Quantum Electronics **32** (10) 925 – 928 (2002)

LASER APPLICATIONS AND OTHER TOPICS IN QUANTUM ELECTRONICS

PACS numbers: 06.30.Bp; 06.60.Jn
DOI: 10.1070/QE2002v032n10ABEH002319

Absolute length measurements with a femtosecond laser

E.V. Baklanov, A.K. Dmitriev

***Abstract*. It is proposed to use radiation from a femtosecond laser for absolute length measurements. The possibility of creation of a unified frequency and length standard is shown.**

Keywords: *femtosecond laser, precision measurements, frequency and length standards.*

1. Introduction

A great recent progress in the field of measuring optical frequencies has been achieved due to the use of femtosecond lasers. A periodic train of pulses generated by such lasers contains a set of equidistant frequencies, which allows measuring the absolute values of frequencies in a broad range from radio frequencies to the UV region [1 – 3].

Femtosecond lasers can be used not only for frequency measurements but also for measurements of other quantities. In this paper, we propose to use a femtosecond laser for length measurements and creation of a unified frequency and length standard.

2. Radiation of a frequency-stable femtosecond laser

A femtosecond laser emits a periodic train of pulses, whose spectrum consists of a frequency set

$$\omega_m = m\Delta\omega - \Omega, \tag{1}$$

where m are positive numbers;

$$\Delta\omega = \frac{2\pi c}{l} \tag{2}$$

is the mode frequency interval; l is the distance between pulses corresponding to the total laser cavity length; and Ω is the frequency shift, which is common for all the modes ($0 \leqslant \Omega \leqslant \Delta\omega$) [4, 5].

E.V. Baklanov, A.K. Dmitriev Institute of Laser Physics, Siberian Branch, Russian Academy of Sciences, prosp. akad. Lavrent'eva 13/3, 630090 Novosibirsk, Russia; e-mail: balk@laser.nsc.ru.; dak@laser.nsc.ru

Received 16 May 2002
Kvantovaya Elektronika **32** (10) 925 – 928 (2002)
Translated by M.N. Sapozhnikov

Thus, the frequency of each of the modes is determined by the mode interval $\Delta\omega$ and the shift Ω. It is obvious that these two parameters should be stable during the measurements of absolute frequency values.

The mode frequency interval $\Delta\omega$ can be stabilised by using a highly stable radio-frequency generator with the frequency $\Delta\omega$. This method was first realised in a He – Ne laser [6, 7], and then in a Ti : sapphire laser [8 – 10]. The mode interval $\Delta\omega$ can be also stabilised by coupling to the frequency of a stable laser [10]. The frequency shift Ω can be determined by the method discussed in paper [5] (see details in Section 4).

3. Propagation of a periodic pulse train through an interferometer

Consider a perfect Fabry – Perot interferometer. The intensity of radiation of frequency ω_m transmitted by the interferometer is

$$I_m(L) = I_m \frac{1}{1 + p\sin^2(\omega_m L/c)}, \tag{3}$$

where I_m is the incident wave intensity;

$$p = \frac{4(1-T)}{T^2}; \tag{4}$$

T is the transmission coefficient of the interferometer mirrors; and L is its length. We calculated the transmission of a periodic pulse train by the interferometer, assuming that each spectral component is much narrower than the transmission band of the interferometer. In this case, the components can be considered independently of each other, and the radiation intensity transmitted by the interferometer is described by the expression

$$I(L) = \sum_m I_m(L). \tag{5}$$

By introducing the dimensionless variable $z = L/l$, we can rewrite (5) in the form

$$I(z) = \sum_m I_m \frac{1}{1 + p\sin^2[\pi(m+q)z]}, \tag{6}$$

where

$$q = \Omega/\Delta\omega \qquad (0 \leqslant q < 1). \tag{7}$$

Resonance peaks appear in the Fabry–Perot interferometer upon its detuning approximately by a wavelength, so that sometimes it is convenient to use the variable

$$x = (z - a)m'_0, \tag{8}$$

instead of z, where a is the initial length with respect to which the value of z is changed; and m'_0 is the number of one of the modes. Then,

$$I(x) = \sum_m I_m \frac{1}{1 + p\sin^2\{\pi[x(m+q)/m'_0 + qa + ma]\}}. \tag{9}$$

