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Amplification of squeezed light in the regime of triggered optical superradiance

A.A. Kalachev, V.V. Samartsev

***Abstract*.** **The possibility of amplification of pulses of squeezed light in the regime of triggered optical superradiance is analysed. The kinetic equations are obtained which describe the dynamics of cooperative development of the population inversion and dispersion of the quadrature components of polarisation of optical centres interacting with the triggering-pulse field in the squeezed vacuum state. The dependence of the squeezing degree of the superradiance field on the squeezing degrees of the triggering-pulse field and polarisation of an amplifying medium is determined. It is shown that in the case of a sufficiently strong squeezing of the medium, the intensity of the squeezed quadrature component of the superradiance signal is lower than that of an incoherent spontaneous background. Therefore, the superradiance field can be characterised not only by a classical squeezing (when the dispersions of quadratures are not identical) but also by a quantum squeezing (when the dispersion of one quadrature is smaller than its vacuum value).**

Keywords: *optical superradiance, squeezed states.*

1. Introduction

Nonclassical states of light have attracted much recent attention. Among them, the squeezed states play an important role (see reviews [1 – 4]), i.e., the states in which dispersions of canonically conjugated field variables differ from each other. The use of squeezed light characterised by the quantum squeezing, when the dispersion of one quadrature proves to be smaller than its vacuum value, reduces the noise level in interference measurements below a usual vacuum limit, enhances the transmission of optical communication channels, and also allows one to solve a number of other interesting problems related to the noise suppression in optical systems. The problem of amplification is usually solved in a linear regime. In this case, the intensity of light being amplified should be noticeably lower than the saturating intensity, and a small fraction of the energy stored in the active medium is used for increasing the wave intensity. To obtain a field in the squeezed quantum state at the output of such a linear amplifier, a phase-sensitive amplification is required [5]; otherwise, the gain cannot exceed two.

A.A. Kalachev, V.V. Samartsev E.K. Zavoiskii Physicotechnical Institute, Kazanskii Scientific Center, Russian Academy of Sciences, Sibirskii trakt 10/7, 420029 Kazan', Russia;
e-mail: samartsev@kfti.knc.ru; kalachev@kfti.knc.ru

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In this paper, we analyse the possibility of amplifying squeezed light in the optical superradiance regime, i.e., in the regime of collective spontaneous emission of photons of a system of initially excited particles [6]. In the superradiance regime, a system of N inverted atoms can spontaneously transfer to the ground state during the time, which is inversely proportional to the number of atoms, by irradiating a light pulse whose intensity is proportional to N^2. An important feature of superradiance is that almost all the energy stored in a medium is emitted in a coherent light pulse. In this respect, this phenomenon substantially differs from other cooperative phenomena such as photon echo and free induction decay in which only a small fraction of the stored energy is emitted coherently.

Spontaneous emission of a single atom in the field of a squeezed vacuum was first considered by Gardiner [7], who showed that the decay rates of the in-phase and quadrature components of polarisation of the atom are proportional to the dispersions of the corresponding quadratures of squeezed light. The features of cooperative spontaneous emission in the squeezed-light field were studied in papers [8, 9] for a system of two atoms and in papers [10, 11] for a system of many atoms. In particular, it was shown in paper [11] that in the presence of a squeezed vacuum, the statistics of the delay times of a superradiance signal depends on the signal phase and the degree of initial signal squeezing. It is important to note that, first, in all these papers the systems with dimensions smaller than the wavelength of exciting light were analysed, and, second, it was assumed that atoms interact only with the modes of a squeezed vacuum (ideal interaction). The non-ideal interaction was considered only for one atom [12, 13].

