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Distribution of the laser radiation intensity in turbid media: Monte Carlo simulations, theoretical analysis, and results of optoacoustic measurements

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***Abstract*. The spatial distribution of laser radiation intensity in turbid condensed media is studied analytically, numerically, and experimentally by the optoacoustic method. Based on optoacoustic measurements and Monte Carlo numerical simulations, the relation is obtained between the optical characteristics of a scattering medium and the position of the maximum of the spatial distribution of radiation intensity in the medium. It is shown that, when the anisotropy factor exceeds 0.8, this dependence has a universal type in the ranges of absorption and scattering coefficients typical for biological tissues. The method is proposed for measuring the extinction and absorption coefficients in light scattering media from the temporal shape of an optoacoustic pulse detected in relative units. An approximate method for solving a radiation transfer equation is verified, and the regions of application of the P_3 and P_5 approximation are established.**

Keywords: *scattering media, absorption coefficient, scattering coefficient, Monte Carlo simulation, radiation transfer equation, laser optoacoustics.*

1. Introduction

The experimental methods for determining optical characteristics of turbid media based on the measurement of the intensity of diffusion backscattering of light are currently being widely discussed. The known optical characteristics allow one to calculate the light intensity distribution in the medium, which is important for laser diagnostics and therapy of biological media and tissues. There exist the time-resolved spectroscopic method [1 – 4], the modulation method [5 – 8], and the stationary method with a spatial resolution [9 – 12]. Note, however, that each of these methods has some or other drawbacks, which restrict their applications for studying the optical properties of real turbid media. Therefore, the development of a noninvasive method for measuring the spatial distribution of light intensity in such media is still an important and urgent problem.

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In this paper, we propose to solve this problem using the optoacoustic method [13]. When a laser pulse of duration that is much shorter than the travel time of an acoustic wave in the region of heat release is absorbed in a medium, the profile of an optoacoustic signal resembles the spatial distribution of heat sources in the medium. Moreover, an optoacoustic signal will be absent in a medium that does not absorb light. The main advantage of this method is that information on an object under study is provided not by optical but acoustic waves, which are rather weakly attenuated in biological media. Therefore, the optoacoustic method can be used for diagnostics of light absorbing inhomogeneities in media absorbing and scattering light [14 – 16], as well as for measuring the spatial distribution of the laser radiation intensity and the optical characteristics of turbid media [17 – 19]. In addition, this method is successfully used for *in vivo* tomography of tumors in real biological tissues and objects, which is especially important at the initial stage of the cancer development [15, 16].

The aim of this paper is to find the dependence of the position of the maximum of the spatial distribution of the light intensity in a medium on its optical characteristics. For this purpose, we studied the spatial distribution of the laser radiation intensity under the surface of a scattering medium by three methods: numerically (using Monte Carlo simulations), theoretically (solving the radiation transfer equation), and experimentally (by the optoacoustic method). The solution of this problem will allow us to perform noninvasive measurements of the absorption and extinction coefficients from the temporal profile of an optoacoustic pulse without pressure measurements.

2. Spatial distribution of the laser radiation intensity in a scattering medium

The calculation of the light intensity distribution in an inhomogeneous medium is a very complicated problem, which can be solved only using some simplifying assumptions. We studied media representing suspensions of scattering particles in a homogeneous absorbing liquid assuming that: (i) scattering particles are homogeneously distributed in the liquid volume; (ii) the particles do not absorb light at the laser wavelength; and (iii) the volume concentration of particles is lower than 2 %, so that the absorption coefficient of a suspension was assumed equal to that of the liquid without scattering particles.

Under such assumptions, we can treat the medium as homogeneous and describe its optical properties by macroscopic parameters, namely, the absorption and scattering coefficients μ_a and μ_s, respectively.

Experimental results [17] and Monte Carlo simulations [20] show that the propagation of light in a turbid medium under the condition $\mu_a \ll \mu_s$ has a characteristic feature: the maximum of the spatial intensity distribution is located under the surface of a medium being irradiated at the distance $z_{max} \sim l^*$ (where $l^* = [\mu_s(1-g)]^{-1} = 1/\mu'_s$ is the mean photon free path in the medium; g is the anisotropy parameter; and μ'_s is the reduced scattering coefficient) rather than on its boundary (Fig. 1). The amplitude of this maximum can exceed the incident radiation intensity by a factor of 4–6. At the same time, in the case of strong scattering, the light intensity decays exponentially with the penetration depth with the exponent $\mu_{eff} = (3\mu_a\mu'_s)^{1/2}$ (see, for example, [18, 21]. Therefore, for $\mu_a \ll \mu_s$, the value $z_{max}\mu_{eff} \sim (1/\mu'_s)(3\mu_a\mu'_s)^{1/2} = 3\mu_a/\mu_{eff}$, i.e., it depends only on the ratio μ_a/μ_{eff} and is independent of the absolute values of μ_a and μ_{eff} and the anisotropy factor g. On the other hand, the maximum of a light flux in the case of a homogeneous absorbing non-scattering medium is located on its surface. This means that the maximum will shift to the surface with increasing the ratio μ_a/μ'_s, because in the limit $\mu_a \gg \mu_s$ radiation is absorbed, not having time to be scattered, and $z_{max} = 0$. The question of whether the dependence of $z_{max}\mu_{eff}$ on μ_a/μ_{eff} only is retained in the range $0.05 < \mu_a/\mu_{eff} < 0.35$, which is typical for biological media and tissues [9] is the main question that we solve in this paper.

