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Phase conjugation of speckle-inhomogeneous radiation in a holographic Nd : YAG laser with a short thermal hologram

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***Abstract*.** **A model of the so-called short hologram, which does not exhibit in-depth diffraction deformation of the fine speckle pattern of the recording fields, is studied. The investigation is performed by the example of a thermal hologram recorded by two speckle waves, which is the output mirror of a ring laser produced as a result of this recording. It is shown that the ability of this short hologram to select a wave conjugated to a speckle signal in the mode of the holographic laser depends both on the degree of mutual mixing of the speckles of recording beams in the hologram volume and on the effects of its saturation by the beams. The maximum accuracy of phase conjugation of speckle radiation in the holographic Nd:YAG laser achieved upon the best selection of the conjugate wave by the short thermal hologram was 93 %.**

Keywords: *holographic laser, phase conjugation, speckle field, short hologram, thermal hologram.*

1. Introduction

Consider a ring laser in which one of the mirrors is a dynamical hologram recorded by a wave $\boldsymbol{E}_1$ coupled into the loop scheme and a wave $\boldsymbol{E}_3$ (produced from the wave $\boldsymbol{E}_1$ after its passage over the elements of the loop scheme) (Fig. 1). Under specific parametric conditions, an oscillation wave $\boldsymbol{E}_2, \boldsymbol{E}_4$ may propagate in the direction opposite to the signal wave $\boldsymbol{E}_1, \boldsymbol{E}_3$, which is conjugated to the signal wave. Therefore, such a 'holographic laser' serves as a nonlinear-optical phase-conjugate (PC) mirror for the wave $\boldsymbol{E}_1$ [1, 2]. In this case, the PC mirror is capable of accomplishing a high-quality phase conjugation of not only a smooth beam, but speckle radiation as well [3 – 4]. This endows it with a certain universality from the viewpoint of phase conjugation of an arbitrary beam, because the latter may always be additionally 'speckled' after passage through a random phase aberrator placed at the input of such a mirror.

In the first papers devoted to the study of a holographic laser from the viewpoint of phase conjugation of speckle

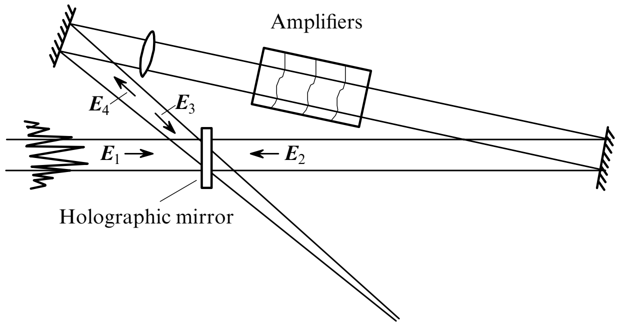

Figure 1. Ring laser with a holographic mirror produced by the speckle-inhomogeneous radiation $\boldsymbol{E}_1, \boldsymbol{E}_3$ coupled into the loop scheme.

radiation, a thermal hologram was recorded, i.e., a hologram that is produced due to the heating δT caused by weak absorption of the signal radiation $\boldsymbol{E}_1, \boldsymbol{E}_3$, in a liquid medium, leading to the inhomogeneous perturbation of its refractive index $\delta n \sim (dn/dT)\delta T$ [5, 6]. These experiments were characterised by a moderate quality of phase conjugation: for CO_2-laser radiation it was $\sim 20\,\%$ [5], and for a Nd : glass holographic laser only 5 % [6].

Later, the quality of phase conjugation of speckle radiation in a holographic laser by recording the holographic mirror 'on gain gratings' in the laser medium was noticeably improved (up to 80 % – 90 %) [3, 4, 7]. This improvement was partially achieved due to the discovery of new factors of selection of the conjugate wave in the laser mode. In particular, the authors of Refs [3, 7] were first to substantiate the selective capability of the so-called short hologram, for which the in-depth diffraction of the recording waves is insignificant on the speckle scale.

In this paper, we revert to the recording of a thermal hologram, which offers several advantages over the gain-grating hologram. Among the virtues of the thermal hologram is its weak sensitivity to the action of the oscillation wave, whereas the gain-grating hologram is susceptible to this action because of the inertialless response of the nonlinearity of amplification saturation to the laser field. As a result, a self-action of the oscillation wave (by inhomogeneous changing of the hologram) on its own mode structure occurs in the gain grating, resulting in the degradation of this structure. The inertial (acoustic) manifestation of the refractive index perturbation in the thermal hologram is a factor which smoothes the above action. This may be important for several specific phase conjugation problems, in particular, for the experimental realisation of the polarisation model of a holographic laser

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constructed in Ref. [8], which solves the problem of phase conjugation of depolarised speckle radiation.

