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Periodic autowave structures in a wide-aperture laser with inertial phase nonlinearity

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***Abstract*. The formation of a periodic structure of the optical field of a wide-aperture laser with inertial Kerr phase nonlinearity is considered. The conditions for the appearance of one-dimensional periodic wave structures propagating along the aperture are found and their properties are studied.**

Keywords: *wide-aperture laser, phase nonlinearity, Andronov – Hopf bifurcation.*

The one-dimensional transverse structure of the optical field of a wide-aperture laser with an inertial saturating filter was studied in papers [1, 2]. It was shown that, under certain conditions, the additional amplitude nonlinearity of such a laser caused by the presence of the intracavity saturating filter results in the violation of stability of stationary lasing (the appearance of the Andronov – Hopf bifurcation) and in the appearance of quasi-periodic autowaves propagating across the aperture. It was shown in papers [3 – 5] that the autowave profile of the optical field can also appear due to the phase nonlinearity of an active medium caused by the laser frequency detuning. In this paper, we studied the transverse space – time one-dimensional structure of a wide-aperture laser with inertial Kerr phase nonlinearity. Such nonlinearity is typical for semiconductor and solid-state lasers and can also be observed in gas lasers with an intracavity phase filter. We found the conditions for the appearance of an autowave profile, which is similar to that described in papers [3 – 5].

1. Linear analysis of stability

The initial system of equations describing the dynamics of a laser with inertial Kerr phase nonlinearity in the homogeneous field approximation has the form

$$\frac{\partial E}{\partial t} - \mathrm{i}\frac{\partial^2 E}{\partial x^2} = \frac{\nu}{2}(N-1)E + \mathrm{i}\nu(\Phi + \Delta_{\mathrm{cav}})E, \quad (1)$$

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$$\frac{\partial N}{\partial t} = N_{\mathrm{e}} - N(1+I), \quad (2)$$

$$\tau_{\mathrm{ph}}\frac{\partial \Phi}{\partial t} = -\Phi + \alpha I. \quad (3)$$

Here, the dimensionless time t and the transverse coordinate x are related with dimensional quantities t_{d} and x_{d} by expressions $t = t_{\mathrm{d}}/T_{\mathrm{i}}$ and $x = x_{\mathrm{d}}(2k/T_{\mathrm{i}}c)^{1/2}$, where k is the wave number; c is the speed of light; T_{i} is the population relaxation time for levels of an active medium; $E = E_{\mathrm{d}}/E_{\mathrm{s}}$; $I = I_{\mathrm{d}}/I_{\mathrm{s}}$ are the dimensionless amplitude and intensity of the laser field, respectively; E_{s} and I_{s} are the saturation amplitude and intensity, respectively; $N = g/g_{\mathrm{t}}$; $N_{\mathrm{e}} = g_{\mathrm{e}}/g_{\mathrm{t}}$; g, g_{e}, and g_{t} are the gain, the unsaturated gain, and the threshold gain averaged over the cavity length, respectively; $\nu = cT_{\mathrm{i}}g_{\mathrm{t}}$ is the coefficient determining the ratio of the population relaxation time to the photon lifetime in the cavity; Φ is the phase incursion of the optical field per unit length normalised to the threshold gain g_{t}; α is the dimensionless coefficient of phase nonlinearity (of any sign); τ_{ph} is the ratio of the relaxation time of the phase incursion to the population relaxation time T_{i}; and $\Delta_{\mathrm{cav}} = (\omega - \omega_{\mathrm{cav}})/cg_{\mathrm{t}}$ is the dimensionless detuning of the laser frequency ω from a mode of an empty cavity with frequency ω_{cav}.

Equation (1) for a slowly varying amplitude E can be obtained by averaging a quasi-optical equation in the longitudinal direction z assuming that the field weakly changes during the round-trip transit time $\tau = L/c$ for radiation in the cavity, where L is the round-trip transit length of the cavity [6]. Equation (2) describes a state of the active medium in the two-level approximation.