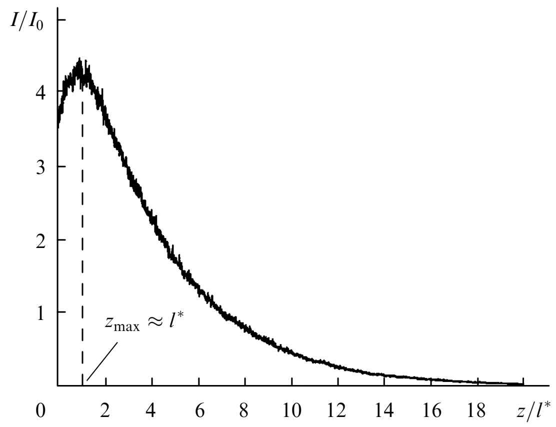

Figure 1. Typical profile of the spatial distribution of the laser radiation intensity in a turbid medium obtained by the Monte Carlo method.

2.1 Monte Carlo simulations

We performed the Monte Carlo simulations of the spatial distribution of the laser radiation intensity in a scattering medium for various values of μ_a, μ_s and g. These coefficients and the refractive indices n_1 and n_2 for transparent and scattering media, respectively, were specified. The values of z_{max} and μ_{eff} were determined from the model profile of the spatial intensity distribution. The error of measuring z_{max} was 3 %–5 % (when 10^5 photons were used), and for μ_{eff} it was less than 3 %. The parameters μ_a, μ_s and g were varied so that the relation $0.05 < \mu_a/\mu_{eff} < 0.35$ was fulfilled. Using these data, we plotted the dependence of $z_{max}\mu_{eff}$ on μ_a/μ_{eff}. The aim of the simulation was to find how this dependence changes with increasing the ratio μ_a/μ_{eff}. The relative errors of measuring $z_{max}\mu_{eff}$ and μ_a/μ_{eff} were 5 % and 8 %, respectively. These dependences are presented in Fig. 2 for different values of the anisotropy parameter g.

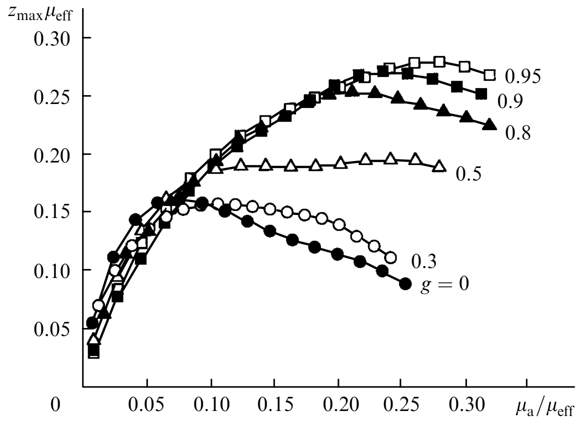

Figure 2. Dependences of $z_{max}\mu_{eff}$ on the ratio μ_a/μ_{eff} for different anisotropy factors g obtained by the Monte Carlo method.

One can see that the value of $z_{max}\mu_{eff}$ decreases with decreasing g when the ratio μ_a/μ_{eff} remains constant. Indeed, when the ratio μ_a/μ_{eff} is fixed, the value of $\mu_a/\mu'_s = 3(\mu_a/\mu_{eff})^2$ also remains constant, while $\mu_a/\mu_s = \mu_a(1-g)/\mu'_s$ increases with decreasing g. Therefore, the probability of absorption of photons under the medium surface will increase. In the limit $\mu_a/\mu_s \gg 1$, the intensity maximum will be located on the medium surface, and $z_{max} = 0$. At the same time, the curves corresponding to $g \geqslant 0.8$ converge within the error of measurement of $z_{max}\mu_{eff}$ for $\mu_a/\mu_{eff} < 0.3$. Thus, Monte Carlo simulation shows that under such conditions the value of $z_{max}\mu_{eff}$ depends only on the ratio μ_a/μ_{eff} but not on the absolute values of μ_a and μ_{eff}. These results cannot give a final answer about the features of the laser radiation intensity distribution in a medium because they were obtained using the Henney–Greenstein function, which approximately describes a real radiation pattern. Below, we compare these results with experimental data and the results of the approximate solution of the radiation transfer equation.

