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Simulation of phase locking of two lasers with hybrid resonators

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***Abstract*. A three-dimensional numerical diffraction model of a slab laser with a hybrid resonator is developed and used for studying the phase locking of two lasers optically coupled through the edge of the output mirror. Numerical analysis of phase locking of industrial CO_2 slab lasers shows that such a locking is possible for an appropriate choice of coupling parameters. It is found that the destructive effect of the active medium caused by an increase in the pump intensity can be minimised. It is shown that as the pump intensity is increased, the lateral radiation pattern of the output radiation improves while the beam is broadened in the transverse direction due to excitation of higher-order waveguide modes.**

Keywords: *slab laser, hybrid resonator, phase locking, unstable resonator, waveguide.*

1. Introduction

The recent advances in the technology of industrial gas lasers have been associated with the design of slab lasers with diffusion cooling. In this case, there is no need for a rapid gas circulation, and the power is increased due to a long gap used in the laser [1]. The single-mode lasing ensuring a high quality of the laser beam is obtained with the help of a hybrid resonator in which the radiation normal to the gap is confined by the walls (waveguide propagation), while radiation along the gap (in the lateral direction) is confined by the mirrors forming an unstable resonator [2]. The problem of producing output radiation for industrial applications has been solved successfully at present [3]. The power of industrial devices can be increased by adding up the powers of several lasers [4]. The optical quality of the resultant laser beam can be improved further by using the optical coupling between resonators and implementing phase locking.

Because of the complex geometry of slab lasers, their numerical simulation necessitates an approach based on the solution of three-dimensional diffraction equation. The problem of phase locking of lasers with unstable resonators was considered by us earlier [5, 6] in the two-dimensional approximation of diffraction optics. Similar calculations were performed in the USA [7] (see also a review of the earlier works in Ref. [8]).

In this work, we have developed a three-dimensional numerical model of a slab laser, which is used for studying the phase locking of two slab lasers optically coupled through the edge of the mirror in an unstable resonator. Note that a similar three-dimensional model describing a CO_2 slab laser with a hybrid resonator was developed earlier in Ref. [9].

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2. Formulation of the problem

Fig. 1 shows the scheme of the system of two slab lasers under consideration. The optical coupling was produced by two plane reflecting mirrors (*3*) which directed the radiation emerging from the edge of the convex mirror (*2*) of one of the resonators to the other resonator. Cylindrical mirrors forming an unstable resonator in the lateral direction may be common for both lasers. It is obvious that the mathematical calculation of the optical fields in this scheme is quite complex. The mathematical problem to be solved can be formulated in the general form in terms of radiation transformation operators. If the functions v and f correspond to beams incident on the output (convex) mirror of each of the resonators, their coupling can be described with the help of two operators. The operator $\hat{P}$ describes the round trip of a beam reflected from the convex mirror in one of the resonators, while the operator $\hat{T}$ describes the propagation of a beam passing through the coupling channel and entering the other resonator, where the beam makes a round trip and is incident on its output mirror. Using these operators, the system of equations for fields v and f can be written in the form

$$\begin{pmatrix} P & e^{i\psi}T \\ e^{i\chi}e^{i\psi}T & e^{i\chi}P \end{pmatrix}\begin{pmatrix} v \\ f \end{pmatrix} = \gamma\begin{pmatrix} v \\ f \end{pmatrix}, \qquad (1)$$

where γ is the eigenvalue of the system of equations; χ is the difference in the optical lengths of the resonators in phase incursions; and ψ is the phase incursion in the coupling channel minus a quantity multiple of 2π (see Refs [6, 9] for a detailed discussion of these parameters). For identical resonators, $\chi = 0$ and the system of equations (1) can be split into two equations

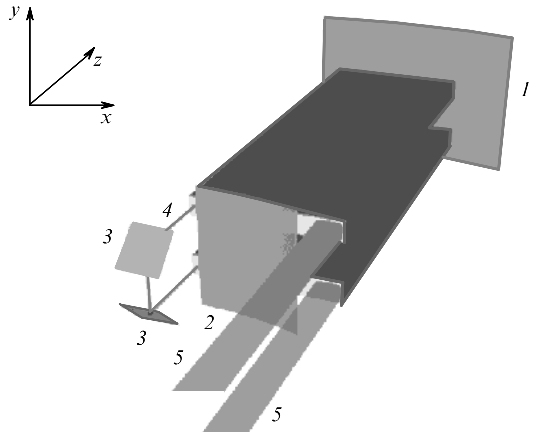

Figure 1. Scheme of the laser system: (*1*) concave mirror; (*2*) convex mirror; (*3*) deflecting mirrors; (*4*) coupling channel; (*5*) output beam.