The system of equations (1) – (3) has two homogeneous equilibrium states. The first state corresponds to the absence of lasing ($E = 0$, $N = N_0$, $\Phi = 0$). The second equilibrium state ($E = E_{\mathrm{c}}$, $N = N_{\mathrm{c}} = 1$, $\Phi = \Phi_{\mathrm{c}} = \alpha I_{\mathrm{c}}$, $\Delta_{\mathrm{cav}} = -\alpha I_{\mathrm{c}}$) corresponds to stationary lasing with the intensity $I_{\mathrm{c}} = |E_{\mathrm{c}}|^2 = N_{\mathrm{e}} - 1$.

To study the stability of stationary lasing, we will seek solutions of system (1) – (3) in the form $E = E_{\mathrm{c}}(1+e)$, $N = N_{\mathrm{c}}(1+n)$, $\Phi = \Phi_{\mathrm{c}}(1+\varphi)$, where e, n, and φ are small perturbations of the corresponding stationary quantities proportional to $\exp(\lambda t - \mathrm{i}qx)$. By substituting these solutions into the initial system, we obtain the dispersion equation

$$b_4\lambda^4 + b_3\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0 = 0, \quad (4)$$

where

$$b_0 = q^4(1+I_c) - 2q^2 v\alpha I_c(1+I_c);$$

$$b_1 = \tau_{ph} q^4(1+I_c) + q^4 - 2q^2 v\alpha I_c + vI_c;$$

$$b_2 = \tau_{ph} q^4 + (1+I_c) + \tau_{ph} vI_c;$$

$$b_3 = 1 + \tau_{ph}(1+I_c);$$

$$b_4 = \tau_{ph}.$$

Figs 1 and 2 show the solutions of equation (4) obtained for the parameters of the medium $N_e = 9.5$ and $v = 8.4$ coinciding with those used in papers [4, 5]. The root λ_3 in Fig. 1 is a real quantity, while the roots λ_1 and λ_2 are complex quantities (only real parts of these roots are shown). In the case of inertialless phase nonlinearity ($\tau_{ph} = 0$), stationary lasing is unstable only for $\alpha > 0$, $q < q_1 = (v\alpha I_c)^{1/2}$ (q is the dimensionless wave number), when the real root of (4) is positive. This is the known instability of the transverse structure of the field in a medium where the refractive index increases with increasing intensity (self-focusing), resulting in the disintegration of a laser beam into separate filaments [7–10].

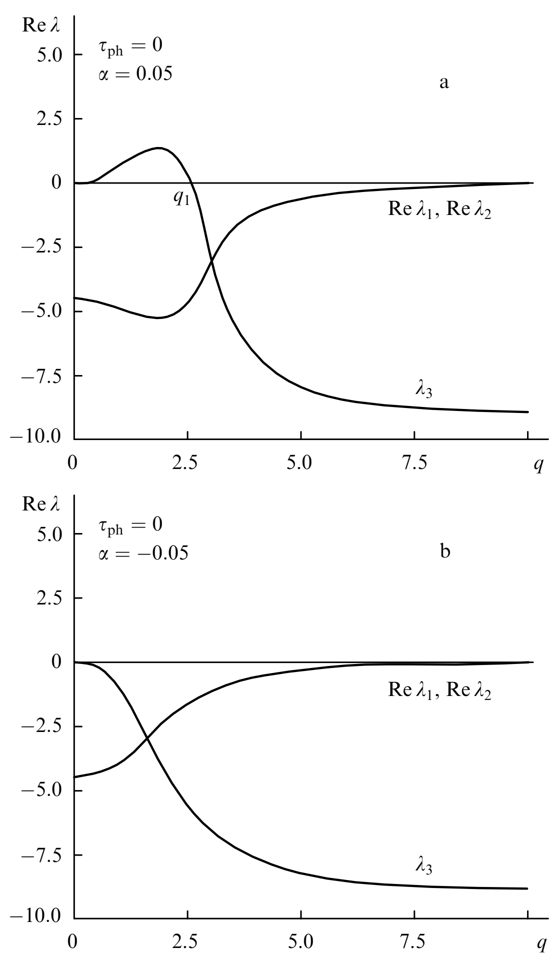

Figure 1. Real parts of the roots of a characteristic equation for $\tau_{ph} = 0$, $\alpha = 0.05$ (a) and $\tau_{ph} = 0$, $\alpha = -0.05$ (b).