2.2 Solution of the radiation transfer equation in the P_3 and P_5 approximations

Consider an analytic approach. Let us assume that a homogeneous transparent and scattering media occupy half-spaces $z < 0$ and $z > 0$, respectively. If scattering dominates over absorption ($\mu_a \ll \mu_s$), multiple scattering of radiation occurs in the medium. The angular radiation spectrum inside the medium produced by an incident light pulse with a planar wave front can be found by solving the radiation transfer equation [22]

$$\boldsymbol{sn}\frac{\partial L(z,\boldsymbol{s})}{\partial z} = -\mu_t L(z,\boldsymbol{s}) + \mu_s \int_{4\pi} L(z,\boldsymbol{s}')p(\boldsymbol{s},\boldsymbol{s}')\mathrm{d}\Omega', \quad (1)$$

where $L(z,\boldsymbol{s})$ is the angular radiation spectrum, i.e., the light intensity at the point z in the direction $\boldsymbol{s}$; $p(\boldsymbol{s},\boldsymbol{s}')$ is the scattering indicatrix; $\mu_t = \mu_a + \mu_s$ is the total extinction coefficient of the scattering medium; $\boldsymbol{sn} = \cos\theta$; θ is the angle between the z axis and the direction of photon propagation inside the medium; $\boldsymbol{s}$ and $\boldsymbol{s}'$ are the unit vectors in the propagation directions of the incident and scattered photons, respectively; $\boldsymbol{n}$ is the unit vector directed along the z axis; and $\mathrm{d}\Omega'$ is the unit solid angle.

As the scattering indicatrix $p(\boldsymbol{s}, \boldsymbol{s}')$ in the case of a strong anisotropy, the Henney–Greenstein function [23]

$$p_{\mathrm{HG}}(\boldsymbol{s}, \boldsymbol{s}') = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g\boldsymbol{ss}')^{3/2}} \tag{2}$$

is commonly used. The intensity of light $I(z)$ at the point z in a strongly scattering medium is related to the ray intensity $L(z, \boldsymbol{s})$ by the expression [22]

$$I(z) = \frac{1}{4\pi} \int_{4\pi} L(z, \boldsymbol{s}) \mathrm{d}\Omega. \tag{3}$$

Therefore, by solving integro-differential equation (1), we can calculated from (3) the intensity of light $I(z)$ at any point of the medium.

Let us represent the ray intensity of light $L(z, \boldsymbol{s})$ inside the medium as a sum of two components [22]: the coherent $L_{\mathrm{coh}}(z)$ and diffusion $L_{\mathrm{dif}}(z, \boldsymbol{s})$. Then, we obtain from equation (1)

$$\frac{\partial L_{\mathrm{coh}}(z)}{\partial z} = -\mu_{\mathrm{t}} L(z), \tag{4a}$$

$$\boldsymbol{sn} \frac{\partial L_{\mathrm{dif}}(z, \boldsymbol{s})}{\partial z} = -\mu_{\mathrm{t}} L_{\mathrm{dif}}(z, \boldsymbol{s}) + \mu_{\mathrm{s}} \int_{4\pi} L_{\mathrm{dif}}(z, \boldsymbol{s}') p(\boldsymbol{s}, \boldsymbol{s}') \mathrm{d}\Omega' + Q(z, \boldsymbol{s}), \tag{4b}$$

where $Q(z, \boldsymbol{s}) = \mu_{\mathrm{s}} \int_{4\pi} L_{\mathrm{coh}}(z) p(\boldsymbol{s}, \boldsymbol{s}') \mathrm{d}\Omega'$ is the source function for the scattered light field. The coherent component of the intensity $L_{\mathrm{coh}}(z)$ can be easily found by solving equation (4a). The boundary condition of reemission for equation (4b) gives the relation between diffuse light directed inside the scattering medium and a fraction of diffuse light reflected from the interface between media [8]:

$$\int_{\cos\theta > 0} L(z = 0, \boldsymbol{s}) P_k(\boldsymbol{sn}) \mathrm{d}\Omega = \int_{\cos\theta < 0} R_{\mathrm{Fr}}(-\boldsymbol{sn}) L(z = 0, \boldsymbol{s}) P_k(-\boldsymbol{sn}) \mathrm{d}\Omega, \tag{5}$$

where $R_{\mathrm{Fr}}(\boldsymbol{sn})$ is the Fresnel reflection coefficient for nonpolarised light [8] and $P_k(\boldsymbol{sn})$ is the Legendre polynomial of the kth order.

The diffusion component $L_{\mathrm{dif}}(z, \boldsymbol{s})$ of the light intensity in (4b), the scattering indicatrix $p_{\mathrm{HG}}(\boldsymbol{s}, \boldsymbol{s}')$ and the source function $Q(z, \boldsymbol{s})$ can be represented as expansions in Legendre polynomials [22, 24]

$$L_{\mathrm{dif}}(z, \boldsymbol{s}) = \sum_{k=0}^{N} \frac{2k + 1}{4\pi} y_k(z) P_k(\boldsymbol{sn}), \tag{6a}$$

$$p_{\mathrm{HG}}(\boldsymbol{s}, \boldsymbol{s}') = \sum_{k=0}^{N} \frac{2k + 1}{4\pi} g^k P_k(\boldsymbol{s}, \boldsymbol{s}'), \tag{6b}$$

$$Q(z, \boldsymbol{s}) = \sum_{k=0}^{N} \frac{2k + 1}{4\pi} q_k(z) P_k(\boldsymbol{sn}), \tag{6c}$$

where $y_k(z)$ and $q_k(z)$ are the coefficients of expansion of functions $L(z, \boldsymbol{s})$ and $Q(z, \boldsymbol{s})$ in Legendre polynomials, respectively.

