LASER APPLICATIONS AND OTHER TOPICS IN QUANTUM ELECTRONICS

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Spectra of quantum systems in superstrong inertial force fields

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***Abstract*. The effect of strong gravitational fields, arising upon cyclic and linear acceleration, on the spectrum of hydrogen-like ions is considered. The possibility of experiments on measuring the frequency shifts of ion energy levels in superstrong inertial force fields by using modern accelerators is shown.**

Keywords: *level shift in the gravitational field, strong gravitational fields, Stark and Zeeman effects.*

Investigations of the properties of quantum systems (molecules, atoms, ions, atomic nuclei) in gravitational fields is of definite interest from the viewpoint of studies of the fundamental properties of matter. The principal problem is the elucidation of the degree to which the quantum systems are affected by a metric field g_{ik} itself and by its most important characteristic – the space–time curvature tensor R^i_{klm}.

Under ordinary conditions, however, the influence of gravitational forces on the objects under study is extremely small because the interaction constant is very small. Indeed, the deviation of the metric tensor components g_{ik} from the Galilei diagonal values $\eta_{ik} = \{1, -1, -1, -1\}$ in the terrestrial gravitational field is as small as 3×10^{-9} [1]. The components of the curvature tensor of this field are also small and do not exceed 1.5×10^{-26} cm^{-2}. Therefore, it is quite problematic to measure under laboratory conditions a weak metric effect on a quantum system, and the more so to detect the curvature tensor effect against its background.

The situation changes radically in the case of quantum systems in strong gravitational fields. Until recently, the properties of quantum objects in strong gravitational fields were investigated bearing in mind the abstract possibility of observation of various gravitational effects in the vicinity of neutron stars or black holes [2].

However, the advancement of astronautics and the progress in the field of accelerator technology give promise that controllable experiments under terrestrial conditions may be possible. One of such possibilities is related to spaceborne experiments. In this case, it is possible to realise the conditions when the quantum system under study resides locally in the geodesic frame of reference. The metric tensor of space within the spacecraft will then coincide with the Galilei metric tensor to a high degree of accuracy [1], and therefore the curvature tensor alone will affect the quantum system in experiments of this type (for more details, see Ref. [3]).

Charged-particle accelerators offer even greater prospects for studying the influence of the metric field on quantum systems. The properties of quantum systems moving in modern accelerators correspond to those that would be observed in gravitational fields producing an acceleration of the order of $(10^{14} - 10^{16})g$. Despite the fact that the inertial force field simulates the gravitational field only to the second partial derivatives of the metric tensor, the study of the behaviour of quantum systems moving in an accelerator makes it possible to verify the basic concepts of the metric nature of inertial force fields.

The influence of accelerated motion of ions on their energy level spectrum is one of the effects that can be simply verified experimentally. This was pointed out already in Ref. [4]. However, the author of Ref. [4] only formulated the problem, without calculating the shifts of ion energy levels during their noninertial motion.

We consider this question in greater detail for accelerators of two types: circular and linear. Let us assume that a hydrogen-like ion with one electron is relativistically uniformly accelerated in a linear accelerator under the action of a constant and uniform electric field $\boldsymbol{E}_0$. The law of centre-of-mass motion of a hydrogen-like ion in the laboratory frame of reference can be written as

$$z = \frac{c^2}{w}\left(1 + \frac{w^2 t^2}{c^2}\right)^{1/2} - 1,$$

where $w = |(Z-1)eE_0|/(M+m)$; M and Ze are the mass and nuclear charge of the ion; and m and e are the electron mass and charge, respectively.

Let us pass to the relativistic uniformly accelerated frame of reference, where the ion is at rest, to find the energy level spectrum of this ion. To do this, we have to solve, as is well known, the Dirac equation written in the noninertial frame of reference:

$$\mathrm{i}\hbar\gamma^n \nabla_n \Psi - \frac{e}{c} A_n \gamma^n \Psi - mc\Psi = 0, \tag{1}$$

where ∇_n is the covariant derivative with respect to the metric of the noninertial frame of reference; γ^n are the

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Dirac matrices; and A_n is the four-dimensional vector-potential of the electromagnetic field. The nonzero components of the metric tensor have the form

$$g_{00} = \frac{1}{1 + w^2t^2/c^2}, \; g_{01} = -\frac{wt}{c(1 + w^2t^2/c^2)^{1/2}}, \quad g_{11} = g_{22} = g_{33} = -1. \tag{2}$$

Because, along with inertial force fields, an electron experiences the nuclear field and the external field $\boldsymbol{E}_0$, the solution of Eqn (1) is an intricate mathematical problem. For this reason, we will find the solution of the Dirac equation only in the weakly relativistic case, when the effect of the external electric field and the inertial force fields on the ion spectrum is weak and the solution can be obtained by the perturbation method. As a result, we obtain the splitting of the ion terms caused by the noninertial character of its motion:

$$\Delta\omega_{\text{ni}} = \frac{3(Z-1)eE_0ma_0}{(M+m)\hbar} = \frac{3mwa_0}{\hbar}, \tag{3}$$

where $a_0 = \hbar^2/me^2 \sim 0.529 \times 10^{-8}$ cm.

