CONTROL OF LASER RADIATION PARAMETERS

PACS numbers: 42.55.Ah; 42.60.Lh
DOI: 10.1070/QE2002v032n08ABEH002272

Study of parameters of simultaneous lasing on two lines sharing an upper level

A.A. Pikulev

***Abstract*. Stationary lasing at two competing lines sharing an upper level is studied. Based on the expressions for the gain obtained earlier, the possible lasing regimes are considered (at one or two lines) and approximate formulas are derived for determining the output power in each line. These formulas are shown to be the generalisation of the Rigrod formula to the case of simultaneous lasing at several lines.**

Keywords: *competition between laser lines, gain, stationary lasing, Rigrod formula.*

1. Introduction

The effect of competition between two and more laser lines sharing an upper level is well known and was described in detail in the literature devoted to the experimental studies of lasing at transitions in inert gases pumped by uranium fission fragments or an electron beam [1 – 4]. Despite this, the effect of competition has not been adequately studied theoretically so far. In paper [5], the expression for the gains at simultaneous lasing on several lines was obtained based on the three-level model of a plasma laser. The study performed in paper [5] showed that two qualitatively different cases of the interaction between two competing lines exist during stationary lasing: symbiosis and quenching. In the case of symbiosis, the output power in both lines increases monotonically with increasing pump power. In the quenching regime, lasing at a line with a higher threshold quenches (sometimes completely) the line at which lasing begins earlier.

Although the numerical methods that allow one to calculate the parameters of simultaneous lasing at any number of laser transitions are popular at present, they are not always can be used. This is explained by the fact that the rate constants of plasma-chemical processes in most gaseous mixtures and the radiation parameters of many levels and laser transitions are unknown or are known only approximately, and it is impossible to calculate reliably the lasing parameters in many cases. For this reason, the solution of the inverse problem of finding the parameters of an active medium from known lasing parameters obtained in experiments is quite urgent.

To do this, it is desirable to have approximate expressions that relate the parameters of a cavity and an active medium with the output power in each line. In the case of one laser line, such expressions were derived in papers [6, 7]. These expressions (especially the Rigrod formula [6]) are widely used for determining the parameters of an active medium from the measured output power.

In this paper, we determine the parameters of stationary lasing at two competing lines sharing an upper level. The main attention is devoted to determining the lasing regime (at one or two lines) and deriving approximate expressions for the output power.

2. Basic relations

Consider stationary lasing at two competing lines sharing an upper level. We denote the upper working level by figure 1 and the lower levels by letters i and j. It was shown in paper [5] that the gain at the $1 \to i$ transition can be found from the expression

$$\alpha_i = \frac{\alpha_i^0 + \beta_{ij} I_j}{1 + (I_i^s)^{-1} I_i + (I_j^s)^{-1} I_j + \delta I_i I_j}, \quad (1)$$

where α_i^0 is the initial gain; I_i^s and I_j^s are saturation parameters; δ is the mutual saturation coefficient; and β_{ij} is the competing parameter. To obtain the expression for the gain at the $1 \to j$ transition, the replacement $i \leftrightarrow j$ should be performed in (1). Let us introduce the notation

$$I_j^0 = -\alpha_i^0 (\beta_{ij})^{-1}, \; I_i^0 = -\alpha_j^0 (\beta_{ji})^{-1}. \quad (2)$$

It follows from (1) and (2) that for $I_j = I_j^0$, the gain $\alpha_i = 0$, so that I_i^0 and I_j^0 can be called the 'zero-gain intensities'.

In the case of stationary lasing in the one-dimensional approximation, the radiation intensity I_i inside the cavity is described by the equations [6 – 9] (radiation propagates along the z axis)

$$\frac{dI_i^+}{dz} = (\alpha_i - \rho_i) I_i^+, \; \frac{dI_i^-}{dz} = -(\alpha_i - \rho_i) I_i^-, \; I_i = I_i^+ + I_i^-, \quad (3)$$

where ρ_i is the distributed loss coefficient; I_i^+ and I_i^- are the intensities of electromagnetic waves propagating from the left to the right and from the right to the left, respectively. Let us denote the reflection coefficients of the left and right cavity mirrors for radiation at the $1 \to i$ transition as r_{1i}

A.A. Pikulev Russian Federal Nuclear Center, All-Russian Scientific-Research Institute for Experimental Physics, prosp. Mira 37, 607190 Sarov, Nizhegorodskaya oblast, Russia; e-mail: pikulev@expd.vniief.ru

Received 8 November 2001; revision received 10 July 2002
Kvantovaya Elektronika **32** (8) 689 – 691 (2002)
Translated by M.N. Sapozhnikov

and r_{2i}, respectively. We assume that the left end of the cavity is located at the point $z = 0$, while the right end is located at the point $z = L$, where L is the cavity length. Equations (3) are integrated under standard boundary conditions.

