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Whispering-gallery waves in optical fibres

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***Abstract*. The process of excitation of whispering-gallery waves (WGWs) in optical fibres (microcavities) with the help of a bitapered fibre is analysed. It is shown that useful information on the WGW modes can be obtained from the spectrograms recorded by scanning the exciting-radiation frequency. Based on the geometrical-optic approximation, the longitudinal sizes of the WGW modes are estimated and it is shown that the ultimate diameter of the fibre exists for optical fibres (microcavities) where a mode can be still excited with the help of a bitapered fibre.**

Keywords: *whispering-gallery modes, optical fibres, microcavities.*

1. Introduction

Interest in the electromagnetic whispering-gallery waves (WGWs) has quickened in recent years. This is explained by several reasons, of which the main one is the development of the technology of high-quality optical fibres of different diameters for various applications. This technology provided the fabrication of spherical microcavities with a quality $\sim 10^9$ made of ultra-pure silica, which allows the use of quartz microspheres (of diameter ~ 300 μm) in narrow-band lasers [1], wavelength-division multiplexers [2], sensor devices [3], and some other fields. The author of paper [4] proposed to use WGWs excited in tapered quartz rods for the shift of the frequency of light with the help of a sound wave propagating along the rod axis, the frequency shift being substantially greater than that commonly achieved using the Bragg diffraction [5]. The experimental realisation of such an acousto-optic cell requires the understanding of all the details of the interaction process.

We have shown [6] that the acousto-optic interaction is essentially a waveguide process. We determined all the conditions at which a wavelength can be changed, and proposed, in particular, to use a tapered quarts rod of a small diameter as an acousto-optic cell. This poses the problem of excitation of WGWs in rods of different diameters.

At present, there exist two methods for exciting WGWs: a prism method and a fibre method in which a piece of a fibre with diameter gradually decreasing from 125 to 2 μm is used. The latter method is quite attractive for experiments on the acousto-optic interaction. For this reason, the aim of this paper is to study in detail excitation and propagation of WGWs in a quartz rod – coupling device system.

2. Analysis of excitation of WGWs in the geometrical-optic approximation

It is known that the field of a wave propagating in an optical fibre of small diameter (less than 7 – 10 μm) emerges outside, which allows one to couple with the WGW field in a quartz rod. A WGW is characterised by the propagation constant $\beta = n^* k$, where n^* is the effective refractive index and $k = 2\pi/\lambda$. An optical wave in a tapered fibre is also characterised by the constant $\bar{\beta}$, which is described by the expression [7]:

$$\bar{\beta}^2 = k^2 n^2 - (2.405)^2/\rho^2, \tag{1}$$

where n is the refractive index of the fibre and ρ is the radius of the tapered fibre in the excitation region.

When $\beta = \bar{\beta}$, a WGW is excited in the rod, whose different modes can be maintained in optical fibres. First of all, these modes are characterised by the radial field distribution. An example of such a distribution for WGWs in a rod [8] is shown in Fig. 1. The WGW mode with one maximum of the field near the interface is the fundamental mode. Modes with two, three, and more maxima are called the higher order modes. The higher the mode order m, the smaller its propagation constant β.

Fig. 2 shows the scheme of excitation of WGW modes with the help of a tapered fibre. Usually, a tunable narrow-band radiation source is placed at the fibre input and a detector at the fibre output. Sometimes, a broadband

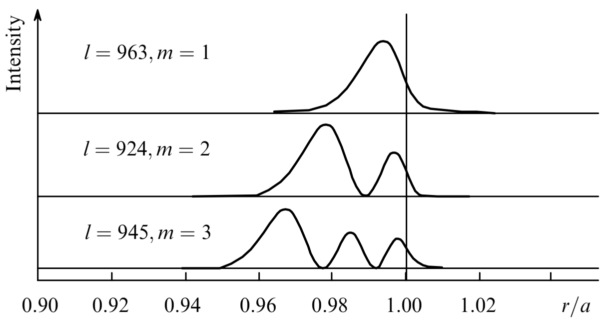

Figure 1. Radial distributions of the field of the WGW modes of the first, second, and third orders in a standard silica fibre of diameter $2r = 125$ μm immersed in ethanol [8] (l is an integer, m is the mode order).