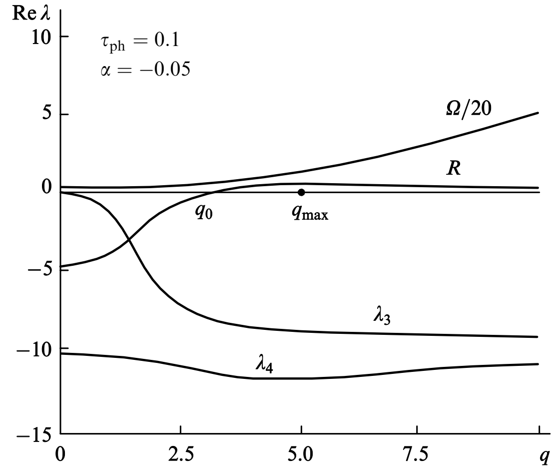

Figure 2. Real parts of the roots of a characteristic equation for $\tau_{ph} = 0.1$ and $\alpha = -0.05$.

The inertia of the phase nonlinearity only reduces the instability increment, while the instability region for q remains invariable.

Another type of instability is observed in a medium with the coefficient $\alpha < 0$ (defocusing nonlinearity). In this case, for $\tau_{ph} \neq 0$, there exists a region of the wave vectors $q > q_0$ (q_0 is the bifurcation value depending on the parameters of a laser) for which two real roots λ_3 and λ_4 of (4) are negative, while the real part R of the complex conjugate roots $\lambda_{1,2} = R \pm i\Omega$ is positive (Fig. 2). This corresponds to the Andronov–Hopf instability with respect to plane waves with frequency Ω propagating across the aperture at the velocity $W = \Omega/q$ [3, 11, 12]. The dependence of the maximum increment R_{max} on α and τ_{ph} for $v = 8.4$ and $N_e = 9.5$ is shown in Fig. 3. The analysis of this dependence shows that for each value of the phase nonlinearity α, the increment first increases from zero to its maximum value with increasing inertia of the phase filter, the maximum value being greater for a greater phase nonlinearity α. As the inertia of the phase filter is further increased, the increment gradually decreases. The gain R_{max} decreases with decreasing inertia v of the active medium and with decreasing the unsaturated gain N_e.

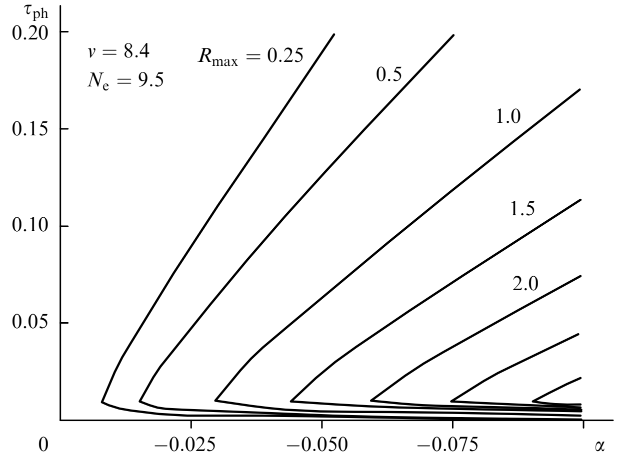

Figure 3. Dependence of the increment R_{max} on the inertia τ_{ph} of the phase filter and the coefficient of phase nonlinearity α.

2. Structure of the laser field in a Fabry–Perot resonator

The analysis of stability of stationary lasing performed above neglected the finite aperture of a real laser and diffraction from mirror edges. We also did not consider the nonlinear perturbation dynamics described by the system of equations (1)–(3). We considered the effect of these factors on the transverse structure of the optical field by using the model of a laser system proposed in papers [2, 13]. The phase nonlinearity was simulated by introducing an intracavity phase filter. The active medium and phase filter were taken into account by introducing infinitely thin filters (*3*) and (*4*) placed near a semiconductor mirror (*1*) (Fig. 4). We assumed that mirror (*2*) is totally reflecting and infinite. The edges of the first mirror were smoothed, so that the profile of the reflection coefficient in regions $-a \leqslant x_d \leqslant -a+2b$ and $a-2b \leqslant x_d \leqslant a$ depended on the transverse coordinate:

$$r(x_d) = \frac{r_0}{2}\left[1 - \sin\left(\pi\frac{|x_d| - \bar{x}}{2b}\right)\right],$$

where $\bar{x} = a - b$; $2a$ is the width of mirror (*1*); $2b$ is the width of the smoothed band; and r_0 is the reflection coefficient of mirror (*1*) outside the smoothed band.