In this paper, we considered the multimode superradiance of an extended polyatomic system under the conditions when only some working modes prove to be squeezed. In this case, the amplification of a weak light field in a squeezed state corresponds to the triggered superradiance regime [14 – 17], when a weak triggering pulse acting during a short time interval or the entire superradiance process initiates a collective transition of atoms to the ground state. The medium emits a triggered superradiance pulse, whose direction is determined by the direction of the triggering pulse, i.e., the energy of the medium is emitted collectively only in the modes corresponding to the triggering pulse. It is assumed usually that the triggering-pulse field is in a coherent state. In this case, the triggering pulse produces a macroscopic polarisation wave in the medium, which results in a substantial shortening of the delay time of the superradiance pulse. In this paper, we consider the case when the triggering-pulse field is in a squeezed vacuum state. In this case, the delay time of a superradiance field is virtually independent of the triggering-pulse intensity, but the direction of emission and the degree of superradiance squeezing are determined by the parameters of the field being amplified.

2. Basic formulas

Consider a system of N two-level atoms interacting with the electromagnetic field of broadband squeezed light. We will calculate superradiance signals assuming that the linear dimensions of a resonance medium are substantially larger than the wavelength λ of the exciting light. In this case, we can neglect the dipole–dipole interaction between the atoms. We also assume, as usual, that the propagation time of photons through the medium is substantially shorter than the self-induced correlation time, so that equations for the density matrix of the atomic system can be written in the Born–Markov approximations. Finally, we will assume that the self-induced correlation time in the medium is substantially shorter than the inhomogeneous lifetime of optical transitions, so that the inhomogeneous broadening can be neglected.

The state of the medium can be conveniently described with the help of the collective operators

$$R_3 = \sum_j b_{3j}, \quad R_{\boldsymbol{q}}^+ = \sum_j b_j^\dagger \, \mathrm{e}^{\mathrm{i}\boldsymbol{q}\boldsymbol{r}_j}, \quad R_{\boldsymbol{q}}^- = \sum_j b_j \, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\boldsymbol{r}_j}, \quad (1)$$

where $b_{3j} = (|2_j\rangle\langle 2_j| - |1_j\rangle\langle 1_j|)/2$ is the operator of the half-difference of populations of the ground $|1\rangle$ and excited $|2\rangle$ states of the jth atom; $b_j^\dagger = |2_j\rangle\langle 1_j|$ and $b_j = |1_j\rangle\langle 2_j|$ are the rising and lowering operators, respectively. The wave vectors $\boldsymbol{q}$ satisfy the orthogonality condition

$$\Gamma(\boldsymbol{q} - \boldsymbol{q}') = \left| \frac{1}{N} \sum \mathrm{e}^{\mathrm{i}(\boldsymbol{q}-\boldsymbol{q}')\boldsymbol{r}_i} \right|^2 = \delta_{\boldsymbol{q}\boldsymbol{q}'}, \quad (2)$$

i.e., the angles between the wave vectors of polarisation modes are larger than the diffraction angle of emission for each mode.

The basic kinetic equation for the reduced density operator $\rho_{\mathrm{a}}(t)$ of the atomic subsystem in the Born–Markov approximation has the form

$$\frac{\mathrm{d}\rho_{\mathrm{a}}(t)}{\mathrm{d}t} = -\frac{i}{\hbar}\mathrm{Tr}_{\mathrm{f}}[H_{\mathrm{af}}(t), \rho_{\mathrm{f}}(0)\rho_{\mathrm{a}}(t)]$$

$$-\frac{1}{\hbar^2}\mathrm{Tr}_{\mathrm{f}}\int_0^t \left[H_{\mathrm{af}}(t), [H_{\mathrm{af}}(t-\tau), \rho_{\mathrm{f}}(0)\rho_{\mathrm{a}}(t)]\right]\mathrm{d}\tau, \quad (3)$$

where $\rho_{\mathrm{f}}(0) = |0\rangle\langle 0|$ is the field-density operator at the initial instant of time;

$$H_{\mathrm{af}} = \mathrm{i}\omega_0 \sum_{\boldsymbol{k}s} \left(\frac{\hbar}{2\omega V \epsilon_0}\right)^{1/2}$$

$$\times\Big\{(\boldsymbol{\mu}\boldsymbol{\varepsilon}_{\boldsymbol{k}s})R_{-\boldsymbol{k}}^- a_{\boldsymbol{k}s}\exp[-\mathrm{i}(\omega_0+\omega)t]$$

