Quantum Electronics **32** (9) 820–824 (2002)

PACS numbers: 42.55.Lt; 42.60.Mi
DOI: 10.1070/QE2002v032n09ABEH002298

Multimode oscillation of a cw unstable-cavity CO_2 laser with a nonstationary active medium

V.G. Naumov, P.A. Svotin

***Abstract*. A method of analysis of lasing at several transverse modes in a nonstationary active medium with substantial optical inhomogeneities is developed. It is based on the assumption that the round-trip transit time for radiation in the resonator is much shorter than the characteristic time of a change in the spatial distributions of the gain and refractive index. The nonstationary oscillation of a transverse-flow CO_2 laser is analysed by this method in a two-dimensional approximation in the case when the vibrational excitation of an active medium at the cavity input changes in time and phase distortions move.**

Keywords: *multimode lasing, amplitude-phase characteristics, angular divergence, unstable cavity.*

1. Introduction

Both stationary or nonstationary oscillation regimes can be obtained in transverse-flow industrial cw CO_2 lasers with unstable cavity [1]. In certain cases, the nonstationary lasing is manifested in the modulation of the output power and even in the stochasticity of amplitude-phase characteristics of radiation [2]. These effects are caused by substantial optical inhomogeneities of the active media, nonlinear self-action, thermal deformations of mirrors, and lasing at several transverse modes [3].

The simulation of amplitude–phase characteristics of radiation with nonstationary phase inhomogeneities under the conditions when characteristics of lasing modes are changing is, at present, a poorly studied theoretical problem. The calculation of a cavity was traditionally performed by the Fox–Lee method [4]. This method gives satisfactory results in the case when optical inhomogeneities are small and lasing proceeds at a single transverse mode. Methods to analyse the stationary multimode lasing in an unstable cavity with substantial phase inhomogeneities also exist [5, 6]. The method of [6] provides the algorithm to find a stable self-consistent set of modes, each reproducing its amplitude-phase profile after a round trip in the cavity. Quite often, the nonstationary lasing proceeds under such conditions when the round-trip transit time in the cavity is considerably shorter than the characteristic time of a change in the distribution of the gain and refractive index in the cavity. In the case of substantial phase inhomogeneities, we can assume that lasing proceeds at several transverse modes simultaneously.

In this paper, we consider such a nonstationary lasing and propose a novel method for analysis of a nonstationary multimode lasing in the presence of substantial phase inhomogeneities.

2. Method for calculating nonstationary multimode lasing

A model developed in this paper assumes that the time of light propagation through a path of length L from the output mirror of a cavity to the highly reflecting mirror and backwards $\tau_p = 2L/c$ is much shorter than the characteristic time of a change in optical inhomogeneities and the distribution of the gain in a cavity. In this approximation, we assume that the mode structure has been formed and the output radiation is distributed among several self-matched modes. The presence of the mode structure presumes the complex wave amplitude of each lasing mode to be reproduced with a high accuracy after a cavity round trip. In the case of a great number of round trips, a drift of mode characteristics matched with a change in optical inhomogeneities of the active medium, which can be described by the kinetic and gas-dynamics equations, is possible.

The concepts of the self-matching and stability imply that the simultaneous lasing at several transverse modes provides the remaining of the rest of modes, not involved in lasing, under the threshold. The deviation of the gain from unity after a round trip of radiation in the cavity is caused by nonstationary inhomogeneities and amplification of the active medium. In such an approximation, the complex amplitudes U_i^+ and U_i^- of radiation waves of each mode, which describe the light propagation from the highly reflecting mirror to the output mirror of a cavity and backwards, satisfy equations of quasioptics [4].

