

PACS numbers: 42.55.Rz; 42.60.Da; 42.65.Sf
DOI: 10.1070/QE2002v032n07ABEH002265

Phase dynamics of radiation in chaotic lasing regimes of a solid-state ring laser

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***Abstract*. The radiation phase dynamics of a solid-state ring laser with a periodic pump modulation operating in a dynamic chaotic regime is studied theoretically and experimentally. It is found that, in the regime of synchronised dynamic chaos, the optical phases of counterpropagating waves change by π in intervals between two adjacent chaotic radiation pulses. It is shown that spontaneous radiation has a significant effect on the chaotic dynamics of the counterpropagating wave intensities, but its effect on the phase dynamics in a solid-state ring laser is negligible.**

Keywords: *ring laser, phase dynamics, dynamic chaos.*

1. Introduction

Studies of the dynamic chaos in lasers, which have been underway for almost three decades, not only help to better understand the general properties of the chaotic dynamics in a wide class of nonlinear systems but also are of significant applied importance. Much attention is currently paid to investigating the processes of dynamic chaos synchronisation in coupled [1 – 4] and ring [5 – 7] solid-state lasers. The regimes of identical synchronised chaos [1, 2, 5], generalised synchronisation [6], and phase synchronisation [7 – 10] were investigated. The results of these studies are currently used in systems for optical data transmission [11].

Studying the possibilities of utilising chaotic lasing regimes for enhancing the sensitivity in the recording of optical nonreciprocities using ring lasers is of substantial interest. In this case, it is desirable to use the information not only on the intensity dynamics of counterpropagating waves but also on their optical phases.

Investigations of the phase dynamics of chaotic laser radiation is of fundamental interest for the analysis of general problems of phase synchronisation of chaotic oscillations [12 – 14]. Note that, at present, theoretical studies in this field do not unambiguously predict the features of phase dynamics for particular nonlinear systems.

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Received 4 April 2002
Kvantovaya Elektronika **32** (7) 654 – 658 (2002)
Translated by A.S. Seferov

The phase dynamics in chaotic lasing regimes of ring chip lasers was analysed in papers [7, 15]. On the basis of the results of a numerical simulation of the lasing regimes of a solid-state ring laser (SRL), it was predicted [7] that a regime of synchronised chaos, in which regular jumps of optical phases occur during the intervals between adjacent chaotic lasing spikes, can exist in such a laser. Experimental jumps of the difference in the optical phases of counterpropagating waves in a ring chip laser were observed for the first time in paper [15].

The aim of this work is a more detailed theoretical and experimental study of the regime of synchronised chaos accompanied by regular jumps in the phase difference of counterpropagating waves.

2. Recording of the phase dynamics in the regime of dynamic chaos

The evolution of optical phases of nonstationary laser radiation can be recorded by optical heterodyning using an external reference signal. This technique was used to study the phase dynamics in a $^{15}NH_3$ gas ring laser pumped by a CO_2 laser [16].

The phase dynamics of ring lasers can also be investigated using an interference technique, which was successively implemented in paper [15]. In this case, the information on the phase dynamics is contained in a signal of photomixing of the counterpropagating waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$:

$$\boldsymbol{E}_{\text{pm}} = \boldsymbol{E}_1 + \boldsymbol{E}_2 \,. \qquad (1)$$

Since, in a general case, the polarisations of these waves are not identical, it is expedient to select identical (e.g., linear) polarisation components in each wave. The intensity of the photomixing signal of the counterpropagating waves in this case can be represented in the form

$$I_{\text{pm}} = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos\varphi \,, \qquad (2)$$

where $I_{1,2}$ are the intensities of the selected components of the counterpropagating waves with the same polarisation and φ is the difference in the optical phases of the interfering waves.