$$-(\boldsymbol{\mu}^*\boldsymbol{\varepsilon}_{\boldsymbol{k}s})R_{\boldsymbol{k}}^+ a_{\boldsymbol{k}s}\exp[\mathrm{i}(\omega_0-\omega)t]$$

$$+(\boldsymbol{\mu}\boldsymbol{\varepsilon}_{\boldsymbol{k}s}^*)R_{\boldsymbol{k}}^- a_{\boldsymbol{k}s}^\dagger\exp[-i(\omega_0-\omega)t]$$

$$-(\boldsymbol{\mu}^*\boldsymbol{\varepsilon}_{\boldsymbol{k}s}^*)R_{-\boldsymbol{k}}^+ a_{\boldsymbol{k}s}^\dagger\exp[\mathrm{i}(\omega_0+\omega)t]\Big\}$$

is the interaction Hamiltonian; ω_0 and $\boldsymbol{\mu}$ are the frequency and dipole moment of the atomic optical transition, respectively; ω and $\boldsymbol{\varepsilon}_{\boldsymbol{k}s}$ are the frequency and unit polarisation vector of a photon for the mode $\boldsymbol{k}s$, respectively (s is the polarisation index); $a_{\boldsymbol{k}s}$ and $a_{\boldsymbol{k}s}^\dagger$ are the creation and annihilation operators of a photon in the mode $\boldsymbol{k}s$, respectively; V is the field quantisation volume; and ϵ_0 is the dielectric constant. The field of a broadband squeezed vacuum is characterised by the following properties:

$$\begin{aligned}
&\langle a_{\boldsymbol{k}s}\rangle = \langle a_{\boldsymbol{k}s}^\dagger\rangle = 0,\\
&\langle a_{\boldsymbol{k}s}^\dagger a_{\boldsymbol{k}'s'}\rangle = n\delta_{\boldsymbol{k},\boldsymbol{k}'},\\
&\langle a_{\boldsymbol{k}s} a_{\boldsymbol{k}'s'}^\dagger\rangle = (n+1)\delta_{\boldsymbol{k},\boldsymbol{k}'},\\
&\langle a_{\boldsymbol{k}s} a_{\boldsymbol{k}'s'}\rangle = m\delta_{\boldsymbol{k}',2\boldsymbol{k}_{\mathrm{p}}-\boldsymbol{k}},\\
&\langle a_{\boldsymbol{k}s}^\dagger a_{\boldsymbol{k}'s'}^\dagger\rangle = m^*\delta_{\boldsymbol{k}',2\boldsymbol{k}_{\mathrm{p}}-\boldsymbol{k}},
\end{aligned} \quad (4)$$

where $2\boldsymbol{k}_{\mathrm{p}} = 2\omega_0/c$ is the wave vector of the pump field of a squeezed light source; n is the number of squeezed modes; m is the squeezing parameter ($m = |m|\exp(\mathrm{i}\phi)$, $|m| \leqslant [n(n+1)]^{1/2}$, ϕ is the pump-field phase). Taking these relations into account, equation (3) can be written in the form

$$\frac{\mathrm{d}\rho_{\mathrm{a}}(t)}{\mathrm{d}t} = \int \mathrm{d}\Omega_{\boldsymbol{k}(|\boldsymbol{k}|=|\boldsymbol{k}'|=\omega_0/c)}$$

$$\times m\beta_{\boldsymbol{k}}^{++}\{R_{\boldsymbol{k}}^+, R_{\boldsymbol{k}'}^+, \rho_{\mathrm{a}}\} - (n+1)\beta_{\boldsymbol{k}}^{+-}\{R_{\boldsymbol{k}}^+, R_{\boldsymbol{k}}^-, \rho_{\mathrm{a}}\}$$

$$-n\beta_{\boldsymbol{k}}^{-+}\{R_{\boldsymbol{k}}^-, R_{\boldsymbol{k}}^+, \rho_{\mathrm{a}}\} + m^*\beta_{\boldsymbol{k}}^{--}\{R_{\boldsymbol{k}}^-, R_{\boldsymbol{k}'}^-, \rho_{\mathrm{a}}\}, \quad (5)$$