$$(\hat{P} - e^{i\psi}\hat{T})w_- = \gamma w_-, \quad w_- = v - f, \tag{2}$$

$$(\hat{P} + e^{i\psi}\hat{T})w_+ = \gamma w_+, \quad w_+ = v + f \tag{3}$$

for two new functions $w_\pm$, which are symmetric and antisymmetric linear combinations of the functions v and f introduced above. It can be easily verified that the system of equations (2), (3) has two solutions: the inphase solution ($v = +f$) and the antiphase solution ($v = -f$), the synphase solution satisfying Eqn (3) and the antiphase solution satisfying Eqn (2). Note that, when the phase ψ is varied, the solutions are transformed into each other for $\psi' = \psi \pm \pi$. Therefore, it is sufficient to find the solutions of one of the equations (2), (3), by varying ψ from zero to 2π.

In the case of identical resonators, the problem becomes even simpler if we take into account the symmetry of laser configuration relative to a plane that is parallel to the planar waveguides of lasers and is located in the middle between them. The lateral projection of such an equivalent resonator is shown in Fig. 2 where a perfectly reflecting mirror (*3*) placed in the middle of the coupling channel (of length d_c) effectively takes into account the symmetry of the configuration. It is also assumed that deflecting mirrors are perfect reflectors and do not limit the aperture of the coupling beam. This approach, which was used earlier in Ref. [6], considerably simplifies the necessary calculations.

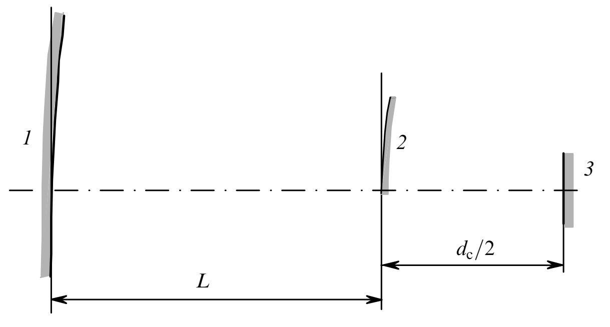

Figure 2. Side projection of the equivalent resonator: (*1*) concave mirror; (*2*) convex mirror; (*3*) equivalent mirror taking into account the symmetry of the scheme.

Note that the approximation concerning the identity of the resonators may not be satisfied easily in actual practice. However, calculations made in such an approximation allow us to determine the key parameters whose choice ensures a reliable selection of the desired mode and a weak sensitivity of phase locking to the requirement concerning the identity of the lasers. The calculations made by us were aimed at studying the spectrum of collective modes of the system, determining the lasing threshold for each mode as a function of the key parameters, finding the optical quality of the resultant output beam, and studying the saturation of the gain and nonuniform refraction. Our study yields the values of the key parameters of the design determining the stability of phase locking.

3. Results of numerical simulation

It was mentioned earlier during our investigations of lasers with unstable resonators coupled by radiation passing through central holes in the mirrors [6] that the phase incursion ψ in the coupling channel is one of the key parameters affecting phase locking. If optical coupling is organised through the edge of the convex mirror opposite to the output, the exact position δ of the optical axis of the unstable resonator relative to this edge (negative values of δ correspond to the emergence of the axis outside the surface of the convex mirror) is of considerable importance and is another key parameter.

The effect of the distance between the mirror edge and the axis of the asymmetric unstable resonator on the laser characteristics is quite well known (see, for example, Ref. [10]). In particular, there exists an optimal distance for which the radiation coupling is minimal though close to the geometrical optics limit. The equivalent scheme in Fig. 2 shows that the use of a system of coupled lasers involves the same problems that were studied earlier [11] when partially reflecting windows were mounted normally to the output beam. It is known that the modes of the composite resonator formed by the resonator mirrors and the outer window also compete in this case. Even a slight reflection from the window makes these modes dominant. Because the optical quality of the output radiation is similar to that of the stable resonator beams because of its multimode character, efforts are made usually to avoid the appearance of such modes in lasing.