Note that the use of photomixing of the counterpropagating waves of a ring laser in phase dynamics studies is more preferable compared to the use of an external heterodyne, because the correlation of technical fluctuations of the counterpropagating waves intensities and phases observed in a ring laser ensures a higher stability of the photomixing signal. It is this technique that was used in this work to measure jumps in the difference of optical phases of counterpropagating waves in regimes of synchronised chaos in a SRL.

3. Experimental

Experiments were carried out with a monoblock $Nd^{3+}:YAG$ SRL pumped by a semiconductor laser. A schematic of the setup is shown in Fig. 1. The chip laser represented a crystalline monolithic block with a spherical entrance surface and three faces of total internal reflection. The geometrical perimeter of the resonator was 2.6 cm, and the angle of resonator nonflatness was 80°. The SRL was pumped by a 250-mW semiconductor laser. The pump power was modulated at a frequency $f_m < 200$ kHz with a modulation depth h varied between 0 % and 100 %. In the absence of the pump modulation, the laser operated in the first-order self-modulation mode at a self-modulation frequency $f_{sm} = 230$ kHz; the frequency of relaxation oscillations was $f_r = 65$ kHz for a pump power exceeding the pump threshold level by 12 %.

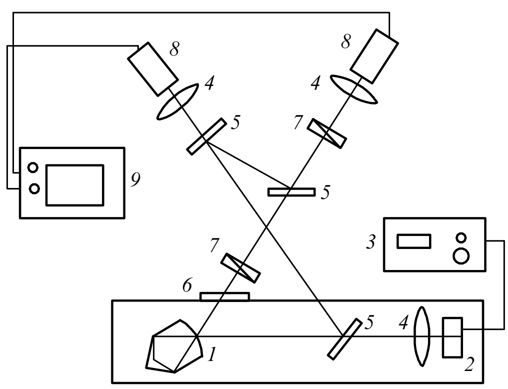

Figure 1. Schematic of the setup: (*1*) laser monolithic block; (*2*) pump diode laser; (*3*) unit for the modulation of pump radiation; (*4*) focusing lenses; (*5*) beamsplitters; (*6*) IKS-6 filter; (*7*) polarisers; (*8*) LFD photodetectors; (*9*) ACK 31-51 electronic oscilloscope.

Different lasing regimes (periodic, quasi-periodic, and dynamic chaotic regimes) were observed in the SRL depending on the pump frequency and modulation depth. In the case of a dynamic chaos, both synchronised (when counterpropagating waves have identical temporal characteristics) and nonsynchronised lasing regimes can be excited. From the viewpoint of a study of the general properties of the phase dynamics of nonlinear systems, the regime of synchronised chaos is of greatest interest [5, 7, 15]. The experiments performed have revealed that this regime exists within a limited range of the pump-power modulation depth $h_{min} < h < h_{max}$, and the bounds of this range depend on the pump modulation frequency. Note that the region of existence of synchronised chaos has a maximum width for $f_m = 29$ kHz.

Consider the main experimental results obtained in the regime of synchronised dynamic chaos at a pump modulation frequency $f_m = 29$ kHz and a modulation depth $h =$ 65 %. Oscillograms of the counterpropagating-wave radiation intensity at the chip-laser output over a time interval of 1100 μs presented in Fig. 2 show that the intensities of these waves have the same chaotic modulation in this regime.

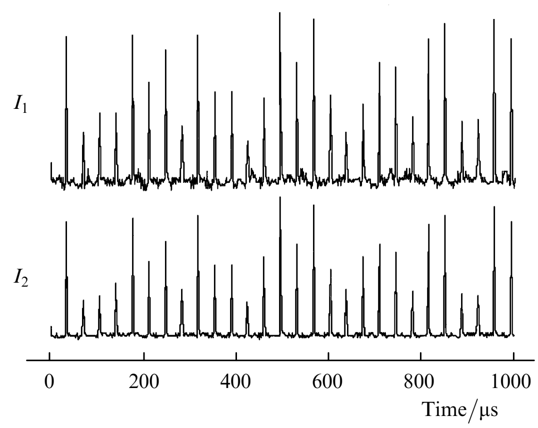

Figure 2. Oscillograms of radiation of counterpropagating waves I_1 and I_2 in the regime of synchronised chaos at a pump modulation frequency $f_m = 29$ kHz, a modulation depth $h = 65$ %, an excess of the pump power above the threshold $\eta_0 = 0.12$ in the absence of modulation, and a self-modulation frequency $f_{sm} = 230$ kHz.