In the optical range, we have $m \gg 1$, so that for $x < 1$, we can write instead of (9), the expression

$$I(x) = \sum_m I_m \frac{1}{1 + p\sin^2[\pi(xm/m'_0 + qa + ma)]}. \tag{10}$$

We assume for simplicity that the emission spectrum contains an odd number $(2N+1)$ of modes of the same intensity $I_m = I/(2N+1)$, where $I = \sum_{m=m_0-N}^{m_0+N} I_m$ is the total intensity of all the modes; and m_0 is the number of the central mode.

Let us analyse the case of $q = 0$. We have, instead of (6),

$$I(z) = \frac{1}{2N+1}\sum_{m=m_0-N}^{m_0+N}\frac{1}{1 + p\sin^2(\pi mz)}. \tag{11}$$

The function $I(z)$ is periodic, with a period equal to unity. We will call maxima at points $z = n$ (n = 1, 2, 3,...) and the corresponding transmission bands the main maxima and bands; $I(n) = I$, i.e., when $z = n$, the interferometer is completely transparent. This is simply explained by the fact that each of the modes emitted by a femtosecond laser exactly coincides with the corresponding mode of the interferometer.

Along with the main transmission maxima, there also exist additional maxima at points $z = n \pm b/d$. When $N/d \gg 1$, the additional maxima are described by the expression

$$I\left(n \pm \frac{b}{d}\right) = \frac{I}{d}, \tag{12}$$

where b/d is a simple fraction. These maxima appear because, when the interferometer length differs from the position corresponding to the main maximum by the value b/d, the number $2N/d$ of radiation modes of the laser exactly coincides with the number of interferometer modes. For example, for $d = 2$, only half of the modes make a contribution to the additional maximum at the point $z = n \pm 1/2$, and for this reason its intensity is less by half than that of the main maximum.

The number of modes contributing to the main and additional maxima is much greater than unity. If this number is of the order of unity, then the corresponding transmission bands can be treated as a background.

We will calculate the amplitudes and widths of the transmission bands of the interferometer from expression (9). For $q = 0$, we have

$$I(x) = \frac{1}{2N+1}\sum_{m=m_0-N}^{m_0+N}\frac{1}{1 + p\sin^2[\pi(xm/m_0 + ma)]}. \tag{13}$$

Let us analyse the behaviour of the maxima of $I(x)$ near the main maximum (a is an integer). In this case,

$$I(x) = \frac{1}{2N+1}\sum_{m=m_0-N}^{m_0+N}\frac{1}{1 + p\sin^2(\pi xm/m_0)}. \tag{14}$$

If the width of the emission spectrum of a femtosecond laser is much smaller than the average emission frequency ($m_0 \gg N$), then, by making the substitution $m \to m_0$ under the sum sign, we have for $x < 1$

$$I(x) = \frac{I}{1 + p\sin^2(\pi x)}. \tag{15}$$

The result is, of course, the same as for monochromatic emission, because in fact we neglected the difference between mode frequencies in the femtosecond laser. The transmission maxima ($I(x_k) = 1$) are located at points $x_k = k$, where $k = 0, \pm 1, \pm 2, \ldots$.

If $x \geqslant m_0/(N\sqrt{p})$, then, for $k \neq 0$, the maxima of the bands, which we now call sidebands, noticeably decrease. Let us find the expression for the amplitudes of these maxima. For $x \leqslant m_0/\sqrt{p}$, the sum in (14) can be replaced by the integral. For $x = x_k$, we have

$$I(x_k) = \frac{1}{2N}\int_{m_0-N}^{m_0+N} \mathrm{d}m \frac{1}{1 + p\sin^2(\pi km/m_0)}. \tag{16}$$

After integration, we obtain

$$I(x_k) = \frac{m_0}{\pi kN(1+p)^{1/2}}\arctan\left[(1+p)^{1/2}\tan\left(\frac{\pi kN}{m_0}\right)\right]. \tag{17}$$

For $\pi kN/m_0 \leqslant 1$, we rewrite (17) in the form

$$I(x_k) = \frac{1}{k\beta}\arctan(k\beta), \tag{18}$$

where $\beta = \pi N(1+p)^{1/2}/m_0$ is the characteristic parameter of the problem. For $k\beta \leqslant 1$, we have

$$I(x_k) = 1 - \frac{k^2\beta^2}{3}. \tag{19}$$

In the main maximum ($k = 0$), the interferometer transmits all radiation, whereas the amplitudes of sidebands decrease proportionally to k^2.