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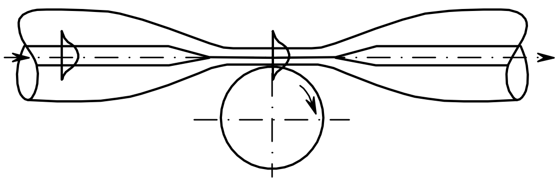

Figure 2. Scheme of excitation of the WGW modes in a cylindrical quartz rod (microcavity) with the help of a tapered part of a fibre (bitapered fibre).

radiation source is placed at the fibre input and a spectrum analyser at the fibre output. To excite WGW modes of different orders, it is necessary to obtain the equality $\beta_m = \bar{\beta}_\rho$ by moving along the tapered fibre.

A typical signal at the fibre output is shown in Fig. 3 [9]. Note that Fig. 3 gives interesting information on the WGW modes excited in the rod. First of all, a mode is excited when the wavelength of the source satisfies the condition

$$\lambda = \frac{2\pi r n^*}{l}, \tag{2}$$

where l is an integer and r is the radius of a cylindrical rode. Condition (2) means that the phase shift $\Phi = 2\pi r\beta$ of the WGW after the round trip in the microcavity should be multiple of 2π, i.e., $2\pi r\beta = 2\pi rkn^* = 2\pi l$. The number l can be expressed in terms of the distance $\Delta\lambda$ between azimuthal WGW modes in the rod to obtain

$$n^* = \frac{\lambda^2}{2\pi r\Delta\lambda}. \tag{3}$$

Thus, we can determine n^* for the WGW mode from Fig. 3. Assuming that this mode is established in a cylindrical rod after an integer number of total internal reflections of light, we can represent n^* in the form

$$n^* = n\sin\theta, \tag{4}$$

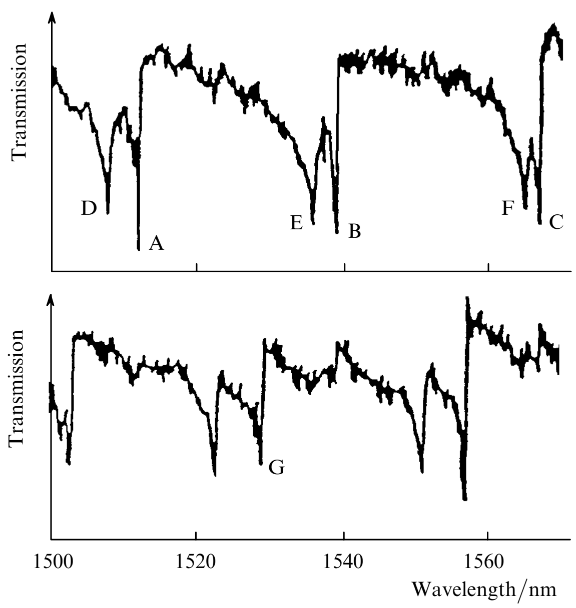

Figure 3. Spectral dependence of the radiation intensity transmitted through a bitapered fibre in contact with a microcavity.

where θ is the angle of incidence of the WGW on the cylinder surface. The number of reflections of the WGW from the cylinder surface is

$$N = \frac{180^\circ}{90^\circ - \theta^\circ}. \tag{5}$$

By using this (geometrical-optic) approach and knowing N and the radius r of a cylindrical rode, we can determine the penetration depth h of the mode under the rod surface:

$$h \approx \frac{r\pi^2}{2N^2}. \tag{6}$$

The value of n^* also can be found from N:

$$n^* = n\left(1 - \frac{\pi^2}{2N^2}\right). \tag{7}$$

Expressions (5)–(7) are valid for large N, i.e., for large radii r.

The trajectory of a WGW in a cylindrical dielectric rod in the geometrical-optic approximation represents a broken line, the number of its breaks being equal to the number N of reflections of the wave from the cylinder surface per total loop of the trajectory. How is the number N of these reflections of a mode of some order determined? It is obvious, first of all, that N satisfies the relation $N \leqslant l = 2\pi rn/\lambda$. For a quartz rod of diameter 125 μm and $\lambda = 1.55$ μm, we obtain $N \leqslant 366$. A mode with such a number of reflections would have a small localisation depth near the cylinder surface. For this reason, it could not be excited because of strong scattering of light from this surface. The scattering losses can be reduced by increasing the localisation depth of the mode; however, in this case, volume absorption losses will increase, if they are present, or volume scattering losses, which are always present. In addition, as h increases, the radiative losses of light in the mode also increase because the angle of incidence of light on the cylinder surface approaches its critical value.