The phase dynamics was studied using faster scans. Fig. 3 shows typical intensity oscillograms for the counterpropagating waves I_1 and I_2 and the photomixing signal I_{pm} that testify to the presence of jumps in the difference of the optical phases in the intervals between adjacent chaotic pulses. One can see that pulses of two types are present in the signal I_{pm}. The peak intensity of pulses of the first type is approximately four times greater than that of the interfering waves. The intensity of pulses of the second type is very low. This indicates that optical oscillations in counterpropagating pulses of the first type are added in phase, whereas in pulses of the second type are added in antiphase. Note that in-phase and antiphase pulses regularly alternate in the course of lasing. Fig. 4 presents the projections of the phase portraits onto the planes I_1, I_2 and I_{pm}, I_1. In the latter case, the phase portrait consisting of two straight lines $I_{pm} = 4I_1$ and $I_{pm} \simeq 0$ also confirms the presence of periodical jumps in the phase difference.

The results obtained show that, in the interval between adjacent chaotic pulses, the phase difference of optical oscillations of counterpropagating waves in a ring chip laser changes jumpwise by π.

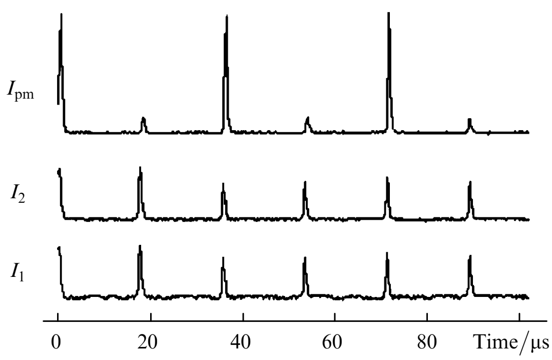

Figure 3. Oscillograms of radiation of counterpropagating waves I_1 and I_2 in the regime of synchronised chaos and the photomixing signal I_{pm} in the case of optical-phase jumps at $f_m = 29$ kHz, $h = 65$ %, $\eta_0 = 0.12$, and $f_{sm} = 230$ kHz.

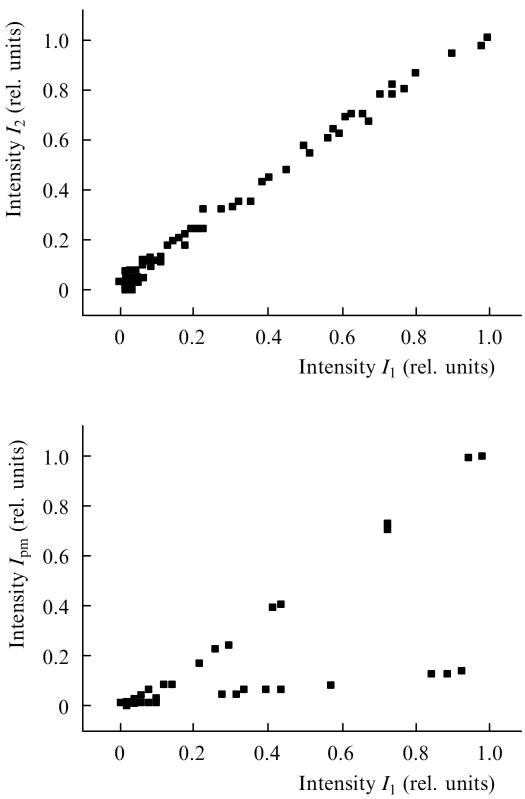

Figure 4. Projections of phase portraits onto the planes I_1, I_2 and I_1, I_{pm} for $f_m = 29$ kHz, $h = 65\,\%$, $\eta_0 = 0.12$, and $f_{sm} = 230$ kHz (experiment).

