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Effect of the intensity modulation on the operation of an adaptive system with an optical feedback

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***Abstract*. The effect of the intensity modulation on the operation of an adaptive system with a shearing interferometer in an optical feedback loop is studied numerically. It is shown that intensity fluctuations at the system input deteriorate the quality of compensation of phase fluctuations. The dependences of the residual compensation error on the statistical characteristics of the radiation phase and intensity fluctuations at the system input are obtained.**

Keywords: *adaptive optics, optical feedback loop, shearing interferometer*.

1. Introduction

The operation of adaptive systems with an optical feedback is based on the transformation of optical distortions in the intensity distribution of light, which is used to control a liquid-crystal (LC) phase modulator [1]. A control light beam was formed in a feedback loop with the help of an interferometer with a reference beam [2], a shearing interferometer [3, 4], the effect of phase visualisation upon diffraction [5], as well as a phase knife [6]. It was assumed in studies of these systems that the light-beam intensity does not exhibit fluctuations. However, this condition is not often fulfilled in practice, and the intensity distribution of light controlling an LC modulator depends not only on phase distortions of the light beam but also on the intensity modulation at the system input. In this paper, we study the influence of intensity fluctuation on the operation of an adaptive system with a shearing interferometer in an optical feedback loop.

2. Analysis of the system operation

Consider the principle of operation of an adaptive system with an optical feedback loop, whose general scheme is shown in Fig. 1.

Let a light beam with phase distortions $F(x, y, t)$ be incident on a system input. The light passes through a semiconductor mirror (*1*), an LC layer of a modulator (*2*), is reflected from the inner mirror of the modulator and is directed to the optical feedback loop with the help of the mirror (*1*). Phase distortions in the optical feedback loop are transformed with a shearing interferometer (*4*) to the control intensity distribution $\hat{I}(x, y, t)$, which is transferred to a photosensitive layer of the modulator with the help of a mirror (*5*). This is accompanied by a change in the refractive index of the LC layer of the modulator, and an additional correcting phase shift is introduced to the system, which is described by the equation of the diffusion type [2]

$$T_0 \frac{\partial U}{\partial t} + U = l_{\text{dif}}^2 \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) + G(\hat{I}), \quad (1)$$

where T_0 is the time constant of the modulator; x and y are the transverse coordinates; l_{diff} is the diffusion length characterising the spatial resolution of the LC modulator; and $G(\hat{I})$ is a statistical characteristic of the modulator.

The initial light beam with phase distortions $F(x, y, t)$ acquires the total phase $H(x, y, t) = F(x, y, t) + U(x, y, t)$ after reflection from the LC modulator. This phase is analysed at the system output after a semitransparent mirror (*3*).

When the control light intensity $\hat{I}$ is lower than the saturation intensity I_s of the LC modulator, the expression [1]

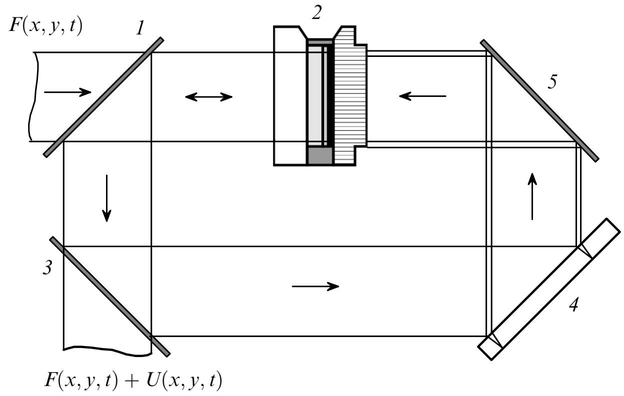

Figure 1. Principal scheme of an adaptive system with a shearing interferometer in an optical feedback loop; (*1, 3, 5*) mirrors; (*2*) LC modulator; (*4*) shearing interferometer.