where $\{A, B, \rho\} \equiv AB\rho - 2B\rho A + \rho AB$;

$$\beta_{\boldsymbol{k}}^{++} = \frac{1}{4\pi}\frac{3}{2}\frac{\beta}{\mu^2}\sum_{s,s'}(\boldsymbol{\mu}^*\boldsymbol{\varepsilon}_{\boldsymbol{k}s})(\boldsymbol{\mu}^*\boldsymbol{\varepsilon}_{\boldsymbol{k}'s'});$$

$$\beta_{\boldsymbol{k}}^{+-} = \frac{1}{4\pi}\frac{3}{2}\frac{\beta}{\mu^2}\sum_{s,s'}(\boldsymbol{\mu}^*\boldsymbol{\varepsilon}_{\boldsymbol{k}s})(\boldsymbol{\mu}\boldsymbol{\varepsilon}_{\boldsymbol{k}s'}^*);$$

$$\beta_{\boldsymbol{k}}^{-+} = \frac{1}{4\pi}\frac{3}{2}\frac{\beta}{\mu^2}\sum_{s,s'}(\boldsymbol{\mu}\boldsymbol{\varepsilon}_{\boldsymbol{k}s}^*)(\boldsymbol{\mu}^*\boldsymbol{\varepsilon}_{\boldsymbol{k}s'});$$

$$\beta_{\boldsymbol{k}}^{--} = \frac{1}{4\pi}\frac{3}{2}\frac{\beta}{\mu^2}\sum_{s,s'}(\boldsymbol{\mu}\boldsymbol{\varepsilon}_{\boldsymbol{k}s}^*)(\boldsymbol{\mu}\boldsymbol{\varepsilon}_{\boldsymbol{k}'s'}^*);$$

$$\beta = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{\omega_0^3\mu^2}{\hbar c^3}$$

is the rate of spontaneous transition in the atom; and $\boldsymbol{k}' = 2\boldsymbol{k}_{\mathrm{p}} - \boldsymbol{k}$. In Eqn (5), the terms describing the frequency shift are omitted, which is a good approximation in the case of an exact resonance.

Let a resonance medium be a cylinder of length L and radius R, whose proportions are determined by the Fresnel number $F = \pi R^2/\lambda L$. If $F \gg 1$, then the diffraction emission angle of axial modes $\alpha_d \approx \lambda/R$ is substantially smaller than the geometrical emission angle $\alpha_g \approx (\lambda/L)^{1/2}$ of the medium, so that superradiance is multimode. Let us assume that squeezed light propagates within a small solid angle $\alpha \geqslant \alpha_d$ along one of the axial polarisation modes of the cylindrical medium. In addition, we assume that the vector $\boldsymbol{\mu}$ is real. Then,

$$\beta_{\boldsymbol{k}}^{++} = \beta_{\boldsymbol{k}}^{--} \approx \beta_{\boldsymbol{k}}^{-+} = \beta_{\boldsymbol{k}}^{+-} = \frac{1}{4\pi}\frac{3}{2}\beta\left[1 - \cos^2(\widehat{\boldsymbol{\mu k}})\right] \equiv \beta_{\boldsymbol{k}}. \quad (6)$$

We will also assume for simplicity that the vector $\boldsymbol{\mu}$ is perpendicular to the symmetry axis of the cylinder. By using Eqn (5) and taking into account (6), we obtain the kinetic equations for average values of dynamical variables

