non-unidirectional energy transfer. For $\Lambda > 12.7$ μm, energy transfer becomes non-unidirectional also for the values of χ near unity, for example, for $\chi > 0.81$ for $\Lambda = 34$ μm, when $\delta_F/\delta_\Gamma = 38.96$ (Fig. 4b).

In the case of the transverse geometry, under conditions considered, energy transfer is non-unidirectional for all values of χ (Figs 4c, d). Because $\Gamma_{\perp}^{+} \gg |\Gamma_{\|}|$ and $|\Gamma_{\|}d| < 0.3$, the rotation of the crystal around the x axis by 180° results in a weak change of the component $\Gamma_{\perp}^{+}$ and the total gain Γ^{+} (curves $\Gamma_{\perp}^{+}$ in Fig. 4d, where $|\Gamma_{\|}d| < 0.5$, virtually coincide). The type of the dependences $\Gamma^{+}(\chi)$ and $\Gamma_{\perp}^{+}(\chi)$ changes similarly with increasing Λ. For $\Lambda = 3.4$ μm (Fig. 4c), the gain Γ^{+} weakly depends on χ, and the maximum value of $\max(\Gamma^{+}) \approx 3$ cm^{-1} is achieved at $\chi = 0$, while its minimum is observed at $|\chi| \to 1$. For $\Lambda = 34$ μm (Fig. 4d), the gain Γ^{+} strongly depends on χ, and the maximum $\max(\Gamma^{+}) \approx 3$ cm^{-1} is achieved for $|\chi| \to 0$, and the minimum $\min(\Gamma^{+}) \approx 0.5$ cm^{-1} is observed for $\chi = 0$.

In the general case, the two-wave gain is determined both by the nonlocal and local components of the field E_{sc}. For $|\chi| \to 1$, expressions (12)–(16) are reduced to the expressions obtained in paper [13] for the two-wave interaction in a crystal in an external constant field. For the conditions under study and $\Lambda = 34$ μm, the relation $E_q \gg E_m \gg E_d$ is valid, and the nonlocal component of the field E_{sc} is small compared to the local component, whose amplitude is $\mathrm{Re}\,E_1 \approx -m_0 E_m$. For this reason, a weak light wave is amplified mainly due to the interaction with the pump wave on the local component of the photorefractive grating.

Comparison of Figs 4a, c and Figs 4b, d shows that the asymmetry of curves $\Gamma^{+}(\chi)$ increases with increasing Λ, which is especially pronounced for the longitudinal geometry (Figs 4a, b). For $\chi \neq 0$, this indicated to the difference between the intensity I_S^{+} for $E_0 = E_m$ and the intensity I_S^{-} for $E_0 - E_m$. For $\chi = 0$ (square-wave field E_0), such a time modulation of the intensity I_S is absent if the incident waves are linearly polarised. For an arbitrary elliptical, in particular, circular [3] polarisation of the incident waves, $I_S^{+} \neq I_S^{-}$ for $\chi = 0$ as well, which is caused by the dependence of the coefficient r_{eff} on the sign of E_0.

Fig. 5 shows the dependences of the relative gain I_S^{+}/I_{S0} and I_S^{-}/I_{S0} on Λ, which was calculated from (12) for the two-wave interaction in a $Bi_{12}TiO_{20}$ crystal of thikness $d = 1$ cm in an external square-wave field with the amplitude $E_m = 10$ kV cm^{-1} for the right-hand circular polarisation of the incident waves. The solid curves in Fig. 5a (the longitudinal geometry) correspond to the weak-wave amplification when the positive direction of the z axis coincides with the [001] crystal axis. The dashed curves are plotted for the ratio $100I_S^{\pm}/I_{S0}$ and correspond to the attenuation of this wave when the positive direction of the z axis coincides with the $[00\bar{1}]$ axis.

The difference in the shape of the solid and dashed curves, as well as the inequality $I_S^{+}/I_{S0} \neq I_{S0}/I_S^{-}$, are caused by the contribution from non-unidirectional energy transfer to the weak-wave intensity. For the transverse geometry and circularly polarised incident waves (Fig. 5b), the rotation of the crystal by 180° around the x axis does not change the weak-wave amplification. Note that in all the cases studied, the intensity I_S^{-} is greater than I_S^{+}. A change in the intensity gain $\Delta = (I_S^{-} - I_S^{+})/I_{S0}$ is most pronounced ($\Delta \approx 91$) for the longitudinal geometry and the grating spacing $\Lambda_{max} = 3.4$ μm, which corresponds to the maximum of the effective space-charge-field amplitude E_{eff}. When the signal wave is attenuated in this geometry, the minimal value $\Lambda_{min} \approx 0.14$ corresponds to the spacing Λ_{max}. For the transverse geometry and $\Lambda = \Lambda_{max}$, the value $\Delta \approx 3$, which virtually corresponds to the maximum value for this case.