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$$G(I) \approx kI - C_1,$$
$$I = \hat{I}/I_s, \tag{2}$$

is valid, where $k = \partial G/\partial I$; C_1 is a constant parameter determined by the threshold intensity of the LC modulator.

When a light beam with a uniform intensity distribution I_0 and phase modulation $H(x, y, t)$ is incident on a shearing interferometer, the light intensity distribution at the interferometer output is described by the expression [3]

$$I \simeq I_0[1 + \gamma \cos(\Delta H + \Delta_0)],$$
$$\Delta H(x, y, t) = H(x + S, y, t) - H(x, y, t), \tag{3}$$

where γ is a contrast of the interference pattern; Δ_0 is the average phase shift determining the position of a working point of the interferometer; S is the transfer displacement of beams in the interferometer. We assume hereafter that $\gamma = 1$.

When the total phase $H(x, y, t)$ is small, the system operation can be analysed in a linear approximation assuming that

$$\cos(\Delta H + \Delta_0) \approx \cos \Delta_0 + \Delta H \sin \Delta_0. \tag{4}$$

In this approximation, the stationary coefficient of suppression of harmonic phase distortions

$$V = \left| \frac{F_\omega + U_\omega}{F_\omega} \right|$$

(U_ω and F_ω are the amplitudes of harmonics) is described by the expression [3]

$$V = \frac{D}{[D^2 + 2K_0(K_0 + D)(1 - \cos \Omega_x S)]^{1/2}},$$
$$D = 1 + l_{\text{dif}}^2(\Omega_x^2 + \Omega_y^2), \tag{5}$$
$$K_0 = -kI_0 \sin \Delta_0,$$

where Ω_x and Ω_y are spatial frequencies along axes x and y, respectively.

The suppression of phase perturbations corresponds to the case of $V < 1$. This inequality is fulfilled simultaneously for all spatial frequencies (the condition of the system stability), when the feedback coefficient $K_0 > 0$ (the sign of K_0 depends on the choice of the operating point Δ_0).

3. Influence of the intensity modulation near the aperture boundary

In practice, the beam width can exceed an aperture of the LC modulator of size A or can be comparable with it. In this case, a region appears near the aperture edge of the shearing interferometer where the light intensity undergoes a jump.

Let us assume that a beam with a plane wavefront and a uniform intensity distribution I_0 with a diameter that exceeds the aperture size arrives at the system input. In this case, the control intensity distribution $I(x, y, t)$ at the system output is described by the relation

$$I(x, y, t) = \begin{cases} I_0\{1 + \cos[\Delta U(x, y, t) + \Delta_0]\} & \text{for } 0 < x < A - S, \\ I_2 & \text{for } A - S < x < A, \end{cases}$$
$$\Delta U(x, y, t) = U(x + S, y, t) - U(x, y, t), \tag{6}$$

where I_2 is the intensity of light reflected from the rear surface of the interferometer. One can see that near the boundary of the LC corrector $x = A$, a region of width S is formed, in which the interfering beams are not crossed and phase distortions are not visualised, and the average light intensity differs from that inside the aperture. Upon transition through the boundary $x = A - S$, the light intensity undergoes a jump equal to $I_0 - I_2$.

We studied the influence of this region on the system operation by solving numerically equation (1) using an implicit scheme. The number of mesh points was chosen so that a step over spatial coordinates was substantially smaller than the diffusion length l_{dif}. We used the boundary conditions of the second kind

$$\frac{\partial U(0, y, t)}{\partial x} = \frac{\partial U(A, y, t)}{\partial x} = 0,$$
$$\frac{\partial U(x, 0, t)}{\partial y} = \frac{\partial U(x, A, t)}{\partial y} = 0. \tag{7}$$

The real behaviour of a photoinduced charge at the boundary can be more complicated, and it depends on the fabrication technology of the LC corrector. Alternatively, we can choose, for example, the zero boundary conditions of the first kind

$$U(0, y, t) = U(A, y, t) = U(x, 0, t) = U(x, A, t) = 0. \tag{8}$$

The numerical experiment showed that the solutions of equation (1) with boundary conditions (7) and (8) differ from each other only near the boundaries. At a distance from the boundary equal to several l_{dif}, their difference becomes insignificant.

