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On the problem of ideal amplification of optical solitons

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***Abstract*. The new possibilities of almost ideal amplification of optical solitons during the incoherent interaction of light pulses with a resonantly amplifying medium are considered. The mechanism of two-photon amplification of optical solitons with an optimal frequency-modulation law is proposed. It is shown that the entirely ideal amplification of solitons cannot be achieved because the law of phase modulation of radiation differs from a parabolic law. The possibility of using the phase cross modulation to produce the required initial phase of amplified solitons is studied.**

Keywords: *optical solitons, phase modulation, cross modulation.*

1. New problems of physics of amplification of optical solitons

Optical solitons are now considered as ideal data carriers in fibreoptic WDM communication systems [1, 2]. The soliton technique also opens up new possibilities for generation of ultrashort light pulses [3 – 6]. The problem of an ideal amplification of optical solitons is a rather old problem of nonlinear optics. Until recently it was accepted, as had been probably first assumed in paper [7], that a nonlinear capture of energy by a soliton occurs in the following way: a soliton system as if divided into two parts, one of which, called a soliton wave, accumulates the energy according to the Zakharov – Shabat concept [8], while the other one, called a non-soliton or dispersing wave, accumulates the energy of redundant waves, 'thrown down' by a soliton [1, 4, 7]. However, recent papers [9 – 12] radically changed this concept. The problem of an ideal amplification of optical solitons was in fact first formulated in papers [9, 10], where a unique possibility of preserving the shape of a soliton pulse in a medium with a spatially inhomogeneous gain was discovered. The model considered in papers [9, 10], which uses a nonlinear parabolic equation, is equivalent to a model based on the nonlinear Schrödinger equation with the self-consistent interaction potential produced by a propagating pulse itself due to self-action effects in a cubic nonlinear medium [1, 4]. The mathematical model [9, 10] and its 'unexpected' solutions in dimensionless standard (soliton) variables have the form [9 – 12]

$$\mathrm{i}\,\frac{\partial\Psi}{\partial Z}+\frac{1}{2}\,\mathrm{sign}(k''_{\omega^2})\,\frac{\partial^2\Psi}{\partial T^2}+\alpha|\Psi|^2\Psi=\mathrm{i}\Gamma(Z)\Psi, \qquad (1)$$

$$\Psi_{\mathrm{c}}(Z,T)=-\frac{\eta\alpha^{-1/2}}{1-2\Gamma(0)Z}\,\mathrm{sech}\left[\frac{\eta T}{1-2\Gamma(0)Z}\right]$$
$$\times\exp\left\{-\mathrm{i}\,\frac{T^2\Gamma(0)}{1-2\Gamma(0)Z}-\mathrm{i}\,\frac{\eta^2 Z}{2[1-2\Gamma(0)Z]}\right\}, \qquad (2)$$

$$\Psi_{\mathrm{d}}(Z,T)=\frac{\eta\alpha^{-1/2}}{1-2\Gamma(0)Z}\,\tanh\left[\frac{\eta T}{1-2\Gamma(0)Z}\right]$$
$$\times\exp\left[\mathrm{i}\,\frac{T^2\Gamma(0)}{1-2\Gamma(0)Z}-\mathrm{i}\,\frac{\eta^2 Z}{1-2\Gamma(0)Z}\right], \qquad (3)$$

where η is a constant, which determines the so-called form-factor of a Schrödinger soliton envelope; $\alpha > 0$ is an arbitrary constant;

$$\Gamma(Z)=\frac{\Gamma(0)}{1-2\Gamma(0)Z} \qquad (4)$$

is the gain of a spatially inhomogeneous active medium; $\Gamma(0) = 1/(2C)$; and C is an arbitrary constant. The solution Ψ_{c} (2) describes the so-called clear soliton [9, 10], while the solution Ψ_{d} (3) describes the dark soliton.

One can see from (2) that the main feature of the soliton amplification is the presence of the phase modulation (frequency chirp). The chirp depends on a distance propagated by a pulse in the amplifying medium. The gain (4) providing the 'ideal' amplification of a soliton is a hyperbolic function of the amplifying medium length. The area under the pulse for a hyperbolically amplified soliton (2) is preserved [2] because a decrease in its duration is determined by a linear function of the medium length, which is typical for soliton pulses [1, 4].