For $k\beta \gg 1$, we have

$$I(x_k) = \frac{\pi}{2k\beta}. \tag{20}$$

It follows from (15) that for $p \gg 1$, the half-width of the main transmission bands $I(x)$ is

$$\gamma = \frac{1}{\pi\sqrt{p}}, \tag{21}$$

coinciding with that in the case of monochromatic emission. For $k\beta \leqslant 1$, the half-width of sidebands is

$$\gamma_k = \gamma\left(1 + \frac{k^2\beta^2}{3}\right). \tag{22}$$

Fig. 1 shows the dependence of the interferometer transmission on its length, calculated from expression (14). The envelope of the maxima is described by expression (18). One can see that the replacement of the sum by the integral is valid in the calculation of the maxima.

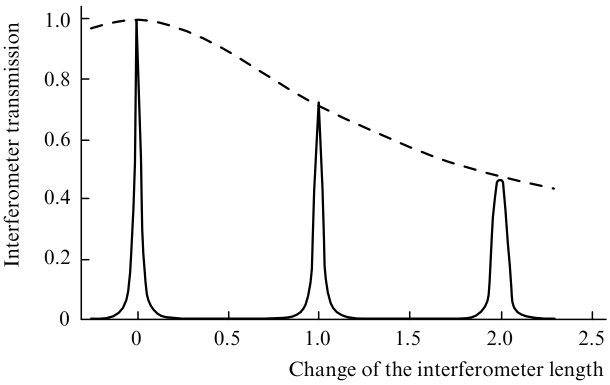

Figure 1. Dependence of the interferometer transmission on its length, plotted by expression (14) for $N = 20$, $m_0 = 1000$, $p = 400$ (solid curve), and the envelope of transmission maxima (18) (dashed curve).

The experimental dependence of the interferometer transmission near the additional maximum $L/l = b/d = 1/2$, which was observed in [11], coincides with the dependence shown in Fig. 1.

4. Measurement of the interferometer length

Consider a femtosecond laser with a tunable mode interval, which is determined only by the laser cavity length l (2). By varying l, we can tune to one of the main transmission maxima of the interferometer, which corresponds to a certain length l_n of the laser cavity. In this case, the mode interval is

$$\Delta\omega_n = \frac{2\pi c}{l_n}, \tag{23}$$

and the interferometer length being measured is

$$L = nl_n. \tag{24}$$

We determine $\Delta\omega_n$ by measuring l_n. To determine L in (24), it is necessary to know the number n of the main maximum. When the interferometer length is large ($n \gg 1$), this problem is nontrivial. Let us use the following approach. Because $nl_n = (n+1)l_{n+1}$, the change in the laser cavity length after tuning to the adjacent interferometer maximum is

$$\Delta l = l_n - l_{n+1} = \frac{l_{n+1}}{n}. \tag{25}$$

By using (23), we obtain

$$n = \frac{\Delta\omega_n}{\Delta\omega_{n+1} - \Delta\omega_n}. \tag{26}$$

By measuring $\Delta\omega_n$ and $\Delta\omega_{n+1}$ for lengths l_n and l_{n+1}, and then rounding off the right-hand side of (26) to an integer, we find n. Knowing n from (26) and l_n from (23), we determine the interferometer length.

From (24) and (25), we obtain

$$L = n(n-1)\Delta l. \tag{27}$$

Relation (27) shows that, by slightly varying the femtosecond laser cavity length, we can increase considerably the interferometer length being measured. For example, if the laser cavity length $l_n = 10$ cm ($n = 10^4$), then $\Delta l = 10$ μm, and the interferometer length being measured (24) is 1 km. A small variation Δl in the laser cavity length can be easily achieved by various methods, for example, using piezoelectric ceramics.