The functions $g(x, z, t)$ and $v(x, z, t)$, involved in the quasioptical equation and taking into account distributions of the gain and refractive index, are determined by a set of differential equations depending on the kinetic model of an active medium. In our approximation, the changes of functions $g(x, z, t)$ and $v(x, z, t)$ within the time τ_p can be neglected, and we can use the stationary equations of quasioptics for a single cavity round trip. The appropriate boundary conditions for the set of quasioptical equations

V.G. Naumov, P.A. Svotin State Research Center of Russia, Troitsk Institute for Innovation and Fusion Research, 142190 Troitsk, Moscow oblast', Russia; e-mail: naumov@triniti.ru; svotin@triniti.ru

Received 7 February 2002; revision received 19 July 2002
Kvantovaya Elektronika **32** (9) 820–824 (2002)
Translated by J.M. Mikhailova

should take into account the nonstationary nature of the active medium; therefore, the boundary conditions are given by

$$U_i^+(x,0,t) = \sigma^{0.5}\exp[-2ikS^{g}(x)]U_i^-(x,0,t),$$
$$U_i^-(x,L,t+\tau) = \sigma^{0.5}\exp[-2ikS^{b}(x)]U_i^+(x,L,t). \quad (1)$$

Here, $S^g(x)$ and $S^b(x)$ are the profiles of surfaces of the output mirror and the highly reflecting mirror, respectively, specified in the coordinate systems where the axis of each mirror directed into a cavity; $\sigma = \sigma(x)$ is the reflectivity of the mirrors; τ is the integration step of the set of differential kinetic equations, used to calculate functions $g(x, z, t)$ and $v(x, z, t)$. Because the parameters of the active medium are considered to be stable within the time τ, we can express the wave $U_i^-(x, L, t+\tau)$ at the output mirror as a function of the wave U_i^+ at instant of time t. The quantity τ is equal to τ_p, however for slow changes of the active medium, the integration step of kinetic equations may be increased ($\tau = s\tau_p$). Under our conditions, the coefficient $s = 2-4$. The results of our calculation did not depend on the value of s. Kinetic equations describing the active medium depend on the total intensity of radiation of different lasing modes $I = \sum I_i$, where $I_i = |U_i^+|^2 + |U_i^-|^2$.

The contribution of the interference of counterpropagating modes to the intensity distribution can be neglected, because in the case of a stationary multimode lasing [6] neither the transverse lasing mode composition, nor the distribution of the radiation power among them are sensitive to this interference. This assumption is based on the fact that the mode competition depends [6] on the relative value of overlap integrals of functions $|U_i^+|^2$ of different modes i in any cross section of a cavity, while the relative value of those integrals is virtually independent of this interference. Thus, in the case of a nonstationary lasing with a stable mode structure, the interference of counterpropagating modes can be neglected.

The accuracy of reproducing of the complex wave amplitude after a round trip in the cavitywas calculated from a minimum of the function

$$f = \int |U^+(x,L,t) - r\exp(-i\varphi)U^+(x,L,t+\tau)|^2 dx \times\left(\int |U^+(x,L,t)|^2 dx\right)^{-1} \quad (2)$$

depending on the complex quantity $r\exp(-i\varphi)$. It is clear that the minimum value of f characterises the reproducibility of the complex wave amplitude after a cavity round trip. To achieve high quality of the mode reproducibility, the condition $f \ll 1$ should be satisfied. The quantity r in a quasi-stationary regime of lasing should satisfy the condition $|r-1| \ll 1$, and the value of φ gives the phase correction related to the change in the lasing mode frequency [4].

Here, we consider multimode lasing in a cw subatmospheric pressure transverse-flow CO_2 laser. The kinetic model of this lasing is analogous to the simplified model [7]. In this model, the vibrational excitation of the upper level of CO_2 molecules and vibrational excitation of N_2 were considered, and the equation for the change of the refractive index v of the active medium was analysed. Hence, the average numbers of vibrational quanta ε per CO_2 molecule, and ε_{N_2} per N_2 molecule were introduced. The equations for ε_{N_2}, ε and v have the form

$$\frac{\partial\varepsilon_{N_2}}{\partial t} + V\frac{\partial\varepsilon_{N_2}}{\partial x} = w_{CO_2-N_2}(\varepsilon - \varepsilon_{N_2}),$$
$$\frac{\partial\varepsilon}{\partial t} + V\frac{\partial\varepsilon}{\partial x} = w_{N_2-CO_2}(\varepsilon_{N_2} - \varepsilon) - \alpha I\frac{\varepsilon}{(1+\varepsilon)^2} - w\varepsilon, \quad (3)$$
$$\frac{\partial v}{\partial t} + V\frac{\partial v}{\partial x} = -\beta\varepsilon v - \alpha_1 I\frac{\varepsilon v}{(1+\varepsilon)^2}.$$