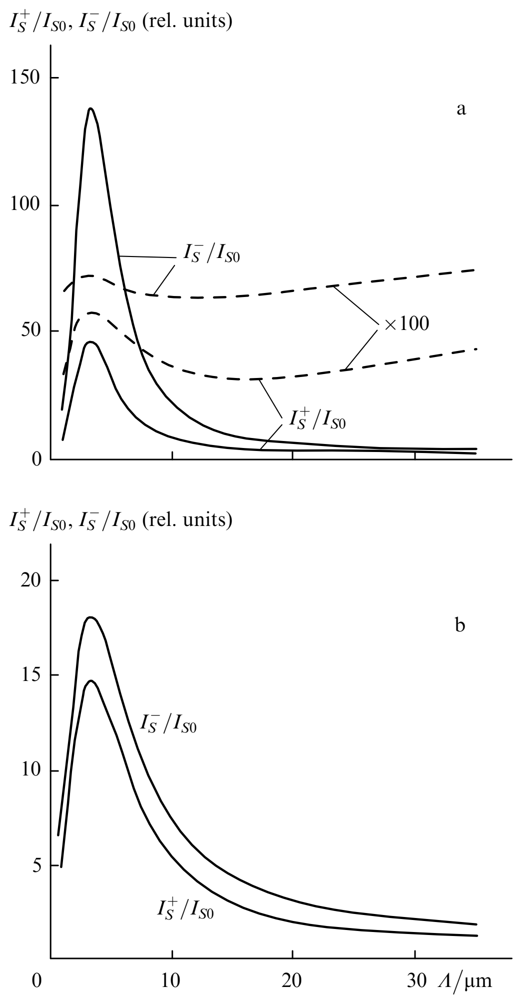

Figure 5. Dependences of the relative gain (solid curves) and attenuation (dashed curves) of the intensity of a weak light wave on the photorefractive grating spacing Λ in a $Bi_{12}TiO_{20}$ crystal of thickness $d = 1$ cm in an external square-wave electric field for circularly polarised of incident waves and the longitudinal (a) and transverse (b) geometry of the interaction.

5. Conclusions

The stationary regime of the symmetric two-wave interaction in a cubic gyrotropic crystal in an external alternating electric field has been analytically described within the framework of adopted approximations. For particular cases of the longitudinal and transverse geometry of the interaction in a $Bi_{12}TiO_{20}$ crystal at a wavelength of 633 nm, the amplitude of the electric field induced in the crystal and the weak-wave intensity gain have been calculated.

It is shown that not only the nonlocal component of a photorefractive grating can be formed, which is typical for a square-wave external field, but also the local component. The local component is produced when the incident waves have an arbitrary elliptical polarisation and the external field has an arbitrary off-duty ratio. When incident waves are linearly polarised, this component is produced only when the external field contains a nonzero constant component. Note that the relation between the amplitudes of the local and nonlocal components of the photorefractive grating changes over the interaction length.

Energy transfer between the light waves interacting in the crystal in an external alternating field contains both the unidirectional and non-unidirectional components, the latter appearing due to the anisotropic diffraction of an intense wave from the inhomogeneous photorefractive grating when the polarisation state changes to the orthogonal state. Both components of the photorefractive grating contribute to the non-unidirectional component. The intensity of a weak light wave in the positive external field differs from the intensity of this wave in the negative field if the incident waves are elliptically or linearly polarised and the external field has a nonzero constant component. Note that the local component of the photorefractive grating in this case is also nonzero.

Within the framework of adopted approximations, when the duration of the interval of positive (negative) values of the external field greatly exceeds the duration of the interval of its negative (positive) values, a photorefractive response of a crystal is similar to that for a crystal in a constant positive (negative) electric field E_0. However, in the case of an alternating external electric field E_0 with period $T \ll \tau_d$, the total current through the crystal is determined by the bias current rather than the conduction current, as in the case of the constant field E_0. For this reason, devices using the refractive response of this type [17] do not require a highly uniform illumination.

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