3.1 Empty resonator modes

To study the collective modes of two coupled hybrid resonators, we choose a number of parameters characteristic of industrial CO_2 lasers, whose variation over reasonable limits does not affect phase locking significantly. These parameters are: the radius $R_1 = 432$ cm of the concave mirror, the radius $R_2 = 216$ cm of the convex mirror, the resonator length $L = 108$ cm, the resonator magnification $M = 2$, the waveguide length $L_g = 100$ cm, the gap $d = 4$ cm between the waveguide and mirror, the width $b = 15$ cm of the active medium, the waveguide thickness $2a = 0.2$ cm, the radiation wavelength $l = 10.6$ μm, and the geometrical length $d_c = 11.2$ cm of the coupling channel.

Using the standard procedure of calculating the optical resonator modes by multiple iteration of the round trip of radiation over the entire system, we can determine the steady-state modes and their eigenvalues characterising the lasing threshold. It was mentioned above that all calculations should be performed by varying the two main parameters δ and ψ. Because the radiation outcoupled from the edge of the mirror (*2*) is reflected back virtually without losses (see Fig. 2), the position of the resonator axis should be varied to both sides of the edge of this mirror.

The removal of the resonator axis beyond the convex mirror enhances coupling between the resonators and hence improves phase locking. However, there is also an adverse effect, which is enhanced as the resonator axis is removed from the convex mirror. This is the formation of a stable resonator between two concave mirrors. The modes of this resonator are localised near the axis, and their field virtually does not leave the resonator. A stable resonator is also formed as a result of inward displacement of the resonator axis. This effect is analogous to the formation of a stable resonator by a plane reflector and the mirrors of the unstable resonator studied earlier [11]. In this connection, the parameter δ was varied in the vicinity of zero. The parameter ψ characterising the small-scale variation of the coupling channel length within a wavelength ($0 \leqslant \psi < 2\pi$) is associated with the interference of the light beam reflected from the convex mirror and the beam emerging from the coupling channel. Hence the influence of this parameter on the formation of optical modes is not obvious and should be studied numerically. Recall that in this case it is sufficient to investigate the inphase mode only. The characteristics of the antiphase mode can then be determined from simple algebraic relations.

We performed a series of calculations with different values of the parameters δ and ψ. Fig. 3 shows the amplitudes (moduli) of eigenvalues of the inphase mode γ_+ of the system under consideration on the ψ, δ plane. One can see that the minima of the amplitudes (and, accordingly, the peaks of the threshold gain) are localised in the vicinity of $\delta = 0$, which is attributed to the diffraction of radiation at the near edge of the mirror. Recall that the geometrical limit for γ_+ is equal to $M^{-1/2} \approx 0.71$.

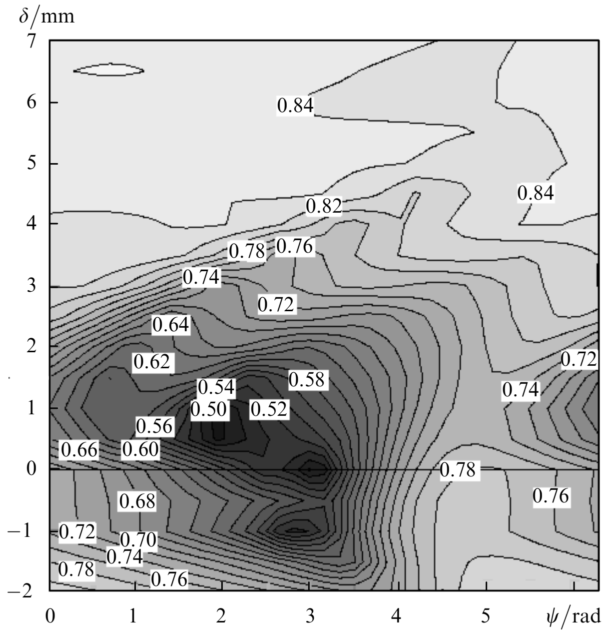

Figure 3. Contour plots of the modulus of eigenvalue of the inphase mode γ_+.