Note that the ion will experience, along with the splitting $\Delta\omega_{\text{ni}}$, the Stark shift $\Delta\omega_{\text{S}}$ of terms caused by the action of the field $\boldsymbol{E}_0$ on the energy levels:

$$\Delta\omega_{\text{S}} = \frac{3eE_0a_0}{\hbar}. \tag{4}$$

It follows from expressions (3) and (4) that

$$\Delta\omega_{\text{ni}} = \frac{(Z-1)m}{(M+m)}\Delta\omega_{\text{S}}. \tag{5}$$

Estimates show that the splitting caused by the inertial force fields for different ions ranges from 10^{-4} to 2.5×10^{-4} of the splitting due to the Stark effect.

Consider now the same ion moving over a circular orbit of radius R_0 with a constant velocity modulus v, which is comparable to the velocity of light, under the influence of an external magnetic field $\boldsymbol{H}_0 = \{0, 0, H_0\}$ in a circular accelerator. According to equations of relativistic mechanics, the motion of the ion centre of mass is described by expressions

$$x = R_0\cos(\Omega t), \; y = R_0\sin(\Omega t), \; z = 0, \tag{6}$$

where

$$\Omega = \frac{(Z-1)eH_0}{(M+m)c}\left(1 - \frac{v^2}{c^2}\right)^{1/2}.$$

Let us pass now to the frame of reference rotating with a frequency Ω. In this frame of reference, the ion centre of mass rests at a distance R_0 from the rotation axis, and the components of the external electromagnetic field take the form

$$\boldsymbol{H}' = \frac{\boldsymbol{H}_0}{(1 - v^2/c^2)^{1/2}}, \quad \boldsymbol{E}' = \frac{[\boldsymbol{v}\boldsymbol{H}_0]}{c(1 - v^2/c^2)^{1/2}}.$$

Therefore, in the rest frame of the ion, its energy levels also experience the Stark effect, in addition to the Zeeman effect. The corrections to the energy levels caused by inertial forces are also small in this case and depend on the frequency Ω. For high frequencies of the ion rotation, when the condition $v = \Omega R_0 < c$ is fulfilled, the energy level shift caused by inertial forces is

$$\Delta\omega_{\text{ni}} = \frac{3ma_0v^2\Omega}{c\hbar} = \frac{3(Z-1)eH_0mv^2a_0}{(M+m)\hbar c^2}. \tag{7}$$

In this case, the level shifts caused by the Stark effect ($\Delta\omega_{\text{S}}$) and the normal Zeeman effect ($\Delta\omega_{\text{Z}}$) are

$$\Delta\omega_{\text{S}} = \frac{3ea_0H_0v}{c\hbar}, \tag{8}$$

$$\Delta\omega_{\text{Z}} = \frac{eH_0}{2mc}. \tag{9}$$

It is easy to verify that

$$\Delta\omega_{\text{ni}} = \frac{6(Z-1)m^2 2a_0}{(M+m)c\hbar}\Delta\omega_{\text{Z}} = \frac{(Z-1)m}{(M+m)c}\Delta\omega_{\text{S}}.$$

It follows from the above expressions that in the weakly relativistic limit, the energy level shifts caused by inertial forces under optimal conditions can amount to $10^{-4} - 10^{-5}$ of the Zeeman and Stark shifts.

If singly charged ^{4}He ions are used in the experiment under discussion, the shift $\Delta\omega_{\text{ni}}$ in a magnetic field $H_0 = 10^5$ Oe produced by superconducting magnets will achieve 1.2×10^9 Hz for $v/c = 0.1$. Therefore, the investigation of the effect of inertial forces on the energy levels of an ion moving in an accelerator is a tractable problem, although an intricate one.

Note that the investigation of the energy levels of noninertially moving ions in Rydberg states is of considerable interest. However, the nonuniformity of the metric tensor within the electron shell can no longer be neglected in this case because of the large dimension of the orbit of the highly excited electron. As a result, the calculation of the energy level shifts of a Rydberg ion moving in an accelerator is an intricate mathematical problem, whose solution will be given in the future.

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References

1. Landau L.D., Lifshits E.M. *The Classical Theory of Fields* (Oxford: Pergamon Press, 1975; Moscow: Nauka, 1988).
2. Novikov I.D., Frolov V.P. *Fizika Chernykh Dyr* (The Physics of Black Holes) (Moscow: Nauka, 1986).
3. Dolgov A.D., Khriplovich I.B. *General Relativity and Gravitation*, **15**, 1033 (1983).
4. Rivlin L.A. *Kvantovaya Elektron.*, **16**, 1070 (1989) [*Sov. J. Quantum Electron.*, **19**, 694 (1989)].