The problem of an ideal amplification of solitons is very important for the development of new soliton information systems [1, 2]. A radically new solution of this problem was proposed in papers [12, 13], where a new concept of a quasi-soliton was introduced for a chirped soliton in a nonlinear

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medium with a periodical spatially inhomogeneous dispersion parameter and a periodic gain. In this way, a quasi-soliton of the nonlinear Schrödinger equation with a parabolic potential appeared [13], and a new, rapidly developing scientific field was initiated – optical solitons in spatially inhomogeneous nonlinear systems and soliton Bloch waves in nonlinear communication lines and soliton lasers [11 – 15]. It was shown that there exist an infinite number of soliton solutions of the Schrödinger equation in active inhomogeneous soliton systems [12]; the existence theorems for chirped solitons were proved, and the method was developed for solving the inverse problem of the scattering theory with a variable spectral parameter [15, 16] for analysis of new problems; as well as the Lax pairs were constructed for new, completely integrated generalised models on the nonlinear Schrödinger equation [14, 15].

However, a number of essential problems of the theory remained unsolved. First of all, equation (1) and its solution (2) posed new problems, the most important of them is how to perform the experiment corresponding to solution (2). It is obvious that a number of fundamental problems are encountered in the experimental realisation of an ideal amplification of solitons. First, the gain in model (1) becomes infinite at the active medium length $Z = 1/[2\Gamma(0)] = C$. Solution (2) has the same singularity at the point $Z = C$. Second, the authors of papers [9, 10] proposed no ideas concerning the experimental realisation of the gain (4) that would be inhomogeneous over the length or at least how to find the gain that would be close optimal.

We formulate and solve the problem of the ideal amplification of a soliton within the framework of model (1) and show that the required gain can be realised experimentally by manufacturing a two-photon fibre amplifier, which excludes effects related to the diffraction and self-focusing of radiation. We also show that the method of modulation of frequency of the initial pulse plays a crucial role in the experimental realisation of the ideal soliton amplifier. We study the possibility of using the phase cross modulation to obtain the required initial phase of amplified solitons. This can be achieved if a control pulse at different wavelength is used simultaneously with the amplified soliton.

2. How can a soliton be perfectly amplified without its transformation to a multi-soliton pulse?

At present, two basic methods for amplifying optical solitons are employed: a rapid nonadiabatic amplification in a line with lumped amplifiers and adiabatic amplification in a distributed active medium (see, for example, books and reviews [1, 2, 4, 17, 18] and references therein).

In the theoretical analysis of the nonadiabatic amplification, it is assumed that the length of an amplifying medium is infinitesimal compared to the characteristic length of the dispersion spread of a pulse. This allows one to consider the amplification of a soliton as a 'point' (delta-like) process, which is equivalent to a simple multiplication of the soliton amplitude by the gain at each point of the medium. A soliton pulse amplified in such a way behaves differently during its propagation in the medium. It can, bypassing the intermediate process of decaying oscillations, either transform to a new soliton or form a bound soliton state, which, as is known, has a complicated periodic temporal structure [18]. These two well-known facts in the theory of Schrödinger solitons were used in one of the first experimental models of a soliton communication line, the so-called a dynamic soliton communication line [18], in which pulses were used with the amplitude exceeding the fundamental-soliton amplitude by a factor of $1.2 - 2$ [4].

In the theoretical analysis of adiabatic amplification, the length of an active medium can no longer be treated as infinitesimal, and the gain is a function of the active-medium length, if for no reason than the inevitable presence of nonzero radiation losses in a fibre. Such a situation is realised, for example, upon Raman pumping of solitons [17, 19, 20].

By analysing the development of theoretical and experimental studies on the amplification of optical solitons, we should note that as a whole the solution of the problem of optimal amplification of a soliton proved to be at a loss. Experiments on the generation of optical solitons are still being performed in homogeneous nonconservative systems, which, however, are described by nonintegrable models. This even in principle does not allow one to expect to amplify a soliton as a whole by retaining its unique properties. The theoretical analysis is based entirely on the perturbation theory [21], which, as we will show below, proves to be invalid already upon amplification of a soliton by a factor of e.