Fig. 2 shows the phase profile calculated upon the feedback closure. One can see that the low-intensity region near the LC corrector boundary $x = A$ causes the phase modulation, which propagates to the opposite edge of the modulator. An increase in the feedback coefficient K_0 and in the displacement S results in a stronger modulation and in the modulation movement to the aperture centre. When the displacement S exceeds the diffusion length, the phase profile represents a step function decaying to the aperture centre (Fig. 2b). In the case of lower displacements, the phase profile is smoothed due to diffusion of a charge in a photoconducting layer of the transparency (Fig. 2a).

The appearance of such a strong phase modulation in the system is extremely undesirable. This effect can be reduced by placing, for example, a filter with a selected transmission coefficient to the region of the beam overlap.

Consider the dependence of phase distortions at the output of the system on the ratio of intensities in two regions. We will estimate the phase modulation from the quadratic error

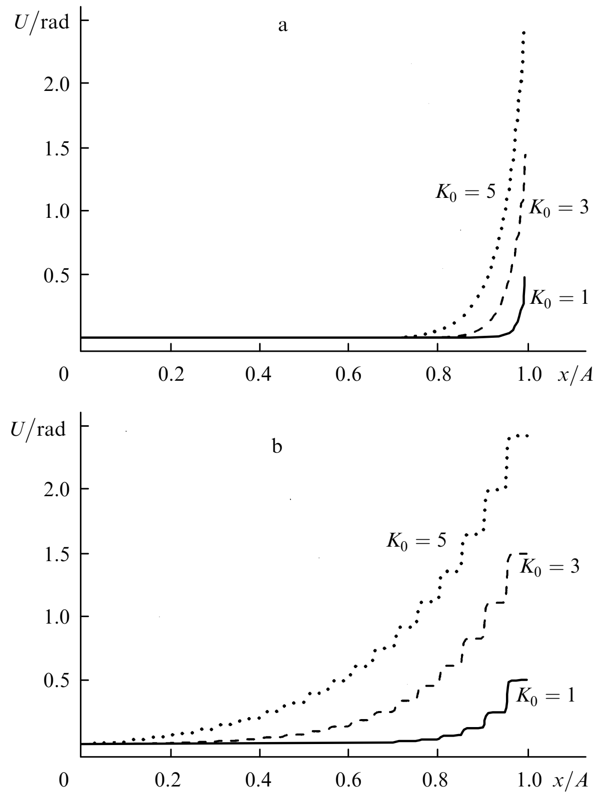

Figure 2. Profiles of phase modulation caused by the intensity jump near the edge of the LC corrector $x = A$ for $S/A = 0.01$ (a) and 0.05 (b) for various feedback coefficients K_0 and $l_{\text{dif}}/A = 0.002$.

$$\sigma_U^2 = A^{-2} \int_A [U(x,y) - \bar{U}(x,y)]^2 \mathrm{d}x\mathrm{d}y, \tag{9}$$

where $\bar{U}(x,y)$ is the aperture-averaged value. Fig. 3 shows the dependence of the error σ_U^2 on the intensity ratio I_2/I_0. One can see that σ_U^2 decreases when this ratio approaches unity. When the intensity ratio is equal to unity, the phase error σ_U^2 is zero in the absence of external phase distortions. The phase modulation, which appears in the system due to the presence of the boundary region, depends on the feedback coefficient K_0. For example, for $K_0 = 3$, the phase modulation with the root-mean-square error $\sigma_U^2 = 0.05$ appears if $I_2/I_0 = 0.6$, and when $K_0 = 5$, it appears when $I_2/I_0 = 0.8$.

4. Influence of a small-scale intensity modulation on the system operation

A beam being corrected can exhibit small-scale intensity fluctuations, which can appear, for example, due to diffraction from small-scale phase inhomogeneities. Let us study the effect of such fluctuations on the system operation.