In the case of stationary lasing, the relations [7]

$$\langle\alpha_i\rangle = \langle\rho_i\rangle + \frac{1}{2L}\ln\frac{1}{r_{1i}r_{2i}} = k_i,$$
$$\langle\alpha_j\rangle = \langle\rho_j\rangle + \frac{1}{2L}\ln\frac{1}{r_{1j}r_{2j}} = k_j, \quad (4)$$

are satisfied, where k_i and k_j are the total loss coefficients, and $\langle\ldots\rangle$ means averaging over the optical cavity length. We can show that two points z_i^* and z_j^* exist inside the cavity at which the system (4), taking into account (1) and (2), assumes the form

$$\frac{\langle\alpha_i^0\rangle[1 - I_j^*(I_j^0)^{-1}]}{1 + I_i^*(I_i^s)^{-1} + I_j^*(I_j^s)^{-1} + \delta I_i^* I_j^*} = k_i,$$
$$\frac{\langle\alpha_j^0\rangle[1 - I_i^*(I_i^0)^{-1}]}{1 + I_i^*(I_i^s)^{-1} + I_j^*(I_j^s)^{-1} + \delta I_i^* I_j^*} = k_j, \quad (5)$$

where $I_i^* = I_i(z_i^*)$ and $I_j^* = I_j(z_j^*)$ are the 'operating intensities'. The relations (5) are valid when the sign of the initial gains does not vary within the cavity. This condition is satisfied in the presence of pumping.

From system (5), we obtain the relation

$$I_j^* = I_j^0\left(1 - \frac{\langle\alpha_j^0\rangle k_i}{\langle\alpha_i^0\rangle k_j}\right) + \frac{\langle\alpha_j^0\rangle k_i I_j^0}{\langle\alpha_i^0\rangle k_j I_i^0} I_i^*. \quad (6)$$

We assume below that the $1 \to i$ line has a lower threshold than the $1 \to j$ line, i.e, $\langle\alpha_i^0\rangle k_j > \langle\alpha_j^0\rangle k_i$. When this condition is satisfied, the expression in the parentheses in (6) is positive. Let us introduce the notation

$$I_j^{\rm cr} = I_j^0\left(1 - \frac{\langle\alpha_j^0\rangle k_i}{\langle\alpha_i^0\rangle k_j}\right), \varepsilon_{ij} = \frac{1}{\varepsilon_{ij}} = \frac{\langle\alpha_j^0\rangle k_i I_j^0}{\langle\alpha_i^0\rangle k_j I_i^0} = \frac{\langle\beta_{ji}\rangle k_i}{\langle\beta_{ij}\rangle k_j}, \quad (7)$$

where $I_j^{\rm cr}$ is the critical intensity of the $1 \to j$ line at which lasing is terminated [5]. The expression for $I_i^{\rm cr}$ is obtained by interchanging subscripts $i \leftrightarrow j$ in (7).

Let us calculate now the intensities I_i^* and I_j^*. In the case of lasing at one line, we have

$$I_i^{*(1)} = I_i^s\frac{\langle\alpha_i^0\rangle - k_i}{k_i}, I_j^{*(1)} = I_j^s\frac{\langle\alpha_j^0\rangle - k_j}{k_j}, \quad (8)$$

where $I_i^{*(1)}$ and $I_j^{*(1)}$ are the 'operating intensities' during lasing at one line.