The aim of this work was to accomplish a high-quality phase conjugation of linearly polarised speckle radiation in a holographic laser with a thermal hologram on the basis of selective properties of the short hologram model. We focused our attention on the investigation of the dependence of these properties on the effect of thermal hologram saturation by the speckles of the recording waves $\boldsymbol{E}_1, \boldsymbol{E}_3$,

2. Experimental

The scheme of the holographic Nd : YAG laser is shown in Fig. 2. The linearly polarised radiation $\boldsymbol{E}_0$ with a diffraction-limited divergence, which was the TEM_{00} mode of a 1.06-μm Nd : YAG master oscillator (MO), was directed to the input of the loop scheme in the form of a transform-limited pulse with a duration of 30 ns. After passing through the aberrator (*1*), which introduced deep and small-scale phase fluctuations over cross section of the beam $\boldsymbol{E}_0$, the signal radiation arrived at the cell (*2*) [filled with weakly absorbing acetone solution of $Cu(NO_3)_2$], now in the form of a beam $\boldsymbol{E}_1$ with a developed speckle pattern to record the thermal hologram together with a wave $\boldsymbol{E}_3$ in the cell. The phase aberrator (*1*) provided a divergence $\theta_0 \sim 2 \times 10^{-3}$ for the wave $\boldsymbol{E}_1$ [θ_0 is the half-width at the e^{-1} level of the maximum of the angular spectrum of the wave obtained after a plane-wave passage through the aberrator (*1*)].

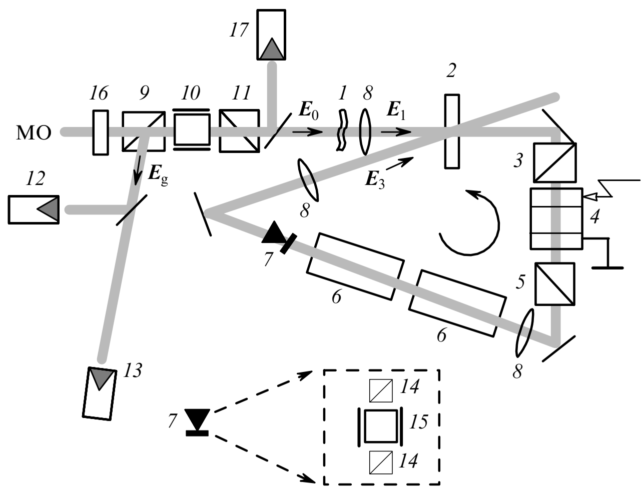

Figure 2. Schematic of a holographic Nd : YAG laser: (*1*) phase aberrator; (*2*) thermal hologram; (*3, 5, 9, 11, 14*) Glan polarisers; (*4*) electrooptical Pockels cell; (*6*) loop Nd : YAG amplifiers; (*7*) Faraday 'optical diode'; (*8*) lenses; (*10, 15*) 45° Faraday rotators; (*12, 13, 17*) calorimeters; (*16*) $\lambda/2$ plate.

Therefore, for the $\boldsymbol{E}_0$ beam radius $a_0 = 1.5$ mm at the input of the aberrator (*1*), the parameter describing the speckle inhomogeneity of the field (the parameter of speckling), $\beta = \rho_1/a_1 = \rho_3/a_3$, is estimated as $\beta \approx 0.11$ in accordance with the expression $\beta = 2\theta_{\rm dif}/\theta_0 = 2/ka_0\theta_0$. Here, $k = 2\pi/\lambda$; $a_{1,3}$ are the radii of the speckle beams $\boldsymbol{E}_1, \boldsymbol{E}_3$ with a Gaussian envelope, which are determined at the e^{-1} level of the maximum of the average intensity at the beam centres; and $\rho_{1,3}$ are the transverse correlation radii determined at the e^{-1} level of the maximum of the transverse correlation function $K_{1,3}(r_1, r_2)$ of the speckle fields $\boldsymbol{E}_1, \boldsymbol{E}_3$ in the doubly Gaussian speckle-beam model [7, 9], where $K(r_1, r_2) \sim \exp[-(r_1 - r_2)^2/\rho^2]$.

The holographic laser regime realised in our work is similar to the *Q*-switching regime of a conventional laser, when a giant pulse several tens of nanoseconds in duration is generated. The $\boldsymbol{E}_2^{\rm mod}, \boldsymbol{E}_4^{\rm mod}$ mode is formed in this substantially transient oscillation regime during a short linear stage, which lasts for several tens of round trips of the oscillation wave in the resonator.

It is important to eliminate the effect of different parasitic seeds and wave competition on this process. In the case of a substantial gain attained by the instant of recording the hologram in the amplifiers (*6*) (small signal of $\sim 10^4$), these factors can significantly perturb the $\boldsymbol{E}_2^{\rm mod}$ mode of the holographic resonator, thereby impairing the accuracy of phase conjugation of the signal field $\boldsymbol{E}_1$. To eliminate these effects, we placed into the loop scheme a Faraday isolator (*7*) and a polarisation unit comprising Glan polarisers (*3*), (*5*) and an electrooptical Pockels cell (*4*).