Here, V is the velocity of a transverse active-medium flow; I is the radiation intensity; $w_{CO_2-N_2}$ and $w_{N_2-CO_2}$ are the rates of transfer of vibrational excitation from CO_2 molecules to N_2 molecules and back, respectively; $w_{CO_2-N_2}N_{N_2} = w_{N_2-CO_2}N_{CO_2}$; $\alpha = \sigma_0(\hbar\omega)^{-1}$; σ_0 is the effective cross section for the resonance stimulated transition; $\hbar\omega$ is the photon energy; w is the frequency of VT relaxation of the upper laser level of the CO_2 molecule; $\alpha_1 = 1.5\alpha\alpha_2$; $\beta = 2.5w\alpha_2$; $\alpha_2 = \hbar\omega k N_{CO_2}/(C_p P)$; k is the Boltzmann constant; C_p is the heat capacity per molecule of the working mixture at normal pressure; P is the pressure; N_{CO_2} is the concentration of CO_2 molecules in the active medium. The gain in the active medium is given by [7]:

$$g(\varepsilon) = \sigma_0 N_{CO_2}\frac{\varepsilon}{(1+\varepsilon)^2}. \quad (4)$$

The set of equations (3) takes into account the changes in the gain along the flow caused by the exchange of the vibrational excitation between CO_2 and N_2 molecules and by the formation of the inhomogeneous distribution of the refractive index. The transverse velocity V of the active-medium flow in the cavity was assumed to be constant.

The finite-difference approximation was used for the combined numerical integration of the set of quasioptical equations and the set of equations (3). Equations of quasioptics were solved by the standard method of splitting over physical parameters [7]. According to this method, the active medium was divided in a cavity volume into layers of equal thicknesses perpendicular to the z axis. In each layer, the characteristics of the active medium along the cavity axis were assumed constant and their temporal evolution was described by the set of equations (3). The transverse distribution of the radiation intensity in each layer was assumed to be equal to the incident intensity distribution, which was determined by the simultaneous solving quasioptical equations and Eqns (3). The finite-difference approximation of the set of equations (3) was taken from [8].

In our work, much attention is focused on the case of nonstationary multimode lasing in an unstable cavity. To choose the computational scheme for these cases, we examine the results of simulation of the stationary multimode stable lasing under the conditions of substantial optical inhomogeneities of the active medium [3]. The stability criterion implies that nonlasing modes decay after repeated round trips in the cavity, while lasing modes maintain the self-matching regime, in which the distribution of the radiation power over modes and mode characteristics remain stable with time [6].

This fact is of fundamental importance in our method of simulation of nonstationary lasing. It points out that perturbations of characteristics of an arbitrary lasing mode in a stationary state, caused by other lasing modes propagating in a cavity, on average suppress each other; therefore, the amplitude-phase characteristics of the modes remain stable within the infinite time interval. Probably, this fact is caused by the dependence of the active-medium parameters on the superposition of intensities of different lasing modes and the difference between their transverse distributions. Therefore, there is a strong probability that slow changes in the phase and gain inhomogeneities will not cause the destruction of the mode structure of radiation, i.e., the complex wave amplitude, which describes each lasing mode, will be reproduced.

The calculations of a stationary multimode lasing depending on the parameters of optical inhomogeneities showed that the parameters of lasing modes change largely continously. However, there is some discrete set of values of these parameters, at which characteristics of lasing modes change stepwise [3]. In this connection, the new method for calculating a nonstationary multimode lasing should take into account the possibility of such a drastic change in the lasing regime. In this work, we consider only changes in the lasing regimes in the case of single-mode and two-mode lasing.

The specific initial condition should be assigned to describe the evolution of a multimode lasing by means of the equations of quasioptics and kinetics, since the multimode structure of radiation cannot be formed spontaneously in the calculation. In our opinion, it is most convenient to determine the stable initial conditions by solving simultaneously equations of quasioptics and the stationary analogue of the set of equations (3) by the method [6]. This allows one to specify the initial static phase distortions in the cavity and its geometrical characteristics, and then to determine the initial number of modes involved in lasing, and their characteristics, which are matched with each other and with optical inhomogeneities of the active medium. The further simultaneous solving of the set of nonstationary equations gave the changes in the amplitude-phase characteristics of all modes matched both with each other and with the active medium. As a rule, mode characteristics slowly change in time because the values of f from (2), estimated after each cavity round trip for each lasing mode i, satisfy the following condition:

$$f_i < 10^{-9} - 10^{-6}. \tag{5}$$

This result confirms that the mode structure of radiation under nonstationary conditions is stable if the parameters of the active medium change gradually. If the rearrangements of the active medium and changes in the radiation characteristics caused by them occur rapidly the reproducibility error of the complex amplitude f_i, which is close to the upper boundary (5), increases. At the same time, the rate of changing of the correction of the mode eigenfrequency (2) increases and the angular divergence and other characteristics of each lasing mode change faster.