The expansion coefficients $q_k(z)$ of the source function can be written in the form [24]

$$q_k(z) = \mu_{\mathrm{s}} g^k I_0 \exp(-\mu_{\mathrm{t}} z), \tag{7}$$

where I_0 is the intensity of incident laser radiation. By substituting (6a), (6b), and (6c) into (4b), multiplying successively by $P_l(\boldsymbol{sn})$ (where $l = 0, 1, \ldots, N$) and integrating over a total solid angle 4π, taking into account the orthogonality of the corresponding Legendre polynomials, we obtain a system of independent differential equations for the expansion coefficients $y_k(z)$ of the diffusion component of the light intensity. The kth order equation will have the form

$$\frac{k}{2k + 1} \frac{\mathrm{d}y_{k-1}(z)}{\mathrm{d}z} + \mu_k y_k(z) + \frac{k + 1}{2k + 1} \frac{\mathrm{d}y_{k+1}(z)}{\mathrm{d}z} = q_k(z), \tag{8}$$

where $\mu_k = \mu_{\mathrm{a}} + \mu_{\mathrm{s}}(1 - g^k)$. The boundary conditions required for the solution of this system can be found from expression (5). By solving the system (8) and substituting the values of $y_k(z)$ into (6a), we can sum up the obtained series and find $L_{\mathrm{dif}}(z, \boldsymbol{s})$ and, hence, by using (3), also find the diffusion component $I_{\mathrm{dif}}(z)$ of the light intensity in the medium.

Because it is impossible in principle to solve a system of an infinite number of differential equations, the number of terms in expansions (6) is truncated. The case $N = 1$ corresponds to the diffusion approximation. In this approximation, the spatial distribution of the light intensity in a medium is described by the expression

$$h(z) = \frac{I(z)}{I_0} = \exp(-\mu_{\mathrm{t}} z) + \frac{3}{2\mu_{\mathrm{eff}} l^*} \times \left\{ \exp(\mu_{\mathrm{eff}} l^*) - \exp\left[-\mu_{\mathrm{eff}} l^* (2\Delta + 1) \right] \right\} \exp(-\mu_{\mathrm{eff}} z), \tag{9}$$

where

$$\Delta = \frac{2}{3} \left(\frac{1 + R_{\mathrm{eff}}}{1 - R_{\mathrm{eff}}} \right);$$

R_{eff} is the effective coefficient of reflection of diffuse radiation from the interface between media [24]. The first term in (9) describes the coherent component, which rapidly decreases with distance from the interface, and at distances $z > (2 \div 3) l^*$ the light intensity is determined only by the diffusion component of the scattered field [the second term in (9)] because the extinction coefficient is $\mu_{\mathrm{eff}} \ll \mu_{\mathrm{s}}$ in the approximation $\mu_{\mathrm{a}} \ll \mu_{\mathrm{s}}$.

The analytic expression for the spatial distribution $h(z)$ of the light intensity in a medium, obtained within the framework of the diffusion theory ($N = 1$) in the region $0 \leqslant z \leqslant (2 \div 3) l^*$, incorrectly describes a real distribution (see, for example, [6, 23, 24]. For this reason, as noted in [24], the spatial distribution of the light intensity within a small near-surface region of a medium should be calculated by using higher expansion orders N in (6).

Assuming that $N = 3$ in (6), we obtain from (8) a system of four differential equations for the expansion coefficients

$y_0(z), \ldots, y_3(z)$ of the diffusion component of the ray intensity of light in the medium

$$\mu_0 y_0(z) + \frac{\mathrm{d}y_1(z)}{\mathrm{d}z} = q_0(z), \tag{10a}$$

$$\frac{1}{3}\frac{\mathrm{d}y_0(z)}{\mathrm{d}z} + \mu_1 y_1(z) + \frac{2}{3}\frac{\mathrm{d}y_2(z)}{\mathrm{d}z} = q_1(z), \tag{10b}$$

$$\frac{2}{5}\frac{\mathrm{d}y_1(z)}{\mathrm{d}z} + \mu_2 y_2(z) + \frac{3}{5}\frac{\mathrm{d}y_3(z)}{\mathrm{d}z} = q_2(z), \tag{10c}$$

$$\frac{3}{7}\frac{\mathrm{d}y_2(z)}{\mathrm{d}z} + \mu_3 y_3(z) = q_3(z). \tag{10d}$$

The boundary conditions (5) have the form

$$4(-2R_1+1)y_0 + 8(3R_2+1)y_1 + 5(-12R_3+4R_1+1)y_2 + 28(5R_4-3R_2) = 0, \tag{11a}$$

$$(-20R_3+12R_1+1)y_0 + 12(5R_4-3R_2) + 5(-30R_5+28R_3 - 6R_1+1)y_2 + 2(175R_6-210R_4+63R_2+4)y_3 = 0, \tag{11b}$$

where all the values of $y_k(z)$ are taken for $z = 0$, and

$$R_k = \int_0^{\pi/2} R_{\mathrm{Fr}}(\theta)\cos^k\theta\sin\theta\,\mathrm{d}\theta. \tag{12}$$

By solving system (10) with boundary conditions (11) and taking into account that $y_k(z\to\infty)\to 0$, we obtain the spatial distribution of the light intensity in the P_3 approximation:

$$h(z) = \frac{I_{\mathrm{dif}}(z) + I_{\mathrm{coh}}(z)}{I_0} = C_1\exp(-\alpha_1 z) + C_2\exp(-\alpha_2 z) + (m_0+1)\exp(-\mu_t z), \tag{13}$$

where $I_{\mathrm{coh}}(z)$ can be found from (4a); $C_{1,2}$ are constants determined from (11); m_0 and $\alpha_{1,2}$ are complicated functions of the optical characteristics μ_a, μ_s and g of the medium, which are calculated by solving this system using a computer.