The departure of the coupling channel length from an integer of wavelengths λ is found to be a function of the generated longitudinal mode. For a gas pressure of 100 Torr and the above value of the resonator length, the gain line contains a large number of longitudinal modes, which makes it possible to vary the parameter ψ over a wide range. Considering the variation in the threshold gain associated with it, we can expect that lasing occurs at a wavelength corresponding to the peak value of γ_+. Because the maximum values of γ_+ and γ_- coincide (for different values of ψ), a competition appears between the inphase and antiphase modes, which cannot be eliminated by varying the resonator parameters. This problem requires an independent solution. Assuming that such a solution has been found, we study the characteristics of the inphase mode by varying the only remaining key parameter. Fig. 4 shows the modulus of the eigenvalues γ_+ and the discrimination of the antiphase mode relative to the inphase mode $\gamma_+^2 - \gamma_-^2$ as functions of ψ for $\delta = 0$ and 1 mm. One can see that the maxima of the dependences of γ_+ and $\gamma_+^2 - \gamma_-^2$ do not coincide.

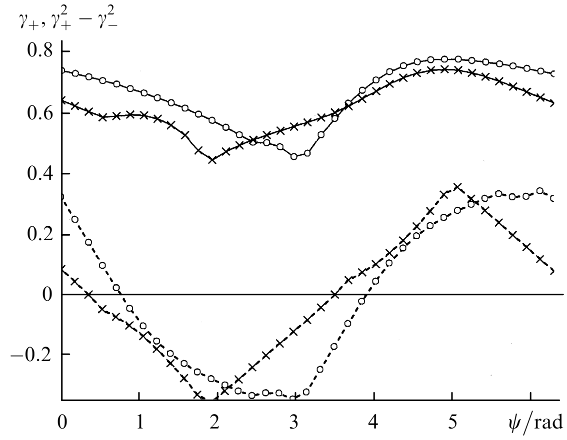

Figure 4. Dependences of the eigenvalues of the inphase mode γ_+ (solid curves) and the quantities $\gamma_+^2 - \gamma_-^2$ (dashed curves) on ψ for $\delta = 0$ (○) and 1 mm (×).

We made a series of calculations of optical modes for the position δ of the optical axis varying in the limits $-1.5 \ldots 2$ mm. The value of ψ corresponding to the maximum value of γ_+ as well as the main characteristics of the inphase mode (see Table 1) were obtained for each position of the axis. The most important mode characteristic is the parameter $(\gamma_+^{\max})^2 - \gamma_-^2$ determining the stability of phase locking. One can see from Table 1 that the discrimination of the antiphase mode is maximum for $\delta = 1$ mm. Note that for all values of δ, the value of γ_+ exceeds the geometrical optics limit for one resonator. This means that

Table 1.

δ/mm	$\gamma_+^{\max}$	$(\gamma_+^{\max})^2 - \gamma_-^2$	θ_x/mrad	ψ_{opt}/rad
−1.5	0.8038	0.154	0.1988	5.0615
−1.0	0.7892	0.2088	0.276	4.7124
−0.5	0.7898	0.202	0.2622	4.7124
0	0.783	0.2586	0.47	4.8869
0.5	0.755	0.3167	0.4132	4.7124
1.0	0.7492	0.3339	0.3888	4.8869
1.5	0.7586	0.2331	0.3083	5.0615
2.0	0.778	0.1752	0.3112	5.4105

mode characteristics are influenced by diffraction and the coupling between resonators. Note also that the lateral divergence angle θ_x at the 0.8 level depends on δ. Unfortunately, the minimum lateral divergence angle is attained at $\delta = -1.5$ mm, when the difference between the losses in the two modes is minimal. In the transverse direction, the structure of the field is controlled by the waveguide, and the optical quality of the beam is insensitive to variations in δ and ψ.

Fig. 5 shows the radiation intensity profiles in the far-field zone in the lateral direction for three values of δ and for optimal values of ψ. The distributions have a complex structure, and the positions of the peaks do not coincide with the optical axis. A characteristic feature of these distributions is the presence of two peaks of comparable heights for $\delta = 0$ and 1 mm. Note that the coupling between resonators does not lead to a sharp increase in the lateral divergence compared to the corresponding value obtained for one resonator.