The Faraday isolator ensured a unidirectional oscillation regime (in the direction opposite to the signal $\boldsymbol{E}_1$) by suppressing the copropagating oscillation wave, thereby eliminating its influence on the production of the $\boldsymbol{E}_2^{\rm mod}$-mode wave. The polarisation unit (*3, 4, 5*) allowed delaying the oscillation pulse $\boldsymbol{E}_2^{\rm mod}$ relative to the signal pulse $\boldsymbol{E}_1$. The delay should be long enough to rule out the possibility that the parasitic wave imparts its spatial structure to the oscillation wave. This parasitic wave emerges due to backscattering of the signal wave itself from the elements of the loop scheme and has the capacity to develop on its own in the ring resonator produced by the signal radiation E_1, E_3.

Such a delay was provided in the following way. The polarisers (*3*) and (*5*) were mutually crossed, and therefore the ring resonator, which was produced almost immediately after the outcoupling of the short signal pulse from the loop scheme, remained closed for lasing. It was opened with the aid of the electrooptical cell (*4*) to which a high-voltage 'step' was applied via a delay (~ 1 μs) timed with the signal pulse to rotate the polarisation by 90° in the cell (*4*).

Note that the Glan polarisers (*3*) and (*5*) were not perfectly crossed as regards the recording of the hologram, i.e., the angle of rotation of one polariser relative to the other was not precisely equal to 90°. Varying this angle about this value afforded passage of the signal wave through the loop scheme such that initially it was significantly attenuated at the output from the polariser (*5*) and then was amplified in the amplifiers (*6*) and transformed to the wave $\boldsymbol{E}_3$ to acquire an average energy density $\langle W_3 \rangle = E_3/\pi a_3^2$, which is approximately equal to the average energy density $\langle W_1 \rangle = E_1/\pi a_1^2$ of the wave $\boldsymbol{E}_1$ (here, $E_{1,3}$ are the energies of the waves $\boldsymbol{E}_{1,3}$). This was required to obtain the best contrast of the holographic grating. Hereafter, the angle brackets denote averaging over a statistical ensemble of realisations of the signal wave $\boldsymbol{E}_1$ (and the holographic laser it produced); the passage to this ensemble is a standard procedure in the treatment of speckle fields [9].

3. Relationship between the quality of phase conjugation of a signal wave in the fundamental mode of the holographic laser and the properties of a short hologram

Despite the suppression of the above parasitic and competitive factors in the holographic resonator, the

$\boldsymbol{E}_2^{\rm mod}$ mode always contains, along with the component $\boldsymbol{E}_{2\rm c}^{\rm mod} \sim \boldsymbol{E}_1^*$ conjugate to the signal wave, a component $\boldsymbol{E}_{2\rm n}^{\rm mod}$ not correlated to it in the fine speckle pattern ($\langle \boldsymbol{E}_{2\rm n}^{\rm mod}\boldsymbol{E}_1\rangle = 0$). It appears mainly because the hologram is reference-free, which means that neither of the two recording speckle waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ serves as a reference wave for the other, i.e., neither of them is spatially homogeneous on the scale of its envelope. As a result, upon reflection from a hologram of even an exactly conjugate wave $\boldsymbol{E}_{2\rm c} \sim \boldsymbol{E}_1^*$ there always emerges, along with the conjugate wave $\boldsymbol{E}_{4\rm c} \sim \boldsymbol{E}_3^*$, a noncorrelated wave $\boldsymbol{E}_{4\rm n}(\langle \boldsymbol{E}_{4\rm n}\boldsymbol{E}_3\rangle = 0)$ in the scattered wave $\boldsymbol{E}_4$. The noncorrelated wave $\boldsymbol{E}_{4\rm n}$ eventually forms the noncorrelated mode component $\boldsymbol{E}_{2\rm n}^{\rm mod}$, which deteriorates the phase-conjugation accuracy in the mode, which is defined as

$$\chi = \frac{\int |\boldsymbol{E}_{2\rm c}^{\rm mod}|^2 {\rm d}r^2}{\int |\boldsymbol{E}_2^{\rm mod}|^2 {\rm d}r^2},$$

where $\boldsymbol{E}_2^{\rm mod} = \boldsymbol{E}_{2\rm c}^{\rm mod} + \boldsymbol{E}_{2\rm n}^{\rm mod}$.

However, the reference-free hologram, which gives rise to the noncorrelated component $\boldsymbol{E}_{2\rm n}^{\rm mod}$, is able to efficiently discriminate against it. This is achieved due to the mutual mixing of the speckles of the recording waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ in the hologram volume. As shown in Refs [6, 7], it is not important how this mixing occurs – 'by diffraction', as in the model of a long hologram [6] [$N \gg 1$, where $N = l/(z_{\rm cor1}z_{\rm cor3})^{1/2}$)], whose length is many times the length $z_{\rm cor1,3} = k\rho_{1,3}^2$ of diffraction spreading of the characteristic speckle inhomgeneities $\rho_{1,3}$ of the recording waves, or 'geometrically', as in the short-hologram model ($N \leqslant 1$) [7].