4. Phase dynamics in the regime of synchronised chaos (numerical simulation)

A simplified SRL model, in which a number of factors capable, in principle, of affecting the behaviour of the optical phases of counterpropagating waves, was used in phase dynamics studies [7]. This model, in particular, neglected the amplitude – phase coupling of counterpropagating waves and the effect of a spontaneous radiation noise.

In this paper, we analyse the SRL phase dynamics using a more rigorous SRL model, in which the radiation dynamics is described by a system of stochastic (Langevin) equations with noise sources determined by quantum fluctuations (i.e., by spontaneous emission in the active medium). Similar equations are used, for example, in studies of quantum fluctuations of SRL radiation in the travelling-wave mode (see, e.g., [17]).

The system of equations describing the SRL lasing dynamics taking into account the amplitude – phase coupling of counterpropagating waves and quantum noise sources has the form

$$\frac{\mathrm{d}E_{1,2}}{\mathrm{d}t} = -\frac{\omega}{2Q_{1,2}}\,E_{1,2} + \frac{\sigma L_{\mathrm{ac}}}{2T}\,(N_0 E_{1,2} + N_{\mp} E_{2,1})(1 + \mathrm{i}\alpha) + \frac{1}{2}\,\mathrm{i} m_{1,2} E_{2,1} + \left(\frac{8\pi\hbar\omega}{V}\right)^{1/2} F_{1,2}\,,$$

$$T_1\,\frac{\partial N_0}{\partial t} = N_{\mathrm{th}}(1+\eta) - N_0[1 + a(I_1 + I_2)] - 2a\mathrm{Re}(N_+ E_1 E_2^*),$$

$$T_1\,\frac{\partial N_+}{\partial t} = -N_+[1 + a(I_1 + I_2)] - aN_0 E_1^* E_2, \tag{3}$$

$$N_- = N_+^*\,.$$

In these equations, $E_{1,2}$ and $I_{1,2}$ are, respectively, the complex amplitudes and intensities of counterpropagating waves; ω is the optical radiation frequency; N_0 is the average (over the resonator length) inverse population; $N_\pm$ are the amplitudes of spatial harmonics of the inverse population induced in the active medium due to a saturation of the amplification by the interfering counterpropagating waves; N_{th} is the threshold inverse population; $T = Ln/c$ is the round-trip time for radiation in the resonator; n is the refractive index of the active medium; L is the perimeter of the ring resonator; L_{ac} is the length of the active element; σ is the cross section of the laser transition; a is the saturation parameter of the active medium; T_1 is the relaxation time of the inverse population; η is the excess of the pump power over the threshold; $Q_1 = Q_2 = Q$ are the quality factors of the resonator for the counterpropagating waves; $m_{1,2}$ are the complex coefficients of the linear coupling between the counterpropagating waves; V is the resonator mode volume; and $F_{1,2}$ are the Langevin sources of spontaneous-emission noises, which are assumed to be δ-correlated and are defined by the following correlation functions:

$$\langle F_{1,2}(t) F_{1,2}(t-\tau)\rangle = 0,$$

$$\langle F_{1,2}(t) F_{1,2}^*(t-\tau)\rangle = \frac{1}{2}\,R_{\mathrm{sp}}\delta(\tau), \tag{4}$$

$$\langle F_{1,2}(t) F_{2,1}^*(t-\tau)\rangle = 0\,,$$

where

$$R_{\mathrm{sp}} = \frac{\sigma L_{\mathrm{ac}} N_2}{VT} \tag{5}$$

is the rate of spontaneous emission into the resonator mode and $N_2 \simeq N_0$ is the population of the upper laser level.