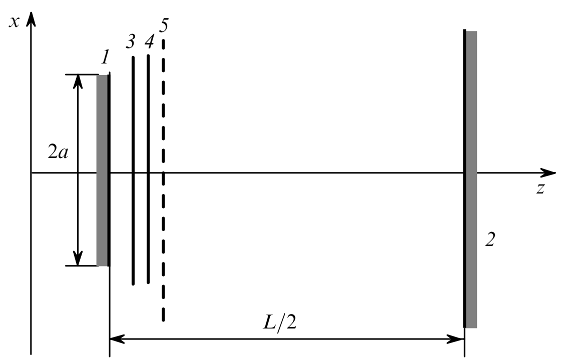

Figure 4. Scheme of a laser used in calculations.

We used the integrated form of the equation for the field:

$$E(x_2) = \frac{\exp(ikL)}{(i\bar{\lambda}L)^{1/2}}\int_{-a}^{a} E(x_1)\exp\left[\frac{ik(x_2 - x_1)^2}{2L} + \frac{(g - g_t)L}{2}\right]dx_1, \qquad (5)$$

where $E(x_1)$ is the initial field in plane (*5*) (Fig. 4); $E(x_2)$ is the field in plane (*5*) after the round trip in the cavity; L is the round-trip length in the cavity; and $\bar{\lambda}$ is the wavelength of light. We calculated a change in the field after the round trip in the cavity from expression (5) and then calculated the effect of the active-medium layers and a phase filter on the field. In this way, we found a change in the field after the round trip in the cavity. Then, we integrated kinetic equation (2) and relaxation equation (3). This iteration procedure gave the states of the medium and field at the time instants multiple to the round-trip transit time in the cavity.

The transverse structure of the field was calculated by varying the parameters α, τ_{ph}, N_e, and v and the Fresnel number $N_F = 2a^2/\bar{\lambda}L$. The loss coefficient depended on the reflection coefficient r_0 and was not varied in calculations ($G_t \equiv Lg_t = 0.21$). Figs 5–9 show the space–time patterns

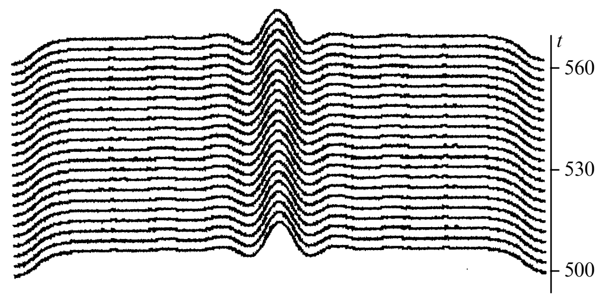

Figure 5. Structure of the optical field for $\tau_{ph} = 0.1$, $\alpha = -0.0045$, $N_F = 300$, $v = 8.4$, and $N_e = 9.5$.

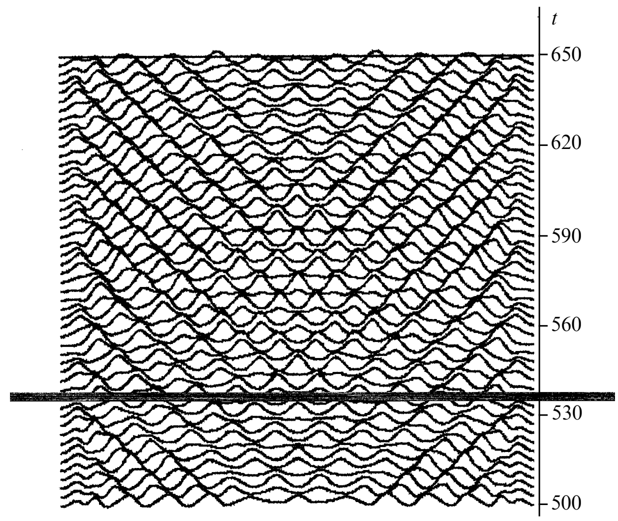

Figure 6. Structure of the optical field for $\tau_{ph} = 0.1$, $\alpha = -0.023$, $N_F = 300$, $v = 8.4$, and $N_e = 9.5$.