Consider the amplification of a Schrödinger soliton in more detail. The propagation of an optical soliton in a resonance active medium upon the incoherent interaction of the pulse with the medium is described by the well-known system of equations: the nonlinear Schrödinger equation (nonlinear parabolic equation) and the equation for the polarisation P of particles of the active medium [22]. This system in the dimensionless standard form is

$$\mathrm{i}\,\frac{\partial \Psi}{\partial z} + \frac{1}{2}\,\frac{\partial^2 \Psi}{\partial \tau^2} + |\Psi|^2 \Psi = \frac{G}{2}\,P, \tag{5}$$

$$\gamma_{\mathrm{a}}\,\frac{\partial P}{\partial \tau} + P(1 + \mathrm{i}\gamma_{\mathrm{a}}\Delta\Omega) = \mathrm{i}\Psi. \tag{6}$$

Here, the running time τ is normalised to the initial pulse duration τ_0 at the entrance to the active medium, the distance propagated by the soliton is normalised to the dispersion length of the pulse spread $L_{\mathrm{d}} = \tau_0^2/|k''_{\omega^2}|$, and the wave amplitude is normalised to the characteristic initial value $|\psi_{\mathrm{s}}| = [8\pi k''_{\omega^2} \times (\tau_0^2 k_0 n_2 c n_0)^{-1}]^{1/2}$; $\gamma_{\mathrm{a}} = T_2/\tau_0$; T_2 is the transverse relaxation time of the high-frequency dipole moment; $\Delta\Omega = \tau_0(\omega_0 - \omega_{12})$; ω_0 is the carrier frequency of the pulse: and ω_{12} is the resonance-transition frequency. The gain parameter in equation (5) is determined by the relation $G = L_{\mathrm{d}}/L_{\mathrm{a}}$ [22], where $L_{\mathrm{a}} = 1/(\sigma_0 N_0)$ is the characteristic gain length; N_0 is the inversion-population density in the absence of the field; and σ_0 is the cross section of the radiative transition at the resonance frequency.

Upon the incoherent interaction of a coherent pulse with a medium, polarisation follows quasi-statically the field, and the contribution to the dispersion of the refractive index has is [3]

$$\delta n_{\mathrm{i}} = \frac{cN_0}{2n_0\omega_0}\,\frac{\sigma_0(T_2\Delta\omega + \mathrm{i})}{1 + (T_2\Delta\omega)^2}, \tag{7}$$

where $\Delta\omega = \omega_0 - \omega_{12}$. When the carrier frequency of the pulse coincides with the frequency of the resonance transition, the quasi-static refractive index becomes purely imaginary. It is known that the mathematical model (5), (6) correctly describes the amplification dynamics of solitons in the picosecond range, when the nonlinear electronic Kerr effect, related to the inertialless addition to the refractive index n_2, dominates. If $G \ll 1$, then equation (5) can be solved using the apparatus of the adiabatic perturbation theory for solitons [21].

In the case of incoherent amplification, when the width of the pulse spectrum is small compared to the width of the gain line [3], system of equations (5), (6) can be transformed as

$$\mathrm{i}\,\frac{\partial\Psi}{\partial z} + \frac{1}{2}\,\frac{\partial^2\Psi}{\partial\tau^2} + |\Psi|^2\Psi$$

$$= \mathrm{i}\,\frac{G}{2}\left(\Psi - \gamma_{\mathrm{a}}\,\frac{\partial\Psi}{\partial\tau} + \gamma_{\mathrm{a}}^2\,\frac{\partial^2\Psi}{\partial\tau^2}\right). \tag{8}$$

The soliton solution of equation (8) with slowly changing parameters has the form

$$\psi_{\mathrm{s}}(z,\tau) = \eta(z)\mathrm{sech}\{\eta(z)[\tau - \tau_{\mathrm{c}}(z) - V(z)z]\}\exp(\mathrm{i}\Phi), \tag{9}$$

$$\Phi = -V(z)[\tau - \tau_{\mathrm{