The second model seems to be preferable to the first one, because it imposes less stringent requirements on the hologram thickness. That is why we used this model in our experiments. In particular, an estimate of the parameter N for the cell (*2*) of largest thickness ($l = 6$ cm) yields, in accordance with the expression $N = l/(z_{\rm cor1}v)$ (where $z_{\rm cor1} = ka_1^2\beta^2, v = a_3/a_1 \approx 1/2$, $a_1 \approx 3$ mm), $N \sim 0.2$, which corresponds to this model.

The quality χ_0 of phase conjugation in the fundamental mode in the short hologram model is described by the expression [7]:

$$\chi_0(\mu) \approx 1 - \mu\left[2\int_0^{1/\mu} {\rm e}^{-x^2}{\rm d}x - \mu\left(1 - {\rm e}^{-1/\mu^2}\right)\right], \qquad (1)$$

where $\mu = \rho_1/(\sqrt{2}l\sin\varphi)$ is the dimensionless parameter of the hologram, which characterises the degree of mutual mixing of the speckles of the recording waves in the hologram, because its inverse quantity is directly proportional to the average number of times each speckle of the recording wave $\boldsymbol{E}_3$ crosses the speckles of the recording wave $\boldsymbol{E}_1$ in the hologram volume.

Therefore, in accordance with expression (1) the phase-conjugation quality χ_0 increases monotonically with decreasing parameter μ. For a fixed hologram thickness l, this takes place with increasing angle φ of convergence of the recording waves. Note that expression (1) is valid for the recording speckle fields with infinite envelopes. As to aperture-limited beams, it holds good down to the convergence angle $\varphi_{\rm opt} \sim 2a_1/l$ (for concurrent convergence geometry), which is optimal from the viewpoint of selective properties of the short hologram, because its further increase no longer leads to the increase in the number of mutual speckle intersections of the recording waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$. To put it otherwise, for the optimal angle $\varphi_{\rm opt}$, the parameter μ attains its minimal value, which is determined only by the parameter of speckling β of the signal field: $\mu_{\rm min} \approx \beta/2\sqrt{2}$. Therefore, for the value of β that was realised in our experiment ($\beta \approx 0.1$) the maximum of phase conjugation quality should be ~ 93 %.

4. Experimental results

The quality of phase conjugation in our experiments was measured in the following way. The oscillation wave $\boldsymbol{E}_{\rm g}$ passed through the aberrator (*1*) and the polarisation unit [consisting of polarisers (*9*), (*11*) and a Faraday rotator (*10*)], which isolated the master oscillator from the holographic laser, and was deflected by the polariser (*9*). A calorimeter (*12*) placed in the near-field zone relative to the polariser (*9*) measured the total energy $E_{\rm g}$ of the oscillation wave and a calorimeter (*13*) located in the far-field zone measured the energy $E_{\rm g}^{\rm c}$ of only its conjugate component. The latter acquired, passing through the aberrator (*1*), a smooth structure of a beam with a diffraction-limited divergence and was spatially filtered from the noncorrelated speckle background $\boldsymbol{E}_{2\rm n}^{\rm mod}$ during its propagation to the calorimeter (*13*). Therefore, the experimental quality of phase conjugation was determined as the ratio of these energies: $\chi_{\rm exp} = E_{\rm g}^{\rm c}/E_{\rm g}$.

Fig. 3 shows the dependences of the phase-conjugation quality $\chi_{\rm exp}$ measured in this way on the energy E_0 of the