The parameter α determines the amplitude – phase coupling. This coupling appears, for example, in the presence of a complex structure of the amplification line (this takes place in an Nd^{3+} : YAG laser in which the amplification line consists of two components shifted relative to each other [18]). The presence of two components in the Nd^{3+} : YAG amplification line leads to a situation in which, even for the lasing at the maximum of the summary amplification contour, there exists a nonzero lasing-frequency detuning with respect to the centre of the laser transition. In the case of a laser with a homogeneously broadened amplification line with a Lorentzian profile, the expression for α has the form

$$\alpha = \frac{\omega - \omega_0}{\gamma}\,, \tag{6}$$

where $\omega - \omega_0$ is the detuning of the lasing frequency relative to the centre of the amplification line and γ is the line FWHM.

The system of Eqns (3) was used to analyse the radiation phase dynamics in the regime of synchronised dynamic chaos in a SRL with a periodic pump modulation. In this case,

$$\eta = \eta_0 + h\cos(\omega_p t), \quad (7)$$

where η_0 is the excess of the pump power over the threshold without modulation and ω_p is the cyclic frequency of the pump modulation.

When the Langevin equations were solved numerically, random forces $F_{1,2}(t)$ were specified at each integration step using a random number generator. In order to describe the action of δ-correlated noise sources, it was convenient to supplement Eqns (3) with two equations for the counterpropagating-wave intensities, which directly take into account the correlation function (4) of the noise sources:

$$\frac{dI_{1,2}}{dt} = -\frac{\omega}{Q} I_{1,2} + \frac{\sigma L_{ac}}{T}[N_0 I_{1,2} + \mathrm{Re}(N_{\mp} E_{2,1} E^*_{1,2})] + \mathrm{Re}(\mathrm{i} m_{1,2} E_{2,1} E^*_{1,2}) + \frac{8\pi\hbar\omega}{V} R_{sp} + \left(\frac{8\pi\hbar\omega}{V}\right)^{1/2} (E^*_{1,2} F_{1,2} + F^*_{1,2} E_{1,2}). \quad (8)$$

The system of Eqns (3), (8) was solved numerically. The complex amplitudes of the fields $E_{1,2}$ and intensities $I_{1,2}$ were determined from (3) and (8), respectively. These parameters were used to determine the photomixing signal intensity from the formula

$$I_{pm} = I_1 + I_2 + 2\mathrm{Re}(E_1 E_2^*).$$

The numerical simulation was executed at chip-laser parameters corresponding to the experimental ones: $T_1 = 240$ μs, $L_{ac} = L = 2.6$ cm, $\eta_0 = 0.12$, $f_r = (2\pi)^{-1}[(\omega/Q)\times(\eta_0/T_1)]^{1/2} = 65$ kHz, $m_{1,2}/(2\pi) = 230$ Hz, $0 \leqslant \alpha \leqslant 0.5$. It was assumed in the calculations that the coupling coefficients are real, and the pump modulation frequency is $f_m = 29$ kHz.

In order to clarify the influence of spontaneous emission on the SRL phase dynamics, the latter was simulated with regard to spontaneous radiation and in its absence. Numerical experiments have shown that spontaneous emission has a significant effect on the counterpropagating-wave intensity dynamics. A typical time structure of radiation in the absence of spontaneous emission is shown in Fig. 5. Despite the fact that all the calculated parameters coincide with the experimental ones (Fig. 3), the laser radiation dynamics obtained in the numerical calculations appreciably differs from the dynamics observed experimentally, although regular jumps of the counterpropagating-wave phase difference take place.

If spontaneous emission was taken into consideration, then, at the parameters mentioned above and a pump modulation depth belonging to the range $25\,\% < h < 100\,\%$, the numerical experiments resulted in a regime of synchronised chaos similar to that observed in the experiment. Fig. 6 shows time dependences of the counterpropagating-wave intensities and the photomixing signal in the presence of spontaneous emission. We see that optical-phase jumps by π take place in the intervals between neighbouring chaotic radiation pulses.