As mentioned above, we should expect that the slow evolution of characteristics of lasing modes can be disrupted by their sufficiently fast changeover. The numerical simulation showed that there are several variants of changing to other lasing modes. In a softest regime of the change, the solving of the set of equations can proceed without interruption, the accuracy of the mode reproducibility decreasing within a short time interval ($f_i \sim 10^{-5} - 10^{-4}$).

In the further solving of the set of equations the values of f_i usually decrease. It is rather simple to take into account the termination of lasing at some of the modes. In this case, we can exclude this mode from lasing at some instant of time when its power is low. The case of the involvement of a new mode into lasing is more interesting. The calculations showed that this case is accompanied by the worsening of the reproducibility of a single or several lasing modes. In the calculation, the criterion for a change in the composition of lasing modes can be the abrupt increase of the reproducibility error f_i of any lasing mode after a cavity round trip. To apply this criterion in the computational program, we may represent it as a parameter

$$f_{\mathrm{r}} = f^{\tau}/f > 1.3 - 1.6, \tag{6}$$

where f^{τ} and f are mode reproducibility errors after two consecutive round trips.

New modes were included into lasing in the following ways. First, the gain distribution and phase inhomogeneities were fixed in the cavity and then highest-Q modes were calculated. Next, the lasing modes were selected among them. If modes with the round-trip gain greater than unity were among the remaining modes, they were included in lasing. Modes were identified by the value of the phase correction related to the change in the eigenfrequency φ (see Eqn (2) and [6]). The power of a new mode involved in lasing was taken to be $10^{-6} - 10^{-7}$ of the power level of lasing modes.

When phase inhomogeneities in a cavity are moderate, we, as a rule, have to include only one mode in lasing. The method to determine whether a new mode should be included in lasing, which was described above, leads to a time delay. Therefore, when we need a high accuracy of the description of the temporal evolution of lasing, we have to watch the Q factor of each nonlasing mode at almost equal time intervals. If some of the nonlasing modes had the round-trip gain exceeding unity, they were included into lasing. More precise but more complicated methods of inclusion of a new mode in lasing may be realised in program, but actually, these methods are similar to the method described above.

When the composition of highest-Q modes substantially differs from the composition of modes involved in lasing, the mode structure does not exist during a certain time interval (the previous structure breaks down, while a new one is not formed yet). This corresponds to the case of the considerable change in the mode composition. One can consider this case if the number of modes is no less than 3–4; we intend to perform this analysis in the future. Note that the present method implies the intrinsic control of the applicability based on the calculation of f from Eqn (2). In this case, we can assume that the generated radiation has no mode structure, if we fail to describe the nonstationary multimode lasing at which all f_i are small.

3. Results of calculations

We performed calculations for the confocal unstable single-pass cylindrical cavity with the gain $M = 2$. The distance L between the highly reflecting and the output mirrors was 540 cm. The cross sizes of the output (b^{b}) and highly reflecting (b^g) mirrors were 4.22 and 8.44 cm, respectively. The optical scheme of a cavity, which fully coincides with the one shown in Fig. 1 of [6], included also an intracavity aperture of size 11.9 cm located at equal distances from the output and highly reflecting mirrors. The active medium was settled in a rectangular of size $aL_{am} = 15 \times 400$ cm at equal distances from the output mirror and the highly reflecting mirror. The computational grid consisted of 2048 mesh points, the step of the grid was $\Delta = a/m = 7.3242 \times 10^{-3}$ cm (here a is the cross size of a rectangle, m is the number of mesh points). Phase inhomogeneities in the cavity volume were produced by periodic distortions of both the surface of the highly reflecting mirror and the phase screen, placed at the diaphragm location. The distortions of surfaces of mirrors per a period d were defined as:

$$H(x+d) = H(x)$$

$$= \begin{cases} A_0 \cos^2(\pi x/d_1), & |x| \leqslant d_1/2 \\ 0, & d_1/2 \leqslant |x| \leqslant d/2\,, \end{cases} \qquad (7)$$

where $d = 0.8$ cm; $d_1 = 0.4$ cm; and A_0 is the amplitude of distortions of the surface of the highly reflecting mirror. Periodic distortions of the phase screen simulated the presence of two flat deflecting mirrors, positioned at an angle of 45° to the cavity axis, in the cavity scheme. Periodic distortions at each deflecting mirror were specified by expression (7) and the parameter x_n, which determined the initial shift of these distortions. During the calculations, the position of distortions at the highly reflecting mirror did not change, so that static phase distortions in the cavity scheme could be described by the parameter $H_0 = 2\pi A_0/\lambda$ and the parameters x_{n1} and x_{n2}, which specified the initial shift of phase distortions on deflecting mirrors.

The parameters of the set of equations (3) were specified by assuming that the pressure of the active medium is $P = 0.14$ atm; the velocity of the gas flow is $V = 190$ m s^{-1}; the relative concentrations of CO_2, N_2 and He are 5 %, 50 %, and 45 %, respectively; and the effective length of the flow at which the population of the upper lasing level decreased by e times, is 10 cm. The effective values of the cross section for stimulated emission and the frequencies of the vibrational excitation exchange between CO_2 and N_2 were calculated assuming that the translational temperature in the active medium is equal to 350 K.

Consider now the nonstationary lasing in the case, when the average number of vibrational quanta of N_2 and CO_2 molecules in the active-medium flow at the cavity input changes as

$$\varepsilon(t) = \varepsilon_0\left(1 + \frac{c_0 t_0}{t_0 + t}\right), \qquad (8)$$

where $\varepsilon = 0.12$; $c_0 = 0.1$; and $t_0 = 200$ μs. Phase distortions of the surface of the highly reflecting mirror and the phase screen were specified using parameters $H_0 = 0.225$ and $x_{n1}/\Delta = x_{n2}/\Delta = 25$. Under this condition, the two-mode lasing is generally observed. Fig. 1a shows time dependences of the radiation power for each mode within a period of about 1 ms. One can see that the radiation power of each mode changed greatly. Within certain time intervals, the lasing vanished and then appeared again. Within the given time interval, the characteristics of lasing modes changed continuously in general. When a new mode was involved in lasing, the phase correction $\varphi = 2L\Delta\omega/c$

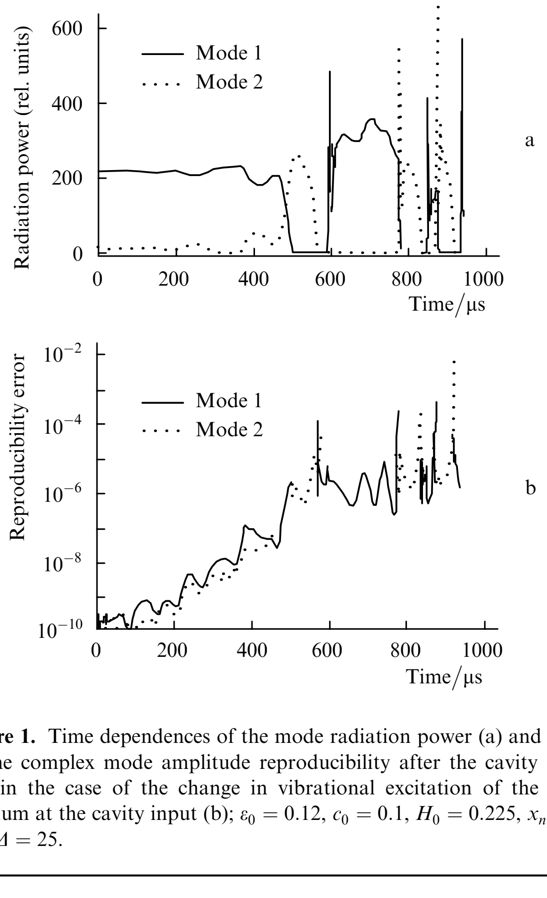

Figure 1. Time dependences of the mode radiation power (a) and errors of the complex mode amplitude reproducibility after the cavity round trip in the case of the change in vibrational excitation of the active medium at the cavity input (b); $\varepsilon_0 = 0.12$, $c_0 = 0.1$, $H_0 = 0.225$, $x_{n1}/\Delta = x_{n2}/\Delta = 25$.