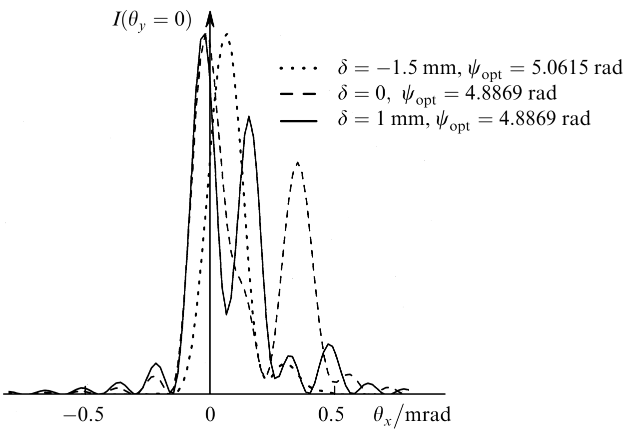

Figure 5. Distribution of the output radiation intensity in the far-field zone along the lateral direction for different values of δ and ψ_{opt} corresponding to the minimum losses (see Table 1).

In simplified models of optically coupled lasers, the concept of optical coupling strength is widely used (see, for example, [8]). In the realistic diffraction model, the optical mode is not defined by a single parameter. Therefore, one has to define for numerical computations the parameter characterising the coupling strength.

3.2 Influence of the active medium

To illustrate the manifestations of saturation and nonuniform refraction, whose role increases with pump power, we chose the following pairs of parameters: $\delta = 0$, $\psi = 0$ and $\delta = 1$ mm, $\psi = 5$ rad. Consider separately the gain saturation and the combined action of the gain saturation and heating of the medium. As the pump intensity increases, both factors (gain saturation and refraction) become more important. The effect of gain saturation was simulated specifying the standard dependence $g = g_0(1 + I/I_s)^{-1}$, where I_s is the saturation intensity, and g_0 is the small-signal gain. Saturation is determined by the parameter g_0/g_{th}, where g_{th} is the threshold gain. Fig. 6 shows the dependence of the moduli of eigenvalues on g_0/g_{th} for inphase ($\gamma_+ \equiv 1$) and antiphase modes for the two pairs of parameters chosen above. For parameters $\delta = 0$, $\psi = 0$ the eigenvalue of the antiphase mode increases and intersects the horizontal straight line $\gamma_+ \equiv 1$ for $g_0/g_{th} \approx 2.5$. This means that starting from this pump level, the single-mode regime becomes unstable, i.e., phase locking is not observed any longer. For $\delta = 1$ mm, $\psi = 5$ rad, the phase locking stability is violated neither by saturation nor by nonuniform refraction due to heating of the mixture (see Fig. 6). For calculating the refraction coefficient profile, a gaseous mixture having the composition $CO_2 : N_2 : He = 1 : 1 : 5$ was used under a pressure of 100 Torr. The temperature of the waveguide walls was assumed to be equal to room temperature, and the temperature profile was assumed to be parabolic, while the temperature at the centre of the waveguide was taken as proportional to the discharge power. Further verification proved that the gain profile does not significantly affect the results of computations, and hence the profile was chosen in the form of a 'step' whose height is a linear function of the pump power Q_{pump} with a typical proportionality factor for the conditions being simulated. Figs 6 and 7 show the results of computations taking into account both the gain saturation and nonuniform refraction, presented as functions of Q_{pump}/Q_{th}, where Q_{th} is the threshold pump power. In the entire investigated interval $1 < Q_{pump}/Q_{th} \leqslant 4.7$ of excess over the lasing threshold, the inphase mode was found to be stable. Thus, the parameters of a system of two optically coupled CO_2 slab lasers with hybrid resonators may be chosen in such a way that a reliable phase locking is ensured.

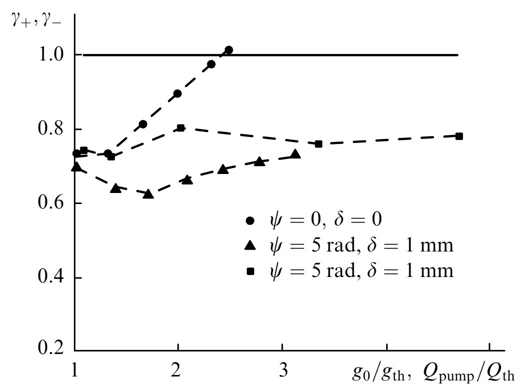

Figure 6. Moduli of eigenvalues of inphase (solid line) and antiphase (dashed curves) modes as functions of the excess over the small-signal gain threshold (●, ▲) or over the pump power (■).