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_3\rangle = -2\beta(N/2 + \langle R_3\rangle + \sum_{\boldsymbol{q}} \mu_q \langle P_{\boldsymbol{q}}\rangle) - 4\mu_0\beta n\langle R_3\rangle,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle P_{\boldsymbol{q}}\rangle = -2\beta(1 - 2\mu_q\langle R_3\rangle + 2\mu_0 n)\langle P_{\boldsymbol{q}}\rangle + 4\mu_q\beta\langle R_3\rangle(N/2 + \langle R_3\rangle)$$

$$-2\mu_0\beta|m|(\mathrm{e}^{\mathrm{i}\phi}\langle R_{\boldsymbol{q}}^+ R_{\boldsymbol{q}'}^+\rangle + \mathrm{e}^{-\mathrm{i}\phi}\langle R_{\boldsymbol{q}}^- R_{\boldsymbol{q}'}^-\rangle) + 8\mu_q\beta n_q\langle R_3\rangle^2, \quad (7)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_{\boldsymbol{q}}^{\pm} R_{\boldsymbol{q}'}^{\pm}\rangle = -2\beta(1 - 2\mu_q\langle R_3\rangle + 2\mu_0 n)\langle R_{\boldsymbol{q}}^{\pm} R_{\boldsymbol{q}'}^{\pm}\rangle$$

$$-2\mu_0\beta|m|\mathrm{e}^{\mp\mathrm{i}\phi}(\langle P_{\boldsymbol{q}}\rangle + \langle P_{\boldsymbol{q}'}\rangle) + 8\mu_q\beta|m|_q\mathrm{e}^{\mp\mathrm{i}\phi}\langle R_3\rangle^2, \{w$$

where

$$P_{\boldsymbol{q}} = \sum_{j\neq j'} b_j^{\dagger} b_{j'} \exp\left[i\boldsymbol{q}(\boldsymbol{r}_j - \boldsymbol{r}_{j'})\right] = \frac{R_{\boldsymbol{q}}^+ R_{\boldsymbol{q}}^- + R_{\boldsymbol{q}}^- R_{\boldsymbol{q}}^+ - N}{2}$$

is the operator corresponding to the intensity of the coherent emission component along the vector $\boldsymbol{q}$;

$$\mu_0 = \beta^{-1}\int \beta_{\boldsymbol{k}} \mathrm{d}\Omega_{\boldsymbol{k}} \approx \alpha^2/4\pi,$$

$$\mu_q = \beta^{-1}\int \beta_{\boldsymbol{k}} \Gamma(\boldsymbol{k} - \boldsymbol{q}) \mathrm{d}\Omega_{\boldsymbol{k}} \approx \alpha_{\mathrm{d}}^2/4\pi$$

are the geometrical parameters characterising the divergence of the squeezed light and medium emission beams along the vector $\boldsymbol{q}$. The values of n_q and m_q differ from zero (and equal to n and m, respectively) only when q corresponds to a squeezed mode. Eqn (7) was derived by using a standard unchanging of correlators $\langle b_{3j} b_{j'} b_{j''}^{\dagger}\rangle = \langle b_{3j}\rangle\langle b_{j'} b_{j''}^{\dagger}\rangle$, $\langle b_{3j} b_{3j'}\rangle = \langle b_{3j}\rangle\langle b_{3j'}\rangle$ $(j \neq j' \neq j'',\ j'' \neq j)$.

Let us introduce the mode correlation function

$$Q_{\boldsymbol{qq}'} = R_{\boldsymbol{q}}^+ R_{\boldsymbol{q}'}^+ \exp(\mathrm{i}\phi) + R_{\boldsymbol{q}}^- R_{\boldsymbol{q}'}^- \exp(-\mathrm{i}\phi) \quad (8)$$

and pass to new variables

$$X_{\boldsymbol{qq}'}^{\pm} = \frac{P_{\boldsymbol{q}} + P_{\boldsymbol{q}'} \pm Q_{\boldsymbol{qq}'}}{4}, \quad (9)$$

which determine the dispersion of the in-phase and quadrature components of the emission field. Then, the system of Eqns (7) can be written in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_3\rangle = -2\beta\left(\frac{N}{2} + \langle R_3\rangle + \sum_{\boldsymbol{q}} \mu_q\langle X_{\boldsymbol{qq}'}^+ + X_{\boldsymbol{qq}'}^-\rangle\right) - 4\mu_0\beta n\langle R_3\rangle,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle X_{\boldsymbol{qq}'}^{\pm}\rangle = -4\beta\gamma_0^{\pm}\langle X_{\boldsymbol{qq}'}^{\pm}\rangle + 4\mu_q\beta\left(\langle X_{\boldsymbol{qq}'}^{\pm}\rangle + \gamma_q^{\pm}\langle R_3\rangle + \frac{N}{4}\right)\langle R_3\rangle, \quad (10)$$

where $\gamma_0^{\pm} = 1/2 + \mu_0(n \pm |m|)$; and $\gamma_q^{\pm} = 1/2 + n_q \pm |m|_q$.