The losses considered above are intrinsic losses of a mode of a dielectric resonator such as a microsphere or a cylindrical rod. These losses are usually characterised by the parameter $\alpha = \Delta A/A$, where A is the amplitude of a WGW and ΔA is the amplitude decrease per loop of the WGW trajectory. Excitation of the mode is also accompanied by losses, without which, however, modes cannot exist. A device for excitation of modes – a prism or a tapered fibre, which are coupled with a microcavity by tunnelling, are characterised by the coupling parameter χ, which determines the WGW leakage from the cavity to the coupling prism or tapered fibre and, vice versa, the transfer of the wave from the coupling device to the cavity. The optimal value of the coupling parameter χ, at which the WGW amplitude in the microcavity is maximum, is achieved when the equality $\chi = \alpha$ [10] and the phase-matching condition $\bar{\beta} = \beta$ are satisfied.

A WGW mode excited in the cavity is in essence a mode of the cavity with a coupling device. The quality of this mode are characterised by the spectral width $\delta\lambda$ (or $\delta\nu$) of the resonance. If the coupling parameter χ is increased, the resonance width also increases and the amplitude of the excited mode decreases. When χ is decreased (for example, by increasing the gap between the prism and microcavity), the resonance width $\delta\lambda$ decreases and the WGW amplitude sharply decreases.

The coupling parameter χ can be always reduced, but it can be increased only up to a certain limit. This limit restricts the magnitude α of losses in the mode which can be still excited using this coupling device. In some papers, the WGW modes were excited by pumping an active medium, which is a part of the cavity. In this case, the spectrum of modes depends on the position of the active medium in the cavity and on the pumping method. Such excitation was analysed in detail in paper [8]. Therefore, we can say that the spectrum of modes excited in the cavity is determined by their Q factor $Q = \lambda/\delta\lambda$ (or losses α) and the excitation method.

Note also that the presence of radiative losses of the WGW modes, which are determined by the change in the refractive index on the curvilinear interface between the media, allows the excitation of a WGW mode by an external light beam directed along the tangent to the cylindrical surface of a dielectric rod [11]. The excited modes were detected in this paper by recording spectrograms of scattered light. It follows from the spectrograms and relations (2)–(7) that WGW modes with $N = 8, n^* \simeq 1.345$ and $h \simeq 5$ μm were excited in standard quartz fibres of diameter 125 μm.

It was shown in paper [9] that the diameter of a fibre and its variations along the axis can be very precisely measured from the spectrograms of excited modes. It was also shown in [9] that the use of a strongly tapered fibre ($\gamma = 10^{-2}$) allows one to excite modes with different radial orders. Dips A, B, and C in the transmission spectrum in Fig. 3 correspond to excitation of the fundamental WGW mode in a quartz rod of diameter 20 μm, while dips D, E, and F correspond to excitation of a higher radial order mode.

It follows from these spectrograms that for the fundamental mode, $n_1^* = 1.426$, $h_1 = 0.125$ μm, and $N = 20$, while for a high-order radial mode, $n^* = 1.35, h \simeq 0.61$ μm, and $N \simeq 9$. According to expression (7), for $N = 9$, the parameter $n^* = 1.355$, and this value is quite close to that obtained from the spectrogram in Fig. 3. According to our estimates, the high-order mode (dips D, E, and F) is the fifth-order radial mode (the mode order is $m \simeq h/h_1$). The spectrograms in Fig. 3 also exhibit distinct dips corresponding to excitation of even higher-order radial modes (up to the limiting order) with the number of reflections $N = 6$, 5, and 4. These modes have large radiative losses, which probably makes possible to excite them with the help of a tapered fibre (taper).

Let us now estimate the longitudinal size of a mode in a conical rod. It is known that a beam trajectory in the ray approximation representing a mode of the conical rod is a spiral on the cone surface with a cusp of the beam and a point of intersection of two counterpropagating beams located in the region of coupling of light to the cone (Fig. 4).