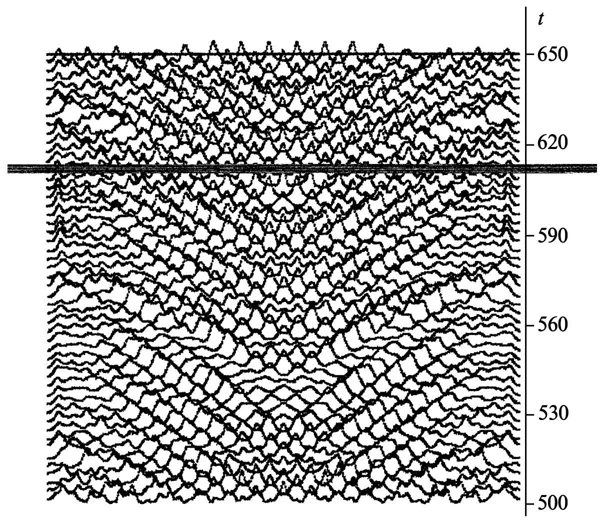

Figure 7. Structure of the optical field for $\tau_{ph} = 0.1$, $\alpha = -0.056$, $N_F = 300$, $v = 8.4$, and $N_e = 9.5$.

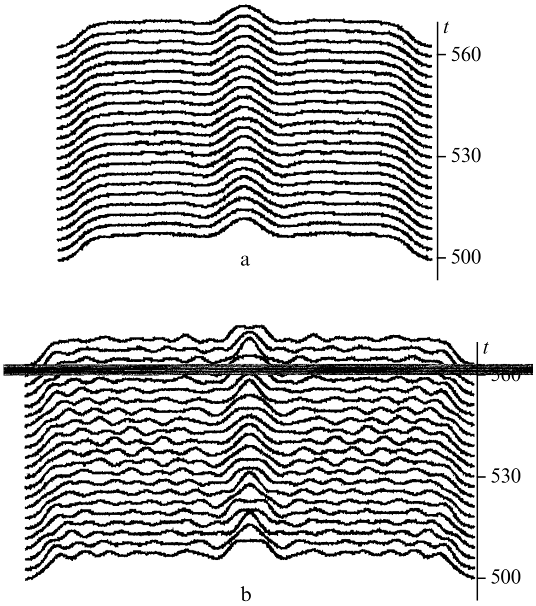

Figure 8. Structure of the optical field for $\tau_{ph} = 0.1$, $\alpha = -0.011$, $N_F = 150$, $v = 8.4$, and $N_e = 9.5$ (a) and for $\tau_{ph} = 0.1$, $\alpha = -0.011$, $N_F = 300$, $v = 8.4$, and $N_e = 9.5$ (b).

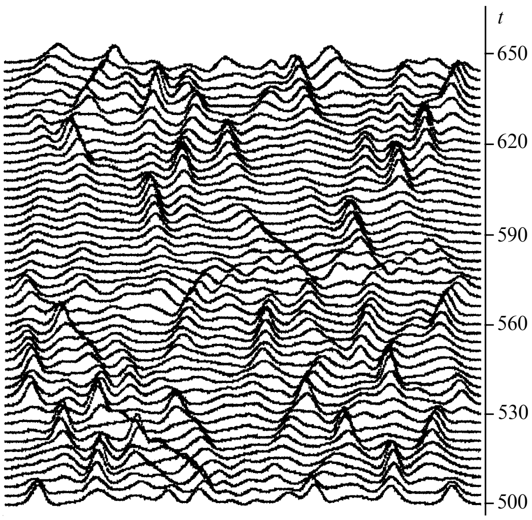

Figure 9. Structure of the optical field for $\tau_{ph} = 0.1$, $\alpha = 0.034$, $N_F = 300$, $v = 8.4$, and $N_e = 9.5$.

of the optical field along the x axis in the case of stationary lasing ($t_d/\tau \geqslant 500$, the dimensionless time is shown on the right in figures).