It is convenient to introduce two quantities

$$D_{\mathrm{f}} = \frac{\gamma_q^+}{\gamma_q^-} \text{ and } D_{\mathrm{m}} = \frac{\langle X_{\boldsymbol{qq}'}^+\rangle + N/4}{\langle X_{\boldsymbol{qq}'}^-\rangle + N/4}, \quad (11)$$

which characterise the degree of squeezing (the ratio of dispersions of quadrature components) of the field and medium, respectively. One can see from the second equation of system (10) that the growth rates of the quadrature components $\langle X_{\boldsymbol{qq}'}^{\pm}\rangle$ substantially differ from each other for large values of D_{f} and D_{m}. Therefore, the degree of squeezing of the triggered superradiance signal depends on the degrees of squeezing of the triggering pulse and the amplifying medium.

3. Basic results

Let us assume that an inverted resonance medium ($\langle R_3\rangle = N/2$, $\langle P_{\boldsymbol{q}}\rangle = \langle P_{\boldsymbol{q}'}\rangle = 0$) is located in a weak field of squeezed light ($n \ll N$). Then, a superradiance pulse is emitted in the direction of polarisation waves with the wave vectors $\boldsymbol{q} = \boldsymbol{k}$ and $\boldsymbol{q}' = \boldsymbol{k}'$. By using the law of conservation of the square of the collective Bloch vector length

$$\langle R_3\rangle^2 + \sum_{\boldsymbol{q}}\langle P_{\boldsymbol{q}}\rangle = \frac{N^2}{4},$$

we find that the total intensity of the superradiance pulse in the approximation $N\mu_q \gg 1$, $N \gg n$ is described in a usual way

$$\langle X_{\boldsymbol{qq}'}^+\rangle + \langle X_{\boldsymbol{qq}'}^-\rangle = \frac{\langle P_{\boldsymbol{q}}\rangle + \langle P_{\boldsymbol{q}'}\rangle}{2} = \frac{N^2}{4}\,\mathrm{sech}^2\left(\frac{t - t_{\mathrm{del}}}{2\tau_{\mathrm{c}}}\right), \quad (12)$$

where $t_{\mathrm{del}} = \tau_{\mathrm{c}}\ln(N\mu_q)$ is the delay time of the superradiance pulse with respect to the pump pulse and $\tau_{\mathrm{c}} = (2\beta N\mu_q)^{-1}$ is the self-induced correlation time of the medium. The fact that the triggering pulse does not contain the coherent component leads to two substantial differences of the triggered superradiance regime under study from a usual regime when the emission field of the triggering pulse is in a coherent state.

First, now the delay time of the superradiance pulse only weakly depends on the triggering-pulse intensity (exact dependences of τ_{c} and t_{del} on the number n of photons are reported in [18]). Second, the mean value of the dipole moment of the medium remains zero during the entire superradiance process. In this sense, the triggered superradiance under study can be called triggered superfluorescence.

Consider now the relation between quadrature components in the superradiance pulse. At the linear stage of the evolution, when $t \ll t_{\rm del}$ and $|\langle R_3(t)\rangle - \langle R_3(0)\rangle| \ll |\langle R_3(t)\rangle|$, it follows from the second equation that the ratio of the growth rate $\langle X^+_{qq'}\rangle$ of the in-phase component to that $\langle X^-_{qq'}\rangle$ of the quadrature component is

$$D_{\rm sr} = \frac{(N + \langle Q_{qq'}\rangle)/4 + \gamma_q^+\langle R_3\rangle}{(N - \langle Q_{qq'}\rangle)/4 + \gamma_q^-\langle R_3\rangle}. \quad (13)$$