The distance H from the coupling point of the beam to its cusp is determined by the relation [6]

$$H = r\frac{1 - \sin(\pi/2 - \varphi)}{\sin\gamma} = \frac{r\varphi^2}{2\sin\gamma}, \tag{8}$$

where φ is the angle of coupling of light into a conical rod. The angle of intersection of beams a and b in the region of the mode excitation on the cone surface is 2φ. This angle should be equal to the divergence of light in the mode, which is determined by the mode dimension $\bar{H}$, i.e.,

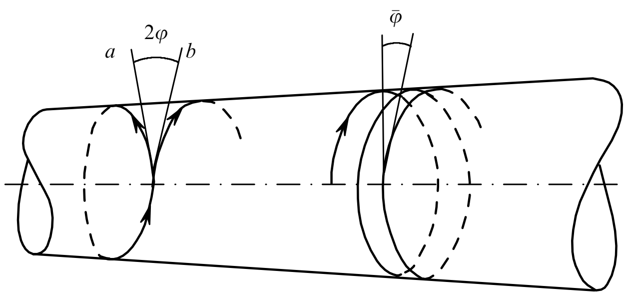

Figure 4. Graphical illustration of the definition of the longitudinal size of modes in a conical fibre microcavity.

$$2\varphi = \frac{\lambda}{n\bar{H}}. \tag{9}$$

Taking into account expressions (8) and (9), and Fig. 4, we have

$$\bar{H} = \frac{1}{2}\left(\frac{2r\lambda^2}{n^2\sin\gamma}\right)^{1/3}. \tag{10}$$

The estimate of the mode size for a conical rod with $r = 10$ μm, $\gamma = 3\times10^{-4}$ and $\lambda = 1.5$ μm gives $\bar{H} \simeq 21.2$ μm. The number w of coils in the spiral can be estimated from the expression

$$w = \frac{\varphi(1+\sqrt{2})}{2\pi\sin\gamma}. \tag{11}$$

For a conical rod of diameter 20 μm, the number of coils in the mode is $w = 31$, the values of w and $\bar{H}$ being independent of the mode order.

The definition of the longitudinal size of a mode introduced above is rather conditional. It is possible to define this size in a different way, for example, assuming that the point of tangency of a tapered fibre and a conical rod is the cusp of the beam representing the mode, while the angle of leakage of light from the mode coincides with the angle $\bar{\varphi} = \lambda/nH^*$. This definition yields the mode size

$$H^* = \frac{1}{2}\left(\frac{4r\lambda^2}{n^2\sin\gamma}\right)^{1/3}, \tag{12}$$

which is larger than the mode size defined earlier only by a factor of 1.26. The WGW mode excited in a conical rod is a leakage mode because the light energy supplied to is leaking from the mode along the cone axis toward its apex. The Q factor of these modes is low, but their longitudinal size can be estimated experimentally.

The conicity of a cylindrical rod is $\gamma = 0$, and it seems that the mode size is infinite. However, this is not the case. The WGW mode size in a cylindrical rod is determined by the diffraction losses of light in the excited beam and by the coupling parameter χ characterising a coupling device. Let us assume that the WGW mode has the longitudinal size W_0 at the end of the excitation region. The divergence angle of light in the mode is $\bar{\varphi} = \lambda/nW_0$, so that the longitudinal size of the mode at the end of its path $L = 2\pi r$ will be $W_1 = 2\pi r\lambda/nW_0 + W_0$. The coefficient of diffraction losses $\Delta A/A$ in this case is $\alpha = (A_0 - A_1)/A_0$, where A_0 and A_1 are the mode amplitudes at the beginning ($L = 0$) and the end ($L = 2\pi r$) of the path, respectively, i.e.,

$$\alpha = 1 - \left(\frac{W_0}{W_1}\right)^{1/2}. \quad (13)$$

As mentioned above, in the case of optimal coupling, $\alpha = \chi$, and the longitudinal size of the mode can be expressed in terms of the coupling parameter χ:

$$W_0 = \left(\frac{\pi r \lambda}{n}\right)^{1/2} (1-\chi)\left(\chi - \frac{\chi^2}{2}\right)^{-1/2}. \quad (14)$$

Thus, if the coupling parameter is $\chi = 0.1$, then the WGW mode can be excited in a standard fibre of diameter 125 μm whose limiting size is $W_0 = 42$ μm; for $\chi = 10^{-3}$, we have $W_0 = 456$ μm.

Note also another circumstance, which follows from expression (14), namely, that the mode size increases with the rod diameter.

When a taper is used to excite WGWs, the possibility of excitation of some mode is determined by the limiting value of the coupling parameter χ, which is limited both by the coupling-region length and (mainly) by the diameter of an optical mode in a thin (3 – 5 μm in diameter) tapered fibre. If the diameter of this mode achieves ~ 10 μm ($W_0 = 10$ μm), then the coupling parameter χ should be 0.56, which is quite a large value. As the rod diameter is increased up to 400 μm, the value of χ increases up to 0.74.