In the cases when the required variation Δl in the cavity length is quite large and cannot be easily realised for technical reasons, we can use additional maxima. This reduces the required variation in the cavity length, but complicates measurements.

The interferometer length can be also measured by using sideband maxima (Fig. 1). A change in the laser cavity length for neighbouring sidebands is of the order of the wavelength and is l^2/m_0L. However, in this case, the problem of identifying the sideband maximum appears.

Fig. 2 shows a scheme for measuring the interferometer length with a femtosecond laser. Radiation from the femtosecond laser (*1*) is incident on the Fabry–Perot interferometer (*2*). Reflected radiation is detected by a photodetector (*3*). A signal from a synchronous detector (*4*) controls a frequency synthesiser (*5*). The laser frequency is locked to the interferometer by the frequency modulation method. An audio-frequency generator (*6*) modulates the femtosecond laser frequency to produce a probe signal. Tuning to one of the transmission maxima of the interferometer is performed with the help of a phase-locked frequency control unit (*7*). A photodetector (*8*) is used for separating the mode interval $\Delta\omega$.

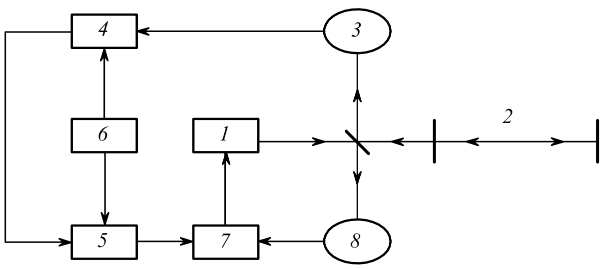

Figure 2. Principal scheme for measuring the interferometer length: (*1*) femtosecond laser; (*2*) Fabry–Perot interferometer; (*3*, *8*) photodetectors; (*4*) synchronous detector; (*5*) frequency synthesiser; (*6*) audio-frequency generator; (*7*) phase-locked frequency control unit.

In the presence of the frequency shift ($q \neq 0$), we have, instead of (24),

$$L = \frac{nl_n}{1 + q/m_0}. \tag{28}$$

The relative contribution of the frequency shift to the error of the length measurement is small. Thus, if we assume that $m_0 = 10^6$, then the parameter $q/m_0 < 10^{-6}$. However, when measuring the interferometer length with the error less than q/m_0, it is also necessary to determine q and m_0. This can be avoided by eliminating the frequency shift q in the emission spectrum of a femtosecond laser. Fig. 3 shows a possible scheme for compensating q. One of the low-frequency modes $\omega_m = (m-q)\Delta\omega$ of the laser is doubled in a nonlinear crystal (*4*). Emission at the frequency $2\omega_m = 2(m-q)\Delta\omega$ is mixed with emission at the frequency ω_{2m}, and the difference frequency $\Omega = q\Delta\omega$ is separated at the output of photodetector (*2*) [5]. Acoustooptic modulator (*5*) shifts each mode by Ω, providing the absence of the frequency shift Ω in emission.

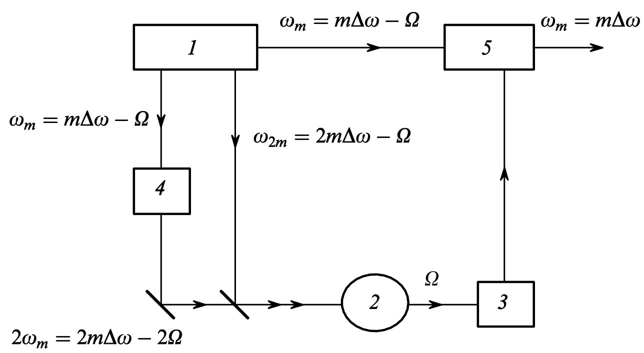

Figure 3. Principal scheme for compensating the frequency-shift $\Omega = q\Delta\omega$: (*1*) femtosecond laser; (*2*) photodetector: (*3*) amplifier; (*4*) frequency doubler; (*5*) acoustooptic modulator.