As a result, the excited medium emits a superradiance pulse with the dispersions of quadrature components differing from each other by a factor of $D_{\rm sr}$. The value of $D_{\rm sr}$ substantially depends on whether the medium is in a squeezed state or not. If the medium is initially completely inverted ($\langle R_3\rangle = N/2$, $\langle Q_{qq'}\rangle = 0$ and $\langle P_q\rangle = 0$), then $D_{\rm sr} = (1+2\gamma_q^+)/(1+2\gamma_q^-) \approx 2\sqrt{D_{\rm f}}$. Therefore, the degree of squeezing of a superradiance pulse emitted by a resonance medium is equal only to the square root of the degree of squeezing of the incident field.

Let us assume now that a medium is initially in a squeezed state, i.e., the dispersions of its quadrature polarisation components are not identical. According to [19], we write

$$\langle Q_{qq'}\rangle = N\sin\theta, \quad \langle R_3\rangle = -\frac{N}{2}\cos\theta, \quad \langle P_q\rangle = 0, \quad (14)$$

where the angle θ is close to $\pi/2$ in the case of strong squeezing. Then, we can write $D_{\rm m} = \cot^2(\eta/2)$, where $\eta = 2\theta - \pi/2$, and

$$D_{\rm sr} = \frac{\gamma_q^-\cos(\eta/2) + 2\sin(\eta/2)}{\sin(\eta/2) + 2\gamma_q^-\cos(\eta/2)}\left(D_{\rm f}D_{\rm m}\right)^{1/2} \sim \left(D_{\rm f}D_{\rm m}\right)^{1/2}. \quad (15)$$

Therefore, if the degree of squeezing of a medium is of the same order of magnitude as that of a triggering pulse, then $D_{\rm sr} \sim D_{\rm f}$, and squeezed light can be amplified in the superradiance regime without decreasing the degree of its squeezing. If the degree of squeezing of the medium is larger than that of the triggering pulse, then squeezed light is not only amplified but the degree of its squeezing also increases. For sufficiently large values of $D_{\rm m}$ (close to the value of $N\mu_q$), we can obtain $\langle X^-_{qq'}\rangle + N/4 < N/4$ at the output, i.e., the intensity of the squeezed quadrature of the superradiance signal proves to be smaller than that of the incoherent spontaneous background. Therefore, the superradiance signal can be characterised not only by classical squeezing (when the dispersions of quadratures are not identical) but also by quantum squeezing (when the dispersion of one quadrature is smaller than the vacuum value).

4. Conclusions

We have shown in this paper that squeezed light pulses can be amplified in the regime of triggered optical superradiance, the degree of superradiance field squeezing being substantially dependent on the degree of squeezing of both the triggering-pulse field and polarisation of the amplifying medium. To amplify squeezed light in the optical superradiance regime without decreasing the degree of squeezing, it is necessary to prepare an active medium in a squeezed state. Unlike the generation of squeezed light, the preparation of a squeezed medium is a nontrivial task (see papers [19–23]). In the general case, the preparation of a squeezed state of a two-level atomic system can be described by the Hamiltonian of the type $H = {\rm i}(g^*R^-R^- - gR^+R^+)$. As shown in paper [19] for a two-atomic case, the interaction described by such a Hamiltonian results in a periodic squeezing of the medium. This means that the dispersion of one of the quadrature components of macroscopic polarisation tends to zero at certain instants of time. It is in this state that the inverted medium ($\langle R_3\rangle > 0$) is capable of amplifying the trigger pulse without decreasing the degree of squeezing. Although the above Hamiltonian looks like the Hamiltonian describing parametric amplification, its physical realisation involves great problems.

We assume that this Hamiltonian can describe the situation when a system of two-level atoms interacts with the pump field whose frequency is twice as a large as the optical transition frequency of atoms. In this case, the interaction should be cooperative because one photon with a double frequency can be absorbed only by two atoms. By switching off the pump field at the instants of a maximum squeezing and simultaneously applying the triggering pulse, we can achieve a substantial squeezing of the triggered superradiance pulse.

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