The numerical calculations performed have demonstrated that the amplitude–phase coupling has virtually no effect on the radiation time structure and phase dynamics for the aforementioned values of the counterpropagating-wave parameters, if the parameters α and h lie within the ranges: $0 \leqslant \alpha \leqslant 0.5$ and $25\,\% < h < 100\,\%$.

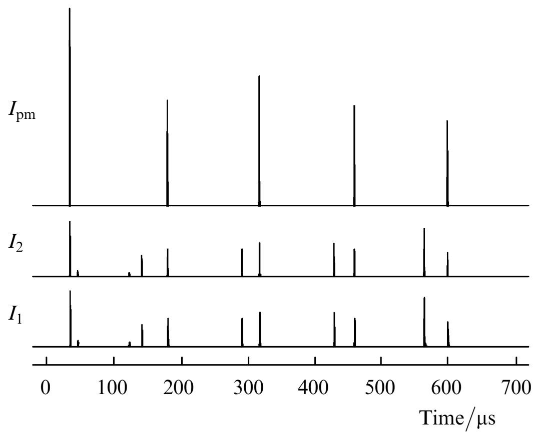

Figure 5. Time-dependent counterpropagating-wave and photomixing signal intensities obtained by a numerical simulation in the absence of spontaneous emission at $f_m = 29$ kHz, $h = 65\,\%$, $\eta_0 = 0.12$, and $f_{sm} = 230$ kHz.

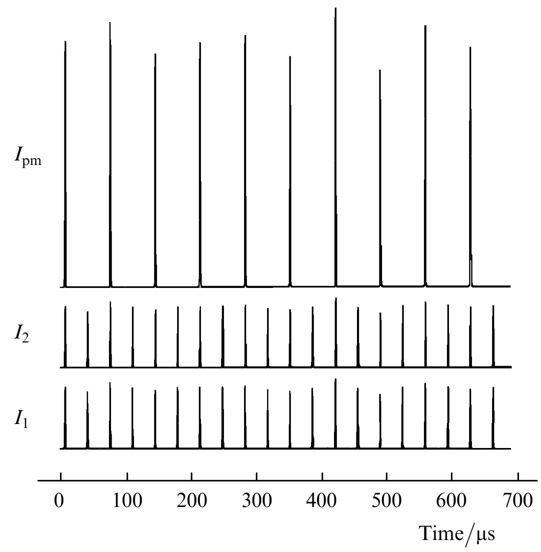

Figure 6. Time-dependent counterpropagating-wave and photomixing signal intensities obtained by a numerical simulation with regard to spontaneous emission at $f_m = 29$ kHz, $h = 65\,\%$, $\eta_0 = 0.12$, and $f_{sm} = 230$ kHz.

5. Conclusions

In this work, a synchronised dynamic chaos in counterpropagating waves and the phase dynamics of their radiation have been studied theoretically and experimentally in a nonself-sustained diode-laser-pumped chip laser. The model of a two-directional SRL takes into account the influence of spontaneous-emission noises on the lasing dynamics and ensures a proper description of the time characteristics of a ring chip laser and its phase dynamics.

The studies performed have revealed that, in the regime of synchronised dynamic chaos arising in a SRL under a pump power modulation, the optical phases of counterpropagating waves change by π in the intervals between two neighbouring pulses of chaotic radiation. It is shown that spontaneous radiation exerts an appreciable effect on the chaotic dynamics of the counterpropagating-wave intensities, but its effect on the SRL phase dynamics is insignificant.

Acknowledgements. The authors thank N.I. Naumkin, S.S. Sidorov, and V.V. Firsov for their help in conducting experiments. This work was supported by the Russian Foundation for Basic Research (Grant Nos 02-02-16391, 02-02-16438, and 02-02-06202) and Russian – Belarussian Foundation for Basic Research (Grant No. 02-02-81006 Bel. 2002).

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