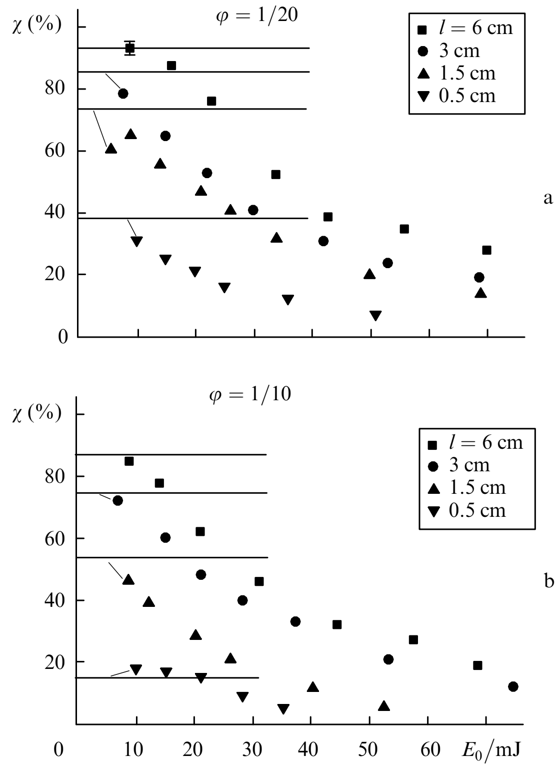

Figure 3. Experimental dependences of phase-conjugation accuracy $\chi_{\rm exp}$ on the input wave energy E_0 for holograms of different thickness l and two angles φ of convergence of the recording beams. The horizontal straight lines determine the phase-conjugation accuracy χ_0 of a speckle beam with the parameter of speckling $\beta \approx 0.11$, which was estimated from expression (1) for the corresponding hologram thickness l and the angle of convergence φ of the recording waves.

input wave $\boldsymbol{E}_0$ for two convergence angles of the recording beams $\boldsymbol{E}_1, \boldsymbol{E}_3$ ($\varphi \sim 1/20$ and $\varphi \sim 1/10$) and four cells of different thickness ($l \sim 0.5, 1.5, 3$, and 6 cm). The energy E_0 was varied by rotating a $\lambda/2$ plate (*16*) and measured with a calorimeter (*17*).

In experiments, a similar absorption of 15 % was provided in all the cells, i.e. $\exp(-\alpha l) \sim 0.85$, where α is the absorption coefficient of the solution. As a result, the oscillation thresholds of the holographic laser (in input wave energy E_0) were equal for the holograms with different thicknesses, because their energy reflectivities R for the mode wave were similar (for a fixed E_0). For $R \ll 1$, the reflectivity was estimated by the following expression:

$$R \approx \tau v^2 (\alpha l g)^2 \langle W_1 \rangle \langle W_3 \rangle,$$

where $\langle W_3 \rangle \approx \langle W_1 \rangle \approx E_0/\pi a_1^2; \tau \sim 0.5$ is the parameter which characterises the degree of coherence, the mutual polarisation state, and the temporal overlap of the recording waves $\boldsymbol{E}_1, \boldsymbol{E}_3; g = k_0 (\partial n/\partial T)_p / \rho c_p$ is a nonlinear parameter; and ρ, c_p are the density and specific heat capacity of the holographic medium. For acetone, g is ~ 17 cm^2 J^{-1} [$k_0 = 2\pi/1.06$ μm, $(\partial n/\partial T)_p \sim -5 \times 10^{-4}$ K^{-1}, $\rho c_p \sim 1.7 \times 10^6$ J m^{-3} K^{-1}].

The experimental threshold in the E_0 energy equal to 8 – 10 mJ corresponds, according to the above data, to a reflectivity $R_{\rm th} \sim 10^{-3}$. This value of R can be regarded as the optimal one, because the maximum quality of phase conjugation obtained for the four holograms near the threshold was in good agreement with that predicted by expression (1), while the accuracy $\chi_{\rm exp}$ of phase conjugation deteriorated with increasing E_0 (and, accordingly, R) (Fig. 3). This was caused by the neglect of thermal-hologram saturation by the speckle recording beams $\boldsymbol{E}_1, \boldsymbol{E}_3$ in the derivation of expression (1): the aberration microlenses localised in the speckle inhomogeneities of the hologram manifest themselves increasingly stronger as the energy of the recording beams increases.

Therefore, the amplification margin in the amplifiers (*6*) should be equal to $\sim 10^4$, if the total losses (the Fresnel, aperture, and polarisation ones) for the oscillation wave in the ring resonator are accounted for (in our experiment these losses were responsible for approximately a ten-fold reduction in its energy in the loop scheme).

This permits the oscillation threshold to be attained without thermal-hologram saturation ($R_{\rm th} \sim 10^{-3}$) by the recording speckle fields. It is the existence of this amplification margin that allowed us to achieve the maximum phase-conjugation quality (~ 93 %) which can provide the short hologram model for a parameter of signal field speckling $\beta \sim 0.1$. We believe that the absence of such an amplification margin, which compels one to deal with a saturated ($R_{\rm th} > 10^{-3}$) thermal hologram, was one of the principal reasons for the low phase-conjugation quality of speckle radiation in Refs [5, 6].