In the case of the P_5 approximation, a greater number of terms $y_k(z)$ are taken into account in expansion (6a). The general solution of the obtained system can be found in the form

$$h(z) = \frac{I_{\mathrm{dif}}(z) + I_{\mathrm{coh}}(z)}{I_0} = \tilde{C}_1\exp(-\tilde{\alpha}_1 z) + \tilde{C}_2\exp(-\tilde{\alpha}_2 z) + \tilde{C}_3\exp(-\tilde{\alpha}_3 z) + (m_k+1)\exp(-\mu_t z), \tag{14}$$

where unknown constants $\tilde{C}_{1-3}$ and parameters m_k and $\tilde{\alpha}_{1-3}$ can be found from the corresponding boundary conditions [see expression (5)], as in the case of the P_3 approximation.

Thus, the spatial distribution $h(z)$ of the light intensity in a medium with the known optical characteristics μ_a, μ_s and g can be calculated by solving the radiation transfer equation in the diffusion, P_3, and P_5 approximations.

3. Theoretical model of a pulsed optoacoustic effect in a scattering medium

Under the assumptions made in Section 2, a medium under study can be described by some 'effective' parameters: the specific heat c_p, the sound speed V_0, the thermal expansion coefficient β, and the temperature conductivity χ. If the relaxation time $\sim 1/(\mu_{\mathrm{eff}}^2\chi)$ of a thermal field in the heated region is much longer than the laser-pulse duration τ_L, then the diffusion of heat during laser heating of a medium can be neglected. Acoustic perturbation caused by absorption of a short laser pulse ($\mu_{\mathrm{eff}}V_0\tau_L \ll 1$) in a medium can be calculated by representing the light intensity in the form $I_0 f(t)H(z) = E_0\delta(t)H(z)$, where E_0 is the power density of the incident laser radiation. In this case, the time dependence of pressure in an acoustic wave produced in the absorbing medium has the form [13, 18].

$$P(\tau) = \frac{\beta V_0^2}{2c_p}\mu_a E_0 \begin{cases} H(-V_0\tau), & \tau < 0, \\ R_{\mathrm{ac}}H(V_0\tau), & \tau > 0. \end{cases} \tag{15}$$

Here, $\tau = t - z/V_0$; $R_{\mathrm{ac}} = (1-Z)/(1+Z)$ is the coefficient of reflection of the ultrasonic wave from the scattering medium-transparent medium interface; Z is the ratio of acoustic impedances of the absorbing and transparent media. Therefore, the optoacoustic signal (15) represents a compression wave followed by a dilatation (for $Z > 1$) or a compression (for $Z < 1$) wave.

One can see from expression (15) that the leading edge of the optoacoustic signal $P(\tau < 0)$ is proportional to the spatial distribution $H(z)$ of the light intensity in the medium, the time scale of variation in P and the spatial scale of variation in H being related via the sound speed as $z = -V_0\tau$. Upon a direct detection of optoacoustic signals (in an absorbing medium) [14, 18], the instant of time $\tau = 0$ corresponds to the arrival of a signal excited on the surface $z = 0$ of the medium under study to an acoustic detector. In the case of an acoustically rigid boundary, pressure $P(\tau)$ has a local minimum at $\tau = 0$ (Fig. 3), which corresponds to a local minimum of the distribution $H(z)$ of the light intensity in the medium at $z = 0$ [18]. Because the time dependence of the leading edge $P(\tau < 0)$ of the optoacoustic signal is determined by the spatial dependence $H(z)$, by measuring pressure at the leading edge of the optoacoustic signal and normalising it to $(\beta V_0^2/2c_p)\mu_a E_0$, we can calculate the spatial distribution of the light intensity in the medium [17]. Measurements in media with different coefficients μ_a and μ_s' provide information on the properties of the distribution $H(z)$. In addition, the function $H(z) = h(z)$ at distances $z \geqslant (2\div 3)l^*$, and therefore [see (14), (15)]

$$P(z) = \frac{\beta V_0^2}{4c_p}\mu_{\mathrm{eff}}E_0\left\{\exp(\mu_{\mathrm{eff}}l^*) - \exp\left[-\mu_{\mathrm{eff}}l^*(2\Delta+1)\right]\right\}\exp(-\mu_{\mathrm{eff}}z). \tag{16}$$

The coefficients μ_a and μ_s' can be calculated from pressure at the leading edge of an optoacoustic signal detected with a high time resolution [18]. However, one can see from expression (16) that the value of μ_{eff} in the medium can be obtained from the approximation of the leading front of the optoacoustic signal (Fig. 3) detected in relative units. The position of the maximum $z_{\max} = -\tau_{\max}V_0$ of the spatial distribution of the light intensity is also determined from