Let us represent the intensity distribution $I_b(x,y)$ in the incident beam in the form

$$I_b(x,y) = \bar{I}_b[1 + i_b(x,y)], \tag{10}$$

where $\bar{I}_b$ is the average light intensity and $i_b(x,y)$ is the modulation coefficient. In this case, the intensity distribution at the output of a shearing interferometer will be determined by the expression

$$I = \frac{1}{2}[I_b(x,y) + I_b(x+S,y)] \times \left\{ 1 + 2\left[\frac{[I_b(x,y)I_b(x+S,y)]^{1/2}}{I_b(x,y) + I_b(x+S,y)} \right] \cos(\Delta H + \Delta_0) \right\}. \tag{11}$$

Let us assume that a beam with a plane wavefront $[F(x,y) = 0]$ and random intensity fluctuations arrives at the system input. We will characterise a random intensity field by the correlation radius r_{int} and the root-mean-square dispersion σ_{int}. Intensity fluctuations will cause phase distortions $U(x, y)$ of the initially plane wavefront of the incident beam. Fig. 4 shows the phase error σ_U^2 averaged over 300 random realisations of the intensity as a function of the ratio r_{int}/A. The random intensity field was simulated with the help of no less than one hundred Gaussians with a random distribution over the aperture. The correlation radius of a random field in this model is determined by the width of Gaussians, while the amplitude of Gaussians determines the dispersion.

One can see from Fig. 4 that the root-mean-square phase error increases with increasing the correlation radius of intensity fluctuations. Such a behaviour of σ_U^2 can be explained as follows. Because the incident beam has a plane

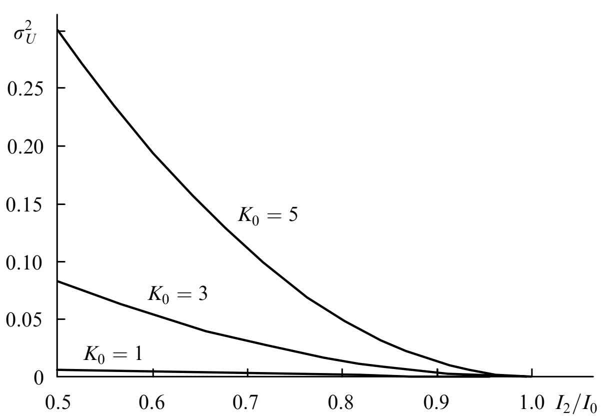

Figure 3. Dependences of the root-mean-square error σ_U^2 on the intensity ratio I_2/I_0 for different K_0, $l_{\text{dif}}/A = 0.002$ and $S/A = 0.02$.

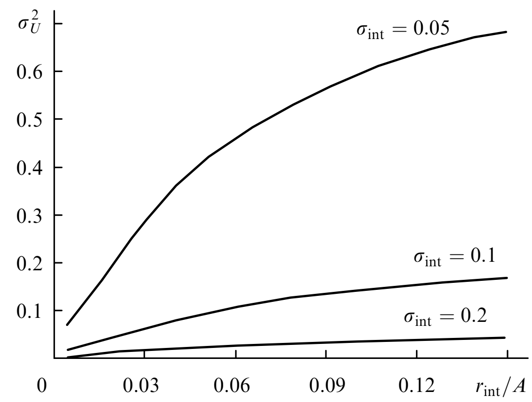

Figure 4. Dependences of the root-mean-square error σ_U^2 on the normalised correlation radius r_{int}/A for different root-mean-square intensity dispersions σ_{int}, $K_0 = 5$, $l_{\text{dif}}/A = 0.005$ and $S/A = 0.02$.

wavefront, the intensity modulation in the feedback loop at the first instant of time will be caused only by fluctuations of the beam intensity. The beam intensity fluctuations will produce the phase modulation in the photosensitive layer of the LC transparency, the characteristic size of the phase modulation being coincident with the characteristic scale of intensity variations. These phase distortions are visualised during the second passage in the feedback loop, and the beam intensity is additionally modulated, resulting in a partial compensation of phase distortions. The quality of this compensation depends on the characteristic size of phase inhomogeneities [3]. For this reason, the phase error σ_U^2 increases with increasing the correlation radius $r_{\rm int}$, the root-mean-square dispersion $\sigma_{\rm int}$ of the intensity at the system input being invariable.