In the case of lasing at two lines, the intensities I_i^* and I_j^* can be calculated fron system (5). By substituting expression (6) to the second equation of system (5), we obtain the quadratic equation

$$A_{ij}(I_i^*)^2 + B_{ij}I_i^* + C_{ij} = 0, \quad (9)$$

where

$$A_{ij} = \delta k_j\varepsilon_{ji}; B_{ij} = k_j\left(\frac{1}{I_i^s} + \frac{\varepsilon_{ji}}{I_j^s} + \delta I_j^{\rm cr}\right) - \langle\beta_{ji}\rangle;$$
$$C_{ij} = k_j\left(1 + \frac{I_j^{\rm cr}}{I_j^s}\right) - \langle\alpha_j^0\rangle. \quad (10)$$

If $(B_{ij})^2 \geqslant 4A_{ij}C_{ij}$, the solutions of equation (10) have the form

$$I_i^* = \frac{-B_{ij} \pm \left[(B_{ij})^2 - 4A_{ij}C_{ij}\right]^{1/2}}{2A_{ij}}, I_j^* = I_j^{\rm cr} + \varepsilon_{ji}I_i^*. \quad (11)$$

For $\langle\alpha_i^0\rangle k_j > \langle\alpha_j^0\rangle k_i$, only two variants are possible when lasing can occur simultaneously at two lines [5]:

(i) The $1 \to j$ line quenches the $1 \to i$ line, i.e., $I_j^0 > 0$ and $I_i^0 < 0$. In this case, when the threshold is exceeded ($\langle\alpha_i^0\rangle > k_i$), lasing at the $1 \to i$ line appears, and then, at $I_i^* > I_i^{\rm cr}$, lasing at the $1 \to j$ line appears, which reduces the lasing intensity at the $1 \to i$ line. For $I_j^* > I_j^{\rm cr}$, no lasing occurs at the $1 \to i$ lines. We take the minus sign in expression (11).

(ii) The $1 \to i$ and $1 \to j$ lines are in symbiosis, i.e., $I_i^0 < 0$ and $I_j^0 < 0$. If $\langle\alpha_i^0\rangle > k_i$, lasing appears at the $1 \to i$ line and then, at $I_i^* > I_i^{\rm cr}$, lasing also occurs at the $1 \to j$ line and then the lasing intensity increases proportionally at both lines. We take the plus sign in expression (11).

These two cases are shown in Fig. 1. The regions where lasing occurs at one line or at two lines simultaneously are shown in Tables 1 and 2 based on Fig. 1.

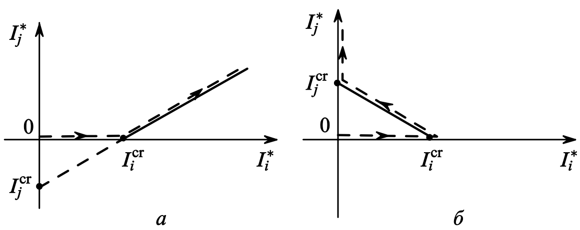

Figure 1. Diagram of lasing at two lines in the case of symbiosis (*a*) and quenching (*b*) (the arrows show the change in the operating lasing intensities during a parametric increase in the pump power).

Table 1. Regions of the lasing diagram in the case of symbiosis.

Line	$0 < I_i^* < I_i^{\rm cr}$	$I_i^* > I_i^{\rm cr}$
$1 \to i$	lasing	lasing
$1 \to j$	no lasing	lasing

Table 2. Regions of the lasing diagram in the case of quenching (the $1 \to j$ line quenches the $1 \to i$ line).

Line	$0 < I_i^* < I_i^{\rm cr}$	$I_i^* > I_i^{\rm cr}, I_j^* < I_j^{\rm cr}$	$I_i^* > I_i^{\rm cr}, I_j^* > I_j^{\rm cr}$
$1 \to i$	lasing	lasing	no lasing
$1 \to j$	no lasing	lasing	lasing

3. Approximate expressions for the lasing intensity

We will derive approximate expressions for the lasing intensity by using the two approximations:

(i) We approximate the distribution of the radiation intensity inside the cavity by a catenary by assuming in (3) that $\alpha_i \simeq \langle\alpha_i\rangle$, $\alpha_j \simeq \langle\alpha_j\rangle$ and expressing the lasing intensity in terms of the average radiation intensity in each line.

(ii) We make the replacement $\langle I_i \rangle \simeq I_i^*$, $\langle I_j \rangle \simeq I_j^*$ in the expressions obtained.