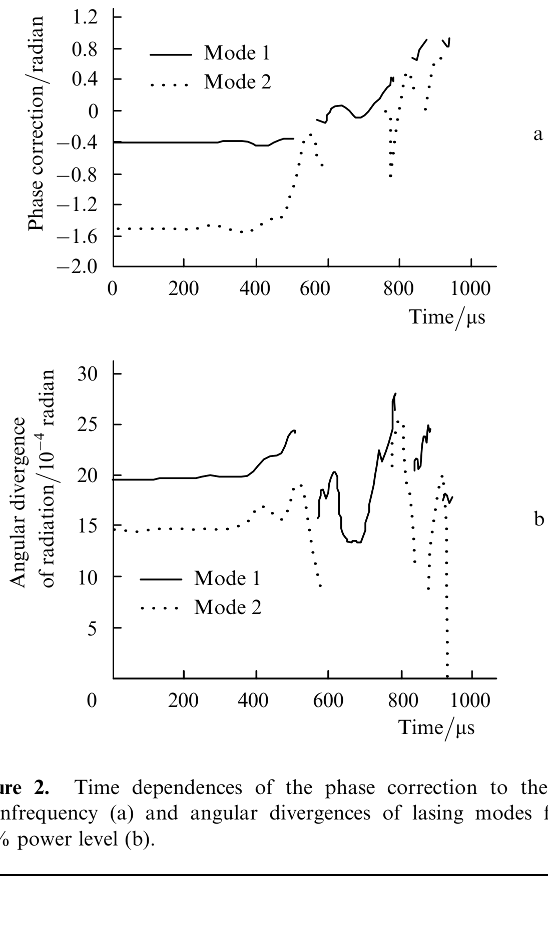

Figure 2. Time dependences of the phase correction to the mode eigenfrequency (a) and angular divergences of lasing modes for the 75 % power level (b).

[see Eqn (2)], where $\Delta\omega$ is the change of the mode eigenfrequency, could exhibit a sudden jump (Fig. 2a). The angular divergence of radiation of lasing modes changed as well (Fig. 2b). At other instants of time, the phase correction φ changed very slowly, which confirms the presence of the mode structure of radiation. In this variant of the simulation, the eventual inclusion of a new mode in lasing was controlled using the above-mentioned criterion (6), which determines the rate of the increasing of the mode reproducibility error after each cavity round trip. The time dependence of the reproducibility error f is shown in Fig. 1b.

The phase inhomogeneities of the active medium can move, resulting in the multimode nonstationary lasing. The movement of inhomogeneities in the plane of the phase screen can simulate the presence of moving phase inhomogeneities. This allows the possibility of simulations of lasing in the presence of nonstationary gas-dynamical density perturbations by the method proposed. The movement of phase inhomogeneities was simulated by varying the coordinates x_{n1} and x_{n2}, which specify the initial position of periodic distortions of surfaces of deflecting mirrors, the initial value of $x_{n1}/\Delta = x_{n2}/\Delta = 10$ and the parameter $H_0 = 0.225$. The average number ε_0 of vibrational quanta in the active-medium flow at the cavity input was equal to 0.12. The velocity of a transverse movement of phase distortions was low enough to neglect the Doppler shift of the lasing frequency. Periodic distortions of mirror surfaces moved with the same velocity in the opposite directions. The temporal increment of the displacement, scaled to the grid steps, was determined by the expression

$$\frac{x_{n1} - x_{n2}}{\Delta} = \int_0^t c_\mathrm{d} \frac{t}{t_0 + t} \,\mathrm{d}t \,. \tag{9}$$

Here, c_d is the velocity of the displacement of phase inhomogeneities within a long time interval; and t_0 is the time necessary to reach this velocity. The results of calculation for $c_\mathrm{d} = 0.14\ \mu\mathrm{s}^{-1}$ and $t_0 = 100\ \mu\mathrm{s}$ are shown in Fig. 3. The calculation presented in Fig. 3a demonstrates the exit of a mode from lasing. Fig. 3b shows the time dependences of phase corrections φ_i, characterising changes in the mode eigenfrequencies, and of the angular divergence of radiation for the 75 % power level of each mode. The error of the mode reproducibility after the cavity round trip is negligible (less than 10^{-7}) for both modes.