Excitation of the WGW modes in cylindrical rods of even greater diameter (above 400 μm) with the help of a thin tapered fibre becomes impossible. To excite the WGW modes in such rods, one should reduce the diffraction losses of light. These losses can be substantially reduced by locally increasing the refractive index along the WGW beam trajectory, i.e., by producing a circular channel waveguide in cylindrical rods [12]. Another method for reducing diffraction losses is the creation of a local curvature of the surface along the cylinder generatrix, i.e., the creation of the so-called circular ridge waveguide for the WGW. It was shown in paper [13] that this method provides the cavity Q factor $\simeq 10^7$.

3. Experiment

We obtained the spectrograms of the WGW modes excited in cylindrical fibres of diameter from 125 to 400 μm. The modes were excited with the help of a tapered fibre fabricated using the following technology: a standard silica fibre of diameter 125 μm was locally heated with an electric arc discharge up to the softening temperature and was stretched with a special computer-controlled device. As a result, we obtained tapered regions of fibres of length 20 mm with the narrow-part (waist) diameter of 3 μm. The bitapered fibre fabricated in this way was fixed in a metal (Invar) holder with a slot in the middle (i.e., under the narrow part of the waist) for placing a silica fibre under study perpendicular to the waist. The fibre under study was fixed on a precision ($\Delta x \sim 0.25$ μm) translation stage along the waist.

A source of radiation coupled into a tapered fibre was a diode-pumped erbium-doped fibre amplifier. A signal at the output of a coupling device was detected with an Anritzu MS96A spectrometer. Fig. 5 shows the typical spectrogram of excitation of a cylindrical optical fibre of diameter 125 μm. We determined from this spectrogram the effective refractive index n^* of the WGW mode and the number N of reflections corresponding to it. Fig. 6 shows the calculated dependence $n^*(N)$ and the dependence $\bar{n}^*(\rho)$ for a thin tapered fibre exciting the WGW [see expression (1)]. Fig. 6 shows the range of values of $n^*(N)$ which can be obtained from the spectrograms by exciting WGWs with the help of a tapered fibre having a waist of a given diameter. We compared the values of n^* and N found from the spectrograms for different fibres with the calculated dependence $n^*(N)$. The values of n^* and N obtained from the spectrogram in Fig. 3 are also shown in Fig. 6. We failed to excite WGWs in a cylindrical fibre of diameter 260 μm and above, which we explain by large diffraction losses of light in these fibres.

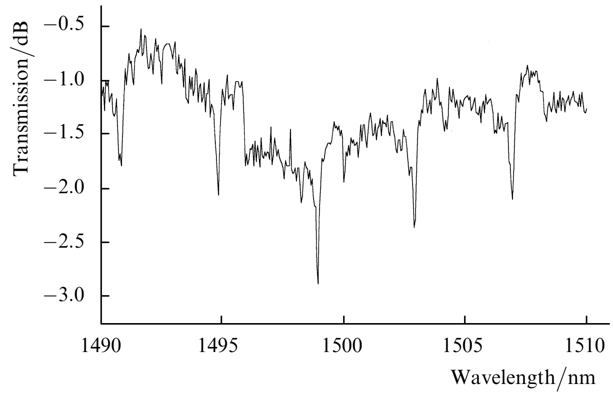

Figure 5. Spectrogram of radiation at the output of a bitapered fibre obtained by exciting WGWs in a standard cylindrical fibre.

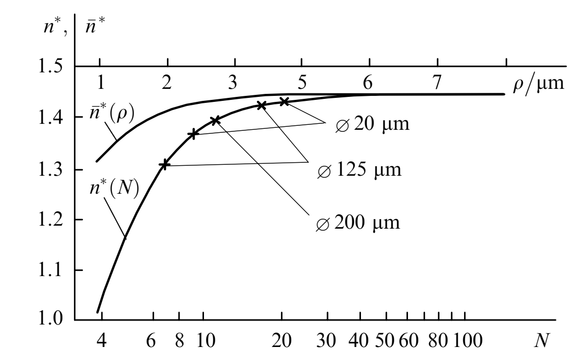

Figure 6. Dependences of the effective refractive index $\bar{n}^*$ of modes in a bitapered fibre on its radius ρ and the dependence of n^* for the WGW modes in a cylindrical fibre microcavity on the number N. The crosses are the experimental values of n^*.

Therefore, based on the data from the literature and our experimental results, we have shown that the geometrical-optic approximation can be used for analysis of the spectrum of whispering-gallery modes excited in cylindrical fibres of large (200 μm) and small (20 μm) diameters.

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