It should be interesting to estimate the effect of the active medium on the variation of optical quality of the output beam. This effect is illustrated in Fig. 7 showing the change in the lateral divergence angle θ_x at the level 0.8, as well as the change in the optical quality of the beam M_y^2 along the y axis, upon an increase in the pump intensity. One can see that the optical quality of the radiation in the lateral direction improves monotonically with gain in the saturated medium model, while it remains unchanged in the transverse direction. If the gas heating and the corresponding refractive index gradients are taken into consideration, no significant deterioration of the optical quality of radiation is observed in the lateral direction. The beam quality in the transverse direction is affected adversely by gas heating which leads to a considerable increase in the value of the parameter M_y^2.

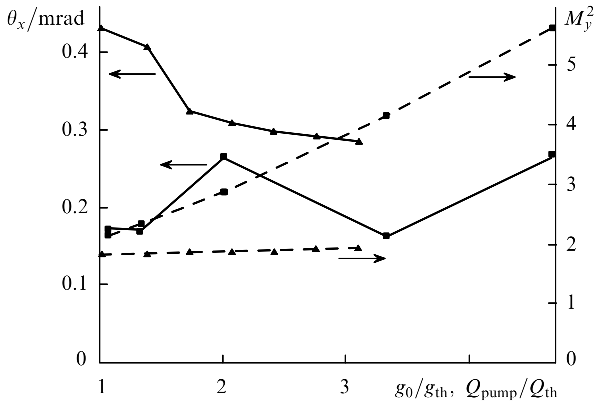

Figure 7. Angle of lateral divergence θ_x of the output beams (solid curves) and the parameter M_y^2 (dashed curves) as functions of the excess over the small-signal gain threshold (▲) or over the pump power (■) for $\delta = 1$ mm, $\psi = 5$ rad.

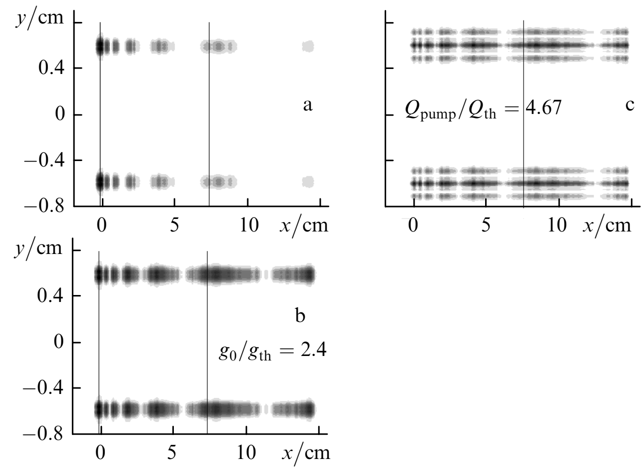

Figure 8. Distribution of the intensity of radiation incident on the output mirror of an empty resonator (a), a resonator filled with a medium with saturable gain (b), and a medium with refractive index gradient and saturable gain (c), for $\psi = 5$ rad and $\delta = 1$ mm. The vertical straight lines show the edges of the convex mirror.

The above-mentioned peculiarities of the effect of active medium on the optical quality of the output radiation are attributed to a change in the field structure in the resonator shown in Fig. 8. A comparison of the distribution of the intensity of radiation incident on the convex mirror of an empty resonator (Fig. 8a) as well as a resonator filled with a medium with saturable gain (Fig. 8b) and a medium with refractive index gradient and saturable gain (Fig. 8c) shows that the field distribution levels out along the gap in the presence of amplification. This leads to an improvement in the quality in the lateral direction. The combined effect of the gain saturation and refractive index gradients leads to a slightly more uniform field distribution along the gap and to the third-order mode excitation in the waveguide. Naturally, the optical quality of higher-order modes is considerably worse than for the fundamental waveguide mode that dominates for low pumping intensities. A comparison of the parameters M_y^2 for a low excitation level in two models also shows that the presence of refractive index gradients in the waveguide deteriorates the optical quality of the output beam of the fundamental mode.

4. Conclusions

Numerical investigations of phase locking of a system of two lasers, carried out for parameters typical of industrial CO_2 slab lasers, show that for an appropriate choice of the length of the optical coupling channel, one can expect the attainment of a stable locking. The effect of the active medium is not reduced to the disappearance of phase locking with increasing pump intensity, but is also manifested in the improvement of the lateral radiation pattern of the output radiation and a considerable broadening of the radiation pattern in the transverse direction due to the excitation of higher-order waveguide modes.

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