According to the first approximation, we set $\alpha_i \simeq \langle \alpha_i \rangle$, $\rho_i \simeq \langle \rho_i \rangle$ in expressions (3). Then, we obtain the radiation intensity at the $1 \to i$ line

$$I_i(z) \simeq I_i^+(0)\left[\exp(\langle \alpha_i \rangle z - \langle \rho_i \rangle z) + \frac{\exp(-\langle \alpha_i \rangle z - \langle \rho_i \rangle z)}{r_{1i}}\right]. \quad (12)$$

The radiation intensity averaged over the cavity length is described by the expression

$$\langle I_i \rangle \simeq -\frac{I_i^+(0)}{\ln(r_{1i}r_{2i})^{1/2}} \frac{1-(r_{1i}r_{2i})^{1/2}}{(r_{1i}r_{2i})^{1/2}}\left[1+\left(\frac{r_{2i}}{r_{1i}}\right)^{1/2}\right]. \quad (13)$$

By using the second approximation, we obtain the expression

$$I_i^{\text{out}} \simeq -I_i^* \ln(r_{1i}r_{2i})^{1/2} \quad (14)$$

for the total output radiation intensity.

The output radiation intensities at the left and right ends of the laser are

$$I_{1i}^{\text{out}} \simeq -\frac{I_i^* \ln(r_{1i}r_{2i})^{1/2}}{1-(r_{1i}r_{2i})^{1/2}} \frac{(r_{1i}r_{2i})^{1/2}(1-r_{1i})}{r_{1i}+(r_{1i}r_{2i})^{1/2}},$$
$$I_{2i}^{\text{out}} \simeq -\frac{I_i^* \ln(r_{1i}r_{2i})^{1/2}}{1-(r_{1i}r_{2i})^{1/2}} \frac{r_{1i}(1-r_{2i})}{r_{1i}+(r_{1i}r_{2i})^{1/2}}, \quad (15)$$

where the subscripts 1 and 2 refer to radiation emitted from the left and right cavity mirrors, respectively. By substituting the intensity $I_i^{*(1)}$ (8) to expressions (15), we obtain the known Rigrod formulas for the case of lasing at one line [6]. Note that Rigrod obtained his formulas from the exact solution of the system of differential equations (3). For the $1 \to j$ line, the same procedure should be performed (the subscript interchanging $i \leftrightarrow j$) with expressions (8) and (15).

4. Conclusions

We have determined the power of stationary lasing at two competing lines sharing an upper level. The study was based on the expressions for the gain obtained for simultaneous lasing at two lines [5]. The main attention was paid to the determination of the lasing regime (at one or two lines) and the derivation of approximate expressions for the lasing power in each line.

The introduction of the concept of the 'operating intensity' allowed us to generalise the results obtained in other papers for the case of lasing at one line to the case of simultaneous lasing at two and more lines. We considered lasing in the cases of competition between two lines in the symbiosis or quenching regimes, derived expressions for determining the 'operating intensities', and developed the method for determining the lasing regime (simultaneous lasing at two lines or at one line). Our analysis showed that the critical radiation intensities introduced in paper [5] serve as boundaries of the regions of lasing regimes.

We derived the expressions for measuring the lasing intensity at two lines, which are generalisations of the Rigrod formulas obtained for lasing at one line.

Acknowledgements. The author thank S.P. Mel'nikov for useful discussions and his help in this study.

References

1. Konak A.I., Mel'nikov S.P., Porkhaev V.V., Sinyanskii A.A. *Kvantovaya Elektron.*, **22**, 225 (1995) [*Quantum Electron.*, **25**, 209 (1995)].
2. Voinov A.M., Konak A.I., Mel'nikov S.P., Sinyanskii A.A. *Kvantovaya Elektron.*, **18**, 1297 (1991) [*Sov. J. Quantum Electron.*, **21**, 1179 (1991)].
3. Zayarnyi D.A., Kholin I.V. *Kvantovaya Elektron.*, **22**, 887 (1995) [*Quantum Electron.*, **25**, 855 (1995)].
4. Suda A., Wexler B.L., Feldman B.J., Riley K.J. *Appl. Phys. Lett.*, **54**, 1305 (1989).
5. Pikulev A.A. *Kvantovaya Elektron.*, **30**, 675 (2000) [*Quantum Electron.*, **30**, 675 (2000)].
6. Rigrod W.W. *IEEE J. Quantum Electron.*, **14**, 377 (1978).
7. Stepanov B.I. *Metody rascheta opticheskikh kvantovykh generatorov* (Methods for Calculations of Lasers) (Minsk: Naulka i Tekhnika, 1966) Vol. 1.
8. Schindler G.M. *IEEE J. Quantum Electron.*, **16**, 546 (1980).
9. Eimerl D.J. *Appl. Phys.*, **51**, 3008 (1980).