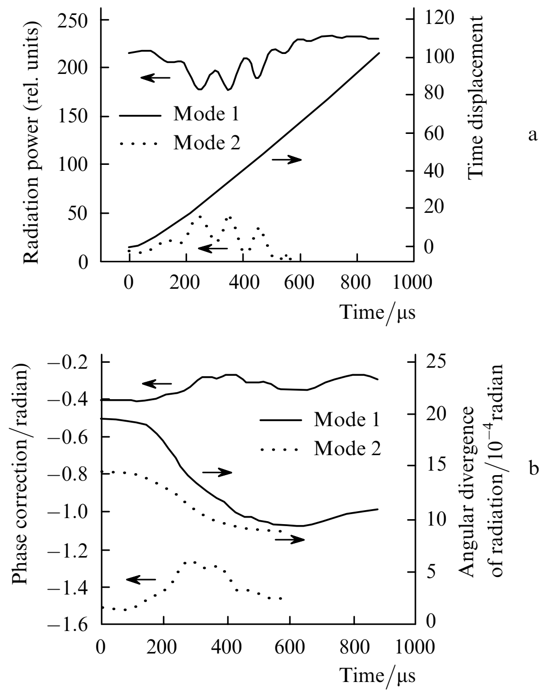

Figure 3. Time dependences of the power of different lasing modes during the movement of phase inhomogeneities on deflecting mirrors (a); and the time dependence of phase corrections to lasing eigenfrequencies and the angular divergence for the 75 % power level (b); $H_0 = 0.225$, $x_{n1}/\Delta = x_{n2}/\Delta = 25$, $\varepsilon_0 = 0.1$.

4. Conclusions

We have developed the method for analysis of the nonstationary lasing at several transverse modes in the case of substantial nonstationary phase and gain inhomogeneities of an active medium. We assumed in our calculations that the round-trip transit time in the cavity is much shorter than the characteristic time of the change in spatial distributions of the gain and the refractive index in the active medium. Our method includes the selection of the initial state of lasing, the scheme of the calculation of nonstationary lasing with a fixed number of lasing modes and the algorithm for including other modes in lasing. By using this method, we calculated a two-mode osillation in a transverse-flow CO_2 laser under conditions of the nonstationary vibrational excitation of the active medium at the cavity input and stimulated the changes in the amplitude-phase characteristics of radiation during the transverse movement of phase distortions.

References

1. Artamonov A.V., Naumov V.G. *Kvantovaya Elektron.*, **4**, 178 (1977) [*Sov. J. Quantum Electron.*, **7**, 101 (1977)].
2. Gurashvili V.A., Zotov A.M., Korolenko P.V., et al. *Kvantovaya Elektron.*, **31**, 821 (2001) [*Quantum Electron.*, **31**, 821 (2001)].
3. Valuev V.V., Naumov V.G., Sarkarov N.E., Svotin P.A. *Kvantovaya Elektron.*, **25**, 16 (1998) [*Quantum Electron.*, **28**, 14 (1998)].
4. Anan'ev Yu.A. *Opticheskie rezonatory i lasernye puchki* (Optical Cavities and Laser Beams) (Moscow: Nauka, 1990).
5. Korotkov V.A., Likhanskii V.V., Napartovich A.P. *Kvantovaya Elektron.*, **17**, 897 (1990) [*Sov. J. Quantum Electron.*, **20**, 820 (1990)].
6. Valuev V.V., Naumov V.G., Svotin P.A. *Matem. Model.*, **7**, 49 (1995) [*Mathematical Modelling*, **7**, 49 (1995)].
7. Elkin N.N., Napartovich A.P. *Prikladnaya optika laserov* (Applied Optics of Lasers) (Moscow: TsNIIatominform, 1988).
8. Samarskii A.A. *Teorya raznostnykh skhem* (Theory of Difference Schemes) (Moscow: Nauka, 1977).