Therefore, the smaller the characteristic spatial size of intensity fluctuations, the weaker they affect the system operation. If $r_{\rm int} < l_{\rm dif}$, then the system is insensitive to fluctuations at all because they are smoothed due to diffusion. As $r_{\rm int}$ increases, the phase error gradually saturates. This is explained by the fact that the phase distortions caused by intensity fluctuations are almost not suppressed, and, hence, the residual error σ_U^2 depends only on the root-mean-square dispersion $\sigma_{\rm int}$ of intensity fluctuations.

Consider now a more interesting case, when a beam with random modulations of the phase $F(x, y)$ and intensity $I_{\rm b}(x, y)$ arrives at the system input. In this case, it is convenient to estimate the quality of compensation from the residual relative compensation error

$$\eta^2 = \frac{\int_A [H(x,y) - \bar{H}(x,y)]^2 {\rm d}x{\rm d}y}{\int_A [F(x,y) - \bar{F}(x,y)]^2 {\rm d}x{\rm d}y}, \tag{12}$$

where $\bar{H}(x, y)$ и $\bar{F}(x, y)$ are the aperture-averaged values. For the fixed parameters of the system, the error η^2 depends on the four statistical characteristics of random fields: σ_F, r_F, $\sigma_{\rm int}$, $r_{\rm int}$. The phase distribution $H(x, y)$ at the system output is determined, as before, by a sum of the phase at the system input and the phase introduced by the LC modulator, i.e., $H(x, y) = F(x, y) + U(x, y)$. However, the phase function depends now not only on the intensity modulation related to the visualisation of phase distortions $F(x, y)$ but also on the external intensity fluctuations $\bar{I}_{\rm b} i_{\rm b}(x, y)$. Fig. 5 shows the dependence of the relative root-mean-square compensation error η^2 on the root-mean-square intensity dispersion $\sigma_{\rm int}$. One can see that the error η^2 increases with increasing $\sigma_{\rm int}$. Note that the larger is the characteristic spatial size of initial beam intensity fluctuations, the stronger they deteriorate the system operation.

Phase distortions in the system are compensated if $\eta^2 < 1$; otherwise, they are enhanced. In the absence of intensity fluctuations in a beam being corrected, when phase fluctuations are sufficiently weak, the relative error η^2 should not depend on the phase amplitude at the system input (in this case, a linear approximation is valid). If the beam intensity is modulated, then at small phase fluctuations at the system input, the wavefront distortions at the system output will be mainly determined by this intensity modulation. Therefore, we can expect that in the presence of intensity fluctuations, a decrease in the phase amplitude will lead to an increase in the relative error η^2. This increase does not mean an increase in the phase dispersion at the system output but is caused by the reduction of the dominator in expression (12). Fig. 6a shows the dependence of the relative

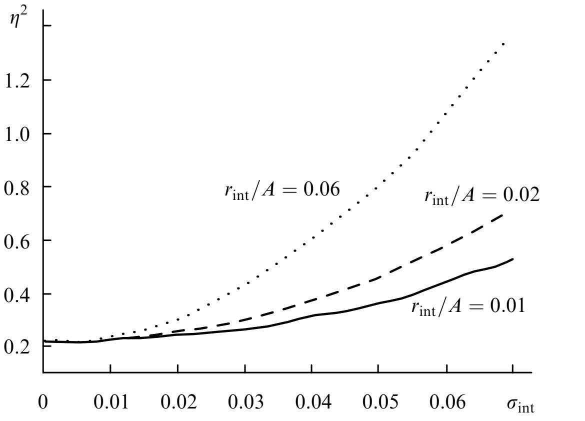

Figure 5. Dependences of the relative residual error η^2 of compensation for random phase distortions on $\sigma_{\rm int}$ for different normalised correlation radii $r_{\rm int}/A$ for statistical characteristics of phase fluctuations $r_F/A = S/A = 0.02$ and $\sigma_F = 0.25$ (r_F and σ_F are the correlation radius and the root-mean-square dispersion of phase distortions).