When the coefficient α was negative, the optical-field structure changed successively, as was earlier observed in papers [4, 5] in the case of a negative frequency detuning: a bright spot ('mound') was observed at the centre of the homogeneous stationary field (Fig. 5), as well as quasi-sinusoidal waves propagating in both directions over the aperture (Fig. 6) and irregular and strongly modulated spatial structures (Fig. 7). Similar space – time patterns of the optical field were observed for different values of N_e, v, and N_F, the appearance of the autowave transverse structure being caused by the increase in the gain R_{max} of the waves (Fig. 3).

The above change of the structures occurs for each set of the parameters N_e, v, and N_F with varying α ($\alpha < 0$) in the following way. For $|\alpha| \to 0$, this is the intensity 'mound' located at the aperture centre (Fig. 5). Its presence does not depend on the inertia of the phase nonlinearity. The stationary field with a bright spot at its centre is an eigenmode of a plane cavity with a large Fresnel number, which is filled with an active medium [2]. An increase in $|\alpha|$, i.e., in the degree of nonlinear defocusing prevents the formation of a diffraction pattern with the intensity 'mound'. In the case of large $|\alpha|$ and the values of τ_{ph} for which $R_{max} > 0.17$, the field profile is strongly modulated by travelling quasi-sinusoidal waves (Fig. 6). A further increase in $|\alpha|$ weakly affects the amplitude of the autowaves, but their space – time structure becomes irregular (Fig. 7).

As the inertia of the phase filter increases (and R_{max} decreases), the amplitude of these waves decreases. In a cavity with a large Fresnel number, the autowave structures can be observed at lower values of R_{max} due to an increase in the aperture width and, hence, in the integrated gain aR_{max} of the autowaves (Fig. 8).

In the case of focusing phase nonlinearity ($\alpha > 0$), no periodic autowave pattern was observed. For small values of α, a pattern with a 'mound' was retained. As α was increased, bright spots appeared and disappeared in different regions of the aperture (Fig. 9). Such field structures were obtained earlier in the case of a positive frequency detuning [4]. They correspond to the appearance and breaking of filaments in a self-focusing active medium [8 – 10].

As a whole, we can conclude that the focusing phase nonlinearity of any origin can cause the filamentation of the transverse structure of a laser field (such studies were performed in many papers [8 – 10]), while the defocusing phase nonlinearity can produce a periodic autowave structure of the field. It is possible that a combination of these types of nonlinearity will permit the control of the laser-field structure, in particular, the suppression of a small-scale self-focusing.

Let us now estimate the Kerr coefficient $|n_2| \approx |\alpha| g_t / I_s k$ for the parameters of the laser system used in this paper. This coefficient characterises a nonlinear addition to the real part of the refractive index ($n = n_0 + n_2 I_d$, where n_0 is the linear refractive index). For $|\alpha| = 0.05$, $g_t = 10^{-3}$ cm^{-1}, $I_s = 1$ kW cm^{-2}, and $k = 6 \times 10^4$ cm^{-1}, we obtain $|n_2| \approx 10^{-9}$ cm^2 kW^{-1}, i.e., the value that is typical for the problems under study [10, 14]. Examples of semiconductor materials with $n_2 < 0$ are presented in papers [15, 16].

3. Conclusions

The main results of the study of the space – time structure of the laser field are as follows.

(i) We have found the regions of the parameters of a phase nonlinearity (α, τ_{ph}) and an active medium (v, N_e) in which a homogeneous stationary lasing becomes unstable with respect to plane waves with the wave number $q > q_0$. We determined the dependence of the instability increment on the parameters of the laser system and showed that this type of instability can be observed only when $\alpha < 0$ and $\tau_{ph} \neq 0$.

(ii) We simulated numerically the distributed model of a laser system in a Fabry – Perot resonator. The dependence of this system on its parameters is in agreement with the results of a linear analysis of the system stability. As negative values of α increase and τ_{ph} increases, quasi-sinusoidal waves begin to propagate over the aperture. A further increase in α (for the same values of τ_{ph}) results in large instability increments R_{max} and a strongly irregular transverse structure of the optical field. As the inertia of the phase nonlinearity increases, the increment first increases and then decreases. The amplitude of the quasi-sinusoidal profile changes similarly. As the Fresnel number increases, the region of optimal parameters (for observation of a periodic profile) is displaced to lower increments R_{max}.

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