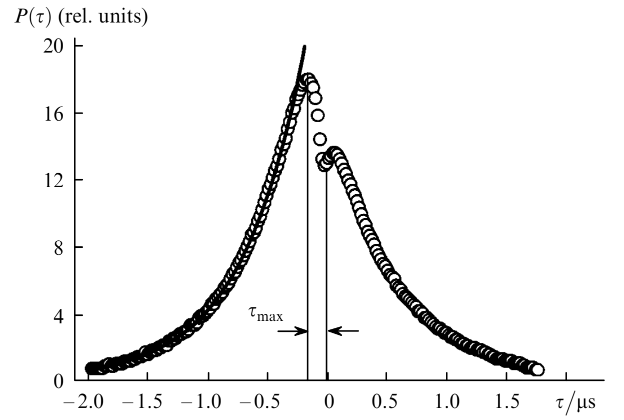

Figure 3. Typical profile of an optoacoustic signal excited in a scattering medium with an acoustically rigid boundary. The curve is the approximation by the function $P(\tau) \sim \exp(\mu_{eff} V_0 \tau)$ for $\tau < 0$.

the shape of the optoacoustic signal. This allows us to plot the dependence of $z_{max}\mu_{eff}$ on μ_a/μ_{eff} (which is similar to that in Fig. 2), where all the quantities can be found experimentally from the profile of the optoacoustic signal.

4. Experimental setup

The spatial distribution of the light intensity and the optical characteristics of strongly scattering media were measured using a setup with a direct detection of optoacoustic signals [14, 18]. Optoacoustic signals were excited by 10−12-ns pulses from a *Q*-switched 1.06-μm Nd:YAG laser. The pulse energy was 50−70 mJ. An optoacoustic signal excited in the medium was detected with broadband piezoelectric detectors made of a PVDP film of thickness 110 and 30 μm. The detectors were absolutely calibrated [25] in ranges 0.05−8 and 0.01−30 MHz, respectively, and their low-frequency sensitivity was 13.5 ± 0.1 and 4.5 ± 0.3 mV Pa^{-1}, respectively. To obtain the acoustically rigid boundary of the medium, a cell was covered by a quartz plate ($Z = 0.12, R_{ac} = 0.79$).

This setup is capable of exciting and detecting acoustic pulses of duration from 200 ns to 10 μs with the pressure amplitude of 2−3 Pa (upon signal averaging over 64 realisations). This provides the measurement of μ_{eff} in the range from 1.5 to 100 cm^{-1} for $\mu_a > 0.05$ cm^{-1}.

5. Media studied

For test measurements, we used a medium with known optical properties − an aqueous suspension of polystyrene microspheres (the radius of particles was $r_0 = 0.38$ μm and their volume concentration was $N_v = 1\,\%$). The values $\mu_s' = 21.8$ cм$^{-1}$ and $g = 0.782$ were calculated using the Mie theory [26] for the known values of r_0 and N_v and the refractive index of polystyrene $n_0 = 1.56$ at 1.06 μm. The absorption coefficient of the initial suspension was assumed equal to that of distilled water $\mu_a = 0.17$ cm^{-1} at 1.06 μm [27] because absorption of polystyrene at this wavelength does not exceed 0.05 cm^{-1} and the concentration of scatterers was low.

We studied suspensions of particles of titanium oxide TiO_2 in water (the average size of particles was less than 1 μm, $N_v = 0.2 - 1.7\,\%$) and 3.5 % fatness milk as scattering media with unknown optical properties. Because of a low volume concentration of particles, the refractive index and thermal parameters of the media were assumed equal to those of water: $n_2 = 1.33$ at 1.06 μm, , $\beta = 1.82 \times 10^{-4}$ K^{-1}, $c_p = 4.18$ J g^{-1} K^{-1}, $\chi = 1.43 \times 10^{-3}$ cm^2 s^{-1}, $V_0 = (1.49 \pm 0.01) \times 10^5$ cm s^{-1} [27]. We also assumed that $g \sim 0.8 - 0.9$.

The absorption coefficient μ_a was varied by adding different amounts of black India ink (0.02−1.2 ml) to a fixed volume of the medium (100 ml). We assumed that the absorption coefficient changed proportionally to the Indian ink concentration, whereas the scattering coefficient remained unchanged. Thus, the absorption coefficient μ_a of media under study was changed from 0.17 to 14.8 cm^{-1}. The reduced scattering coefficient μ_s' was in the range from 18 to 52 cm^{-1} and was measured from the absolute value of pressure produced by an optoacoustic signal [18].

6. Experimental results

To demonstrate the possibility of the experimental study of the properties of the distribution $H(z)$ in a scattering medium, i.e., of correct measurements of z_{max} from the temporal profile of an optoacoustic pulse, we tested the optoacoustic method in a medium with known optical properties − an aqueous solution of polystyrene microspheres. The absorption coefficient of the solution was varied by adding black Indian ink of a certain concentration. The parameters μ_{eff} and $z_{max} = -V_0\tau_{max}$ were measured from the temporal profile of the optoacoustic signal (Fig. 3) up to $\mu_a/\mu_{eff} = 0.21$. The relative error of measurements of τ_{max} was 5 %−6 % in the range $\tau_{max} = 5 - 200$ ns and was determined by a finite frequency band of piezoelectric detectors used in the experiments.