The relative accuracy of measurement of the interferometer length in this method is limited by the accuracy of measurement of $\Delta\omega$. The accuracy can be increased by measuring intermode beats not between the adjacent modes but between the modes separated by large frequency intervals (up to optical frequencies). The beat frequency in this case is $M\Delta\omega$, where M is a large number. For the same absolute error of frequency measurements, the accuracy of measurements of the interferometer length is proportional to M.

Another error is determined by the accuracy of tuning of the interferometer transmission to a maximum. Because the width of the main transmission bands of femtosecond radiation is the same as that for monochromatic emission, the tuning accuracy is the same in both cases. However, in the case of monochromatic emission, the transmission bands of the interferometer are separated by intervals equal to half the wavelength, and as a consequence, femtosecond emission, unlike monochromatic, allows one to measure much greater lengths.

5. Conclusions

We have solved in this paper the problem of propagation of radiation from a femtosecond laser through a perfect Fabry–Perot interferometer. The derived expressions can be used for numerical calculations. The transmission maxima of the interferometer can be used as reference lines for absolute length measurements.

An important property of a stable femtosecond laser is that it can be used for creation of a unified frequency and length standard. If a femtosecond laser is locked to the frequency standard (for example, to a cesium standard), then its emission can be used for synthesis of both frequencies (from radio frequencies to the UV region) and lengths (from microns to kilometres).

Acknowledgements. The authors thank S.N. Bagaev for his interest in this study and A.V. Denisov and V.S. Pivtsov for useful discussions. This work was supported by the Russian Foundation for Basic Research (Grant No. 02-02-16598).

References

1. Udem Th., Reichert J., Holzwarth R., Haensch T.W. *Phys. Rev. Lett.*, **82**, 3568 (1999).
2. Diddams S.A., Jonts D.J., Ye Jun, Cundiff S.T., Hall J.L., Ranka J.K., Windeler R.S., Holzwarth R., Udem T., Haensch T.W. *Phys. Rev. Lett.*, **84**, 5102 (2000).
3. Bagayev S.N., Dmitriyev A.K., Chepurov S.V., Dychkov A.S., Klementyev V.M., Kolker D.B., Kuznetsov S.A., Matyugin Yu.A., Okhapkin M.V., Pivtsov V.S., Skvortsov M.N., Zakharyash V.F., Birks T.A., Wadsworth W.J., Russell P.St.J., Zheltikov A.M. *Laser Phys.*, **11**, 1270 (2001).
4. Ye J., Hall J.L., Diddams S.A. *Opt. Lett.*, **25**, 1675 (2000).
5. Holzwarth R., Udem Th., Haensch T.W., Knight J.C., Wadsworth J., Russel P.St.J. *Phys. Rev. Lett.*, **85**, 2264 (2000).
6. Zakhar'yash V.F., Klement'ev V.M., Pyl'tsyn O.I., Chebotaev V.P. *Kvantovaya Elektron.*, **19**, 604 (1992) [*Quantum Electron.*, **22**, 558 (1992)].
7. Chebotayev V.P., Klementyev V.M., Pyltsyn O.I., Zakharyash V.F. *Appl. Phys. B*, **54**, 98 (1992).
8. Spence D.E., Ducloy J.M., Lamb K., Sleat W.E., Sibbett W. *Opt. Lett.*, **19**, 481 (1994).
9. Bagaev S.N., Zakhar'yash V.F., Klement'ev V.M., Pivtsov V.S., Chepurov S.V. *Kvantovaya Elektron.*, **24**, 327 (1997) [*Quantum Electron.*, **27**, 317 (1997)].
10. Bagayev S.N., Chepurov S.V., Klementyev V.M., Kuznetsov S.A., Pivtsov V.S., Pokasov V.V., Zakharyash V.F. *Appl. Phys. B*, **70**, 375 (2000).
11. Jones R.J., Diels J.-C., Jasapara J., Rudolf W. *Opt. Commun.*, **175**, 109 (2000).