5. Dependence of the selective properties of a short thermal hologram on its saturation by the speckles of recording waves

To explain the experimentally observed lowering of phase-conjugation quality with increasing energy of the speckle beams that record the hologram, we modify somewhat the short-hologram model [described quantitatively by formula (1)] to include the effect of saturation microlenses localised in the speckle-inhomogeneities of the thermal hologram. In the first approximation, this effect causes an additional noise of the conjugate wave $\boldsymbol{E}_{2c}^{\rm mod}(z = l) \sim \boldsymbol{E}_1^*(z = l)$ in the hologram volume when the oscillation wave is outcoupled through the holographic mirror, giving the output wave (Fig. 4)

$$\boldsymbol{E}_{2c}^{\rm out}(z = 0) = \boldsymbol{E}_1^* \exp[\mathrm{i}(\delta\psi_1 + \delta\psi_3)], \tag{2}$$

where

$$\delta\psi_1(r) = g\alpha \int_0^l W_1(r, z)\mathrm{d}z; \ \delta\psi_3(r) = g\alpha \int_0^l W_3(r', z')\mathrm{d}z;$$

$$W_{1,3} \sim |\boldsymbol{E}_{1,3}|^2$$

is the energy density of the recording waves at sections orthogonal to the propagation direction of the corresponding recording wave. Within the framework of the short-hologram model, the dependence of $W_1(r, z)$ and $W_3(r', z')$ on the longitudinal coordinate z or z' is related not to the diffraction deformation of the recording beams, but to the homogeneous attenuation of the field due to absorption in the cell, which can be neglected when $\alpha l \ll 1$, assuming that $W_1(r, z) = W_1(r, 0)$, and $W_3(r', z') = W_3(r', 0)$.

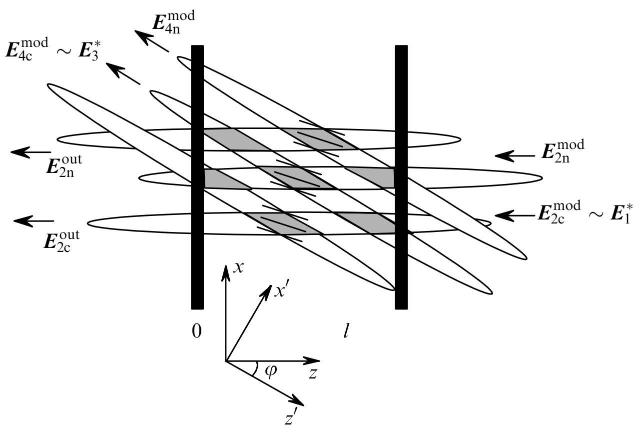

Figure 4. Outcoupling of the $\boldsymbol{E}_2^{\rm mod}(z = l)$ mode wave through a holographic mirror, with the effect that it transforms to the wave $\boldsymbol{E}_2^{\rm out}(z = 0)$, in which the conjugate mode component $\boldsymbol{E}_{2c}^{\rm mod}(z = l)$ has additional noise produced by hologram microlenses localised in its speckle inhomogeneities.

Taking into account this additional noise of the conjugate mode component, the refined expression for the phase-conjugation quality can be written as:

$$\chi_1 = \chi_0 \eta, \tag{3}$$

where

$$\eta = \frac{|\int (\boldsymbol{E}_{2c}^{\rm out} \boldsymbol{E}_1)\mathrm{d}r^2|^2}{\int |\boldsymbol{E}_{2c}^{\rm out}|^2 \mathrm{d}r^2 \int |\boldsymbol{E}_1|^2 \mathrm{d}r^2}$$

is the energy weighting factor of the conjugate component $\boldsymbol{E}_1^*$ in the wave $\boldsymbol{E}_{2c}^{\rm out}$. To calculate the factor η, consider a statistical ensemble of realisations of the signal wave $\boldsymbol{E}_1, \boldsymbol{E}_3$, by estimating the quantity $\langle \eta \rangle$ averaged over the ensemble, which is the less different from η for every specific realisation, the smaller the parameter of speckling ($\beta \ll 1$) of the signal beam: $\eta = \langle\eta\rangle + o(\beta)$.

Furthermore, the effect of the large-scale lens arising from the thermal-hologram saturation by the recording beams on the scale of the their apertures can be neglected, because it is known to be small compared to the effect of microlenses associated with the hologram speckles, and therefore the diameters of waves appearing in the expression for η can be assumed infinite. Then, in accordance with the ergodic hypothesis [9], the expression

$$\langle\eta\rangle = \frac{\left|\langle \boldsymbol{E}_{2c}^{\rm out}\boldsymbol{E}_1\rangle\right|^2}{\left\langle\left|\boldsymbol{E}_{2c}^{\rm out}\right|^2\right\rangle\left\langle\left|\boldsymbol{E}_1\right|^2\right\rangle}$$

is valid.

The substitution of expression (2) for $\boldsymbol{E}_{2c}^{\rm out}$ in the above relation yields, taking into account that the waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ are statistically independent, the following expression for η:

$$\eta = \eta_1\eta_2,$$

$$\eta_1 = \frac{\left|\langle W_1 \exp({\rm i}g\alpha l W_1)\rangle\right|^2}{\langle W_1\rangle^2}; \quad (4)$$

$$\eta_2 = \left|\left\langle \exp({\rm i}g\alpha \int_0^l W_3(r'){\rm d}z)\right\rangle\right|^2.$$

The contributions of the factors η_1 and η_2 caused by phase shifts $\delta\psi_1$ and $\delta\psi_3$, respectively, are not equal. Unlike the inherently inhomogeneous (over the hologram cross section) phase perturbation $\delta\psi_1 \sim W_1(r)$, the degree of phase inhomogeneity $\delta\psi_3 \sim \int_0^l W_3(r'){\rm d}z$ depends strongly on the angle φ of convergence of the recording waves (which also determines the main dimensionless parameter μ of a short hologram).