The experimental dependence of $z_{max}\mu_{eff}$ on μ_a/μ_{eff} is shown in Fig. 4, where the dependence calculated by the Monte Carlo method is also presented for comparison. The relative errors of measurements of values of μ_a/μ_{eff} and $z_{max}\mu_{eff}$ were 3 %−4 % and 6 %−7 %, respectively. One can see that the experimental dependence coincides with the Monte Carlo simulation within the error of measurements. This means that both these methods yield the same results, and we can use the optoacoustic method for measurements in media with unknown optical properties as well.

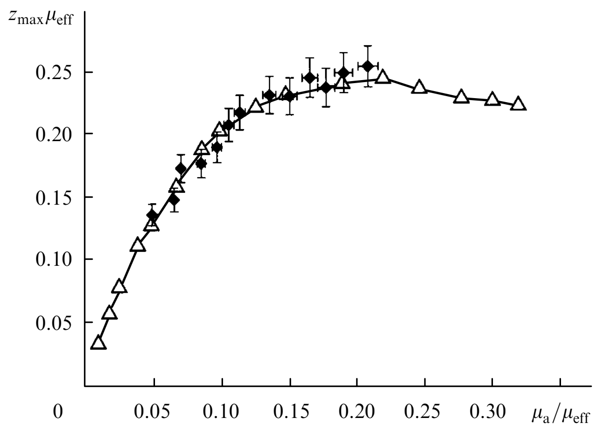

Figure 4. Dependences of $z_{max}\mu_{eff}$ on the ratio μ_a/μ_{eff} for a suspension of polystyrene spheres obtained by the Monte Carlo simulation (△) and experimentally (●).

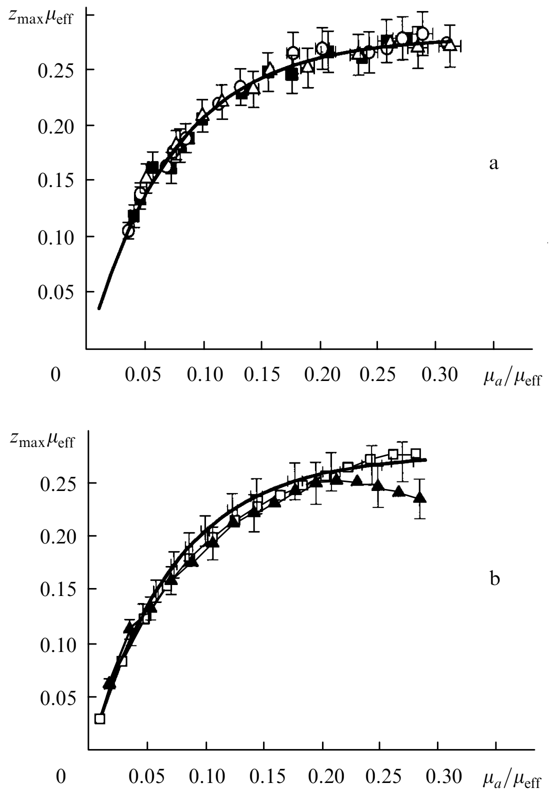

Figure 5. Experimental dependences of $z_{max}\mu_{eff}$ on the ratio μ_a/μ_{eff} for two aqueous suspensions of TiO_2 particles [$\mu_s' = 35.5$ (■) and 51.1 cm^{-1} (○)] and milk [$\mu_s' = 21.6$ cm^{-1} (△)] (a) and Monte Carlo simulations [$g = 0.80$ (▲) and 0.95 (□)] (b). The thick curve is phenomenological dependence (17).

Fig. 5a shows the experimental dependence of $z_{max}\mu_{eff}$ on μ_a/μ_{eff} for media with fixed μ_s': milk ($\mu_s' = 21.6$ cm^{-1}) and two suspensions of TiO_2 with different concentrations ($\mu_s' = 35.5$ and 51.5 cm^{-1}). For comparison, Fig. 5b shows the Monte Carlo simulations for $g = 0.8$ and 0.95. The relative errors of measurements of μ_a/μ_{eff} and $z_{max}\mu_{eff}$ were 3 % – 4 % and 6 % – 7 %, respectively. One can see that the experimental data obtained in different media with different reduced scattering coefficients μ_s' coincide with each other and the Monte Carlo simulations within the error of measurements. Thus, the experimental data confirm the prediction of the Monte Carlo simulation (see Fig. 2) that the quantity $z_{max}\mu_{eff}$ is a function of the ratio μ_a/μ_{eff} and is independent of the absolute values of μ_a and μ_{eff} for $g > 0.8$. This allows us to approximate the experimental points, for example, by the function

$$Y_{fit} = 0.276\left[1 - \exp\left(-13.42\frac{\mu_a}{\mu_{eff}}\right)\right], \quad (17)$$

which is shown by thick curves in Figs 5a, b. Note that the values of μ_{eff} and z_{max} are determined from the leading edge $P(\tau < 0)$ of the optoacoustic signal.