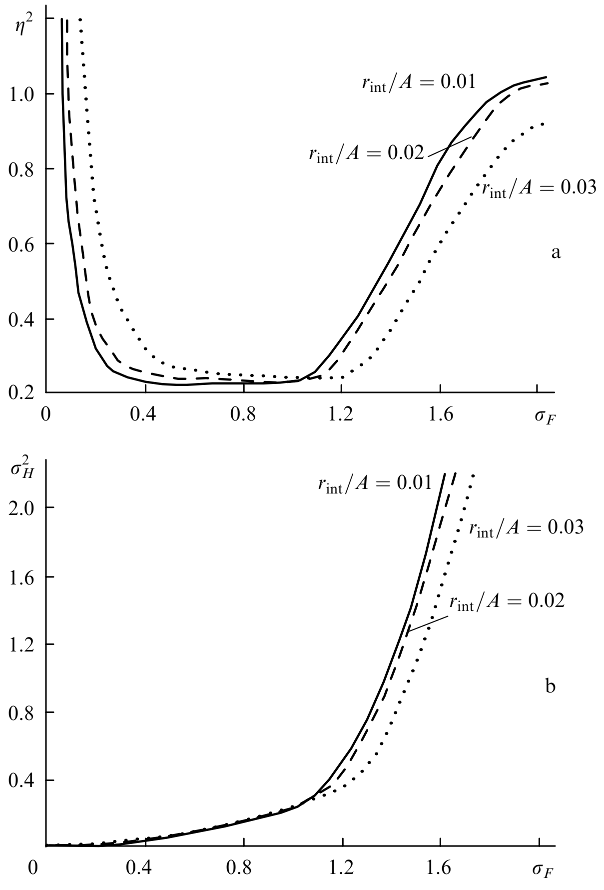

Figure 6. Dependence of the relative residual compensation error η^2 (a) and root-mean-square phase error σ_F (b) on the root-mean-square dispersion σ_F of phase fluctuations at the system input for different $r_{\rm int}/A, r_F/A = S/A = 0.02, \sigma_{\rm int} = 0.035, K_0 = 5$.

residual root-mean-square error η^2 on the root-mean-square dispersion σ_F of phase distortions in the presence of intensity fluctuations. Indeed, the relative error increases with decreasing phase fluctuation at the system input. One can see from Fig. 6b that the phase dispersion σ_H^2 at the system output (when $\sigma_F \to 0$) tends to the value close to zero, which is determined by intensity fluctuations (which are weak in this case). At sufficiently large phase distortions, the ‘useful’ intensity modulation produced due to visualisation of these distortions exceeds intensity fluctuations, which explains a decrease in the relative error η^2 with increasing σ_F. An increase in η^2 for $\sigma_F > 1$ is explained by the restriction imposed on the magnitude of phase distortions, which can be compensated by the system.

5. Conclusions

We have shown in this paper that, when broad light beams are used, a region of low intensity is formed at the edge of the LC corrector, whose width is equal to the transverse displacement S of beams in the interferometer. Our numerical study has shown that a jump in the intensity at the boundary of this region causes phase modulation, which propagates to the aperture centre. This modulation increases with increasing the intensity jump, the feedback coefficient K_0, and the displacement S.

We have simulated numerically the influence of the input-beam intensity fluctuations on the quality of compensation of phase distortions and have shown that this influence decreases with decreasing the correlation radius r_{int} of fluctuations.

The relative residual error of compensation depends on the relation between the phase fluctuation and intensity. In the case of small phase distortions, weak intensity fluctuations result in a strong increase in the relative error η^2.

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