Thus, considering that the speckle fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ obey the Gaussian statistics [9], wherein the energy density is described by the distribution ${\rm d}P(W_{1,3}) = \langle W_{1,3}\rangle^{-1}\times \exp(-W_{1,3}/\langle W_{1,3}\rangle){\rm d}W_{1,3}$, we obtain for the factor η_1

$$\eta_1 = (1+\Psi_1^2)^{-2}, \quad (5)$$

where $\Psi_1 = g\alpha l\langle W_1\rangle$.

For a zero angle of convergence, the factor η_2 becomes $\eta_2(\mu\to\infty) = 1/(1+\Psi_3^2)$, where $\Psi_3 = g\alpha l\langle W_3\rangle$. The relationship between the factor η_2 and the parameter μ will be obtained for a weak hologram saturation ($\Psi_3 \ll 2\pi$). For this purpose, we expand the exponent in expression (4) for η_2 into a series, restricting ourselves to the first three terms sufficient to reveal the sought-for relationship $\eta_2(\mu)$:

$$\eta_2 \approx \Bigg|1 + {\rm i}g\alpha l\langle W_3\rangle - (g\alpha)^2/2 \times \int_0^l\int_0^l \langle W_3[r'(z_1)]W_3[r'(z_2)]\rangle{\rm d}z_1{\rm d}z_2\Bigg|^2. \quad (6)$$

For the Gaussian statistics, the following expansion is valid for the speckle field $\boldsymbol{E}_3$:

$$\langle W_3[r'(z_1)]W_3[r'(z_2)]\rangle = \langle W_3\rangle^2 + |K_3|^2,$$

where $K_3 = \langle W_3\rangle \exp[-(z_1-z_2)^2/2(l\mu v)^2]$.

In view of this, we obtain from expression (6) the following expression for η_2 (omitting the term $\sim\Psi_3^4$ from it):

$$\eta_2(\Psi_3,\mu) \approx 1 - \Psi_3^2[1-\chi_0(v\mu)]. \quad (7)$$

Therefore, in the case of the noisy conjugate mode component outcoupled through the holographic mirror, the phase-conjugation quality $\chi_1(\Psi_{1,3},\mu)$ is estimated, to a first approximation, by the expression

$$\chi_1(\Psi_{1,3},\mu) \approx \chi_0(\mu)\eta_1(\Psi_1)\eta_2(\Psi_3,\mu). \quad (8)$$

A more rigorous calculation, which is beyond the scope of this paper, should also take into account other factors of the influence of the microlenses localised in the speckle-inhomogeneities of the thermal hologram. In particular, when the conjugate mode component $\boldsymbol{E}_{2c}^{\rm mod}(r, z=l)$ is reflected from the holographic mirror in the presence of these microlenses, the fraction of the conjugate component $\boldsymbol{E}_{4c}^{\rm mod}(r', z'=0) \sim \boldsymbol{E}_3^*$ in the scattered wave will decrease with respect to the noncorrelated component $\boldsymbol{E}_{4n}^{\rm mod}$, also impairing the phase-conjugation accuracy in the mode.

This is mainly explained by the fact that, on the one hand, the additional noise (i.e., reduction) of the conjugate component $\boldsymbol{E}_{2c}^{\rm mod} \sim \boldsymbol{E}_1^*$ of the reading wave appears in the hologram volume, which generates in fact the conjugate component $\boldsymbol{E}_{4c}^{\rm mod} \sim \boldsymbol{E}_3^*$ in the scattered wave. On the other hand, the latter also becomes additionally noisy due to the effect of the above-mentioned microlenses.

In the case of a selective short hologram ($\mu \ll 1$), which is not too strongly saturated by the recording waves ($\Psi_{1,3} \ll 2\pi$), the consideration of these additional factors leads to the refinement of expression (8):

$$\chi_2(\Psi_{1,3,}\mu) \approx \frac{f_1(\Psi_{1,3}) - 1 + \chi_0}{f_2(\Psi_{1,3},\mu) - 1 + \chi_0}\eta_1\eta_2, \quad (9)$$

where

$$f_1(\Psi_{1,3}) = \left|\int_0^1 (1-{\rm i}\Psi_3 z)^{-2}[1-{\rm i}\Psi_1(1-z)]^{-2}{\rm d}z\right|^2;$$

$$f_2(\Psi_{1,3},\mu) = \int_0^1\int_0^1 \frac{[1+f(\mu)] - 2{\rm i}\Psi_1(z_1-z_2)[1+2f(\mu)]}{[1-{\rm i}\Psi_3(z_1-z_2)]^2}{\rm d}z_1{\rm d}z_2;$$

$$f(\mu) = \exp\left[-\frac{(z_1-z_2)^2}{\mu^2}\right].$$

Fig. 5 gives the theoretical dependences $\chi_{1,2}[\Psi_{1,3}(E_0)]$ obtained from expressions (8) and (9) for different values of the parameter μ (which correspond to experimental hologram thicknesses and angles of convergence of the recording beams in it). A comparison of these dependences with experimental data (see Fig. 3) testifies to a good qualitative and quantitative agreement between them for a weak saturation of the thermal hologram ($\Psi_{1,3} \leqslant 0.1$) by the recording speckle fields. Also evident is the tendency for a rapid decline in phase-conjugation quality, both in experiment and theory, with increasing parameters $\Psi_{1,3}$.

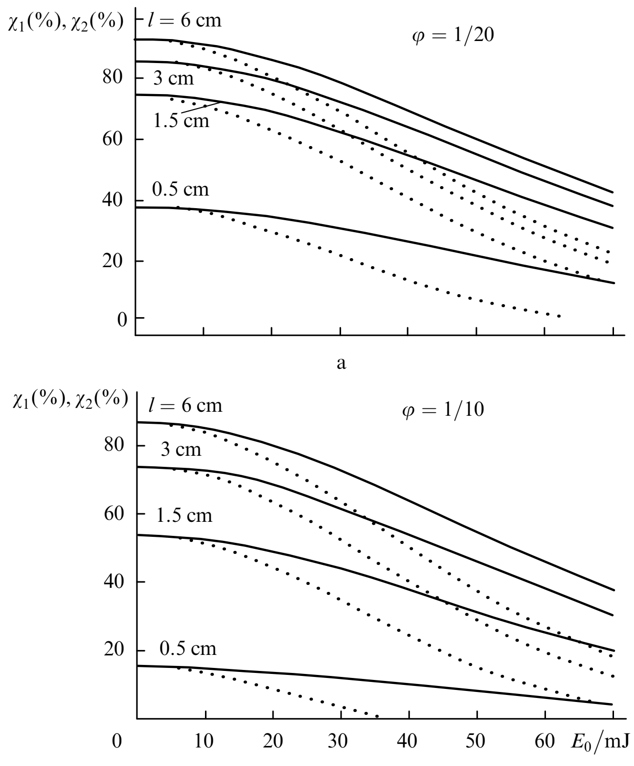

Figure 5. Theoretical dependences of the phase-conjugation accuracy χ_1, χ_2 of a speckle beam (corresponding to $\beta \approx 0.11$) on the input-wave energy E_0 for holograms of different thickness l and two angles φ of convergence of the recording beams. The solid curves are the dependences χ_1 obtained from expression (8), and the dotted curves are the dependences χ_2 (9).

6. Conclusions

The main results of our study are as follows. First, we have shown that the thermal hologram is as good as the gain-grating hologram, provided the effects of its saturation by the speckles of the recording waves are suppressed and its selectivity is ensured within the framework of the short-hologram model. This is confirmed by the fact that the maximum phase-conjugation quality achieved under the conditions of best selection of the conjugate wave in the holographic-laser mode exceeded 90 %. This coincides with the best results obtained in similar schemes employing holograms on gain gratings [3, 4].

Second, the refinement of the short-hologram model made in our work allowed us to analyse the dependence of its selectivity on the specific properties of the thermal hologram related to the effects of hologram saturation by the speckles of the recording fields. As a result of this refinement, the selectivity of the short hologram is characterised, apart from the main parameter μ, by the parameters Ψ_1 and Ψ_3 responsible for the above saturation.

Calculations performed on the basis of the refined model are in good agreement with experimental data. In particular, our analysis of the model and the experimental data showed that microlenses localised in the speckle inhomogeneities of the short thermal hologram has no effect on its selectivity up to energy densities of the recording waves that correspond to $\Psi_{1,3} \sim 0.1$ (or the reflectivity of the holographic mirror of the mode $R \sim \tau v^2 \Psi_1 \Psi_3 \sim 10^{-3}$). The above values of these parameters should be regarded as the optimal ones for a thermal hologram recorded by speckle fields.

Finally, when operating with the thermal hologram, we were able to vary the parameter μ over wide limits (by varying the hologram thickness and the angle of convergence of the recording beams in it), which was hampered in the case of gain-grating hologram whose parameters are limited by the aperture and length of the laser rod. This allowed us to analyse the dependence of the selectivity of a short hologram on the parameter μ in more detail. The analysis revealed (for a nonsaturated thermal hologram, when $\Psi_{1,3} \leqslant 1$) good agreement between the experimental phase-conjugation quality and model calculations (1).

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