The experimental study of the spatial distribution of the laser radiation intensity in strongly scattering media and Monte Carlo simulations showed that the value of μ_a can be obtained from the experimental values of μ_{eff} and z_{max} using the universal dependence

$$\mu_a = -0.074\mu_{eff}\ln(1 - 3.62z_{max}\mu_{eff}). \quad (18)$$

Then, we can calculate the value of the reduced scattering coefficient from the expression $\mu_s' = \mu_{eff}^2/3\mu_a$.

7. Calculations by solving the radiation transfer equation

We have shown in Section 2.2 that the spatial distribution $H(z)$ of the light intensity in a scattering medium can be calculated in the diffusion, P_3 and P_5 approximations if the optical characteristics of the medium are specified. The solution obtained in the diffusion approximation gives $z_{max} = l^*$, which is incorrect because the diffusion approximation itself is valid only at distances $z > (2 \div 3)l^*$ [21, 22]. The dependences of $z_{max}\mu_{eff}$ on μ_a/μ_{eff} obtained in the P_3 and P_5 approximations for the anisotropy factors $g = 0.80 - 0.95$ are presented in Fig. 6. For comparison, phenomenological dependence (17) is also shown in this figure.

One can see from Fig. 6 that the dependences of $z_{max}\mu_{eff}$ on the ratio μ_a/μ_{eff} strongly differ from the phenomenological dependence (17) for $\mu_a/\mu_{eff} > 0.12$ and $\mu_a/\mu_{eff} > 0.15$ for the P_3 and P_5 approximations, respectively. This discrepancy can be caused by several reasons. First, as μ_a increases, the maximum of the light intensity in scattering media under study shifts to their surface and gets into the region where even six terms in expansion (6a) are not sufficient for the adequate description of the angular spectrum of light in the medium. Our results show that the regions of applicability of the P_3 and P_5 approximations can be estimated as $z > (0.6 \div 0.7)l^*$ and $z > (0.4 \div 0.5)l^*$, respectively. To describe correctly the spatial distribution $H(z)$ of the light intensity at smaller distances from the surface of the medium under study, it is necessary to use higher expansion orders in (6).

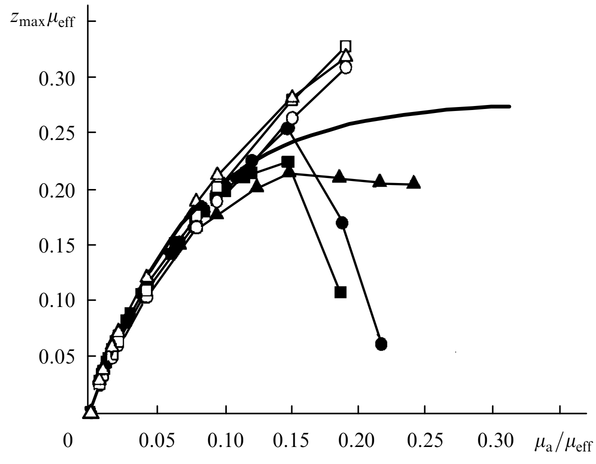

Figure 6. Comparison of phenomenological dependence (17) (thick curve) with the results obtained in the P_3 (open symbols) and P_5 (dark symbols) approximations for $g = 0.80$ (△, ▲), 0.90 (□, ■) and 0.95 (○, ●).

The second source of errors in the analytic approach is the use of the Henney – Greenstein function (2) in the form (6b). Note that it is very difficult to estimate the influence of this factor on the accuracy of description of the function $H(z)$, and this question should be probably studied in a separate paper. It is clear, however, that the accuracy of description of $H(z)$ will improve with increasing approximation order, and the region in which the applied approximation correctly describes the function $H(z)$ will increase.

8. Conclusions

We have studied the spatial distribution of the laser radiation intensity in model scattering media by the Monte Carlo method, by solving the radiation transfer equation, and by the optoacoustic method.

We have investigated the dependence $z_{\max}\mu_{\text{eff}}(\mu_a/\mu_{\text{eff}})$ by the Monte Carlo method for $\mu_a/\mu_{\text{eff}} < 0.32$ and different anisotropy factors g and have found that for $g > 0.8$ the dependences coincide with each other within the calculation error.

We have found by the optoacoustic method that the dependence $z_{\max}\mu_{\text{eff}}(\mu_a/\mu_{\text{eff}})$ for model scattering media (with the anisotropy factor $g > 0.8$) for $\mu_a/\mu_{\text{eff}} < 0.35$ is determined only by the ratio μ_a/μ_{eff} and is independent of the absolute values of μ_a and μ_{eff}, i.e., it has a universal form. The optical characteristics μ_a and μ_s' of homogeneously scattering condensed media can be determined, using this dependence, from the temporal profile of an optoacoustic signal detected in relative units.

We have tested the approximate analytic method for solving the radiation transfer equation in a strongly scattering medium. The dependences $z_{\max}\mu_{\text{eff}}(\mu_a/\mu_{\text{eff}})$ obtained in the P_3 and P_5 approximations coincide for $\mu_a/\mu_{\text{eff}} < 0.12$ and $\mu_a/\mu_{\text{eff}} < 0.15$ respectively, and $g = 0.80 - 0.95$ with optoacoustic measurements and Monte Carlo simulations within the error of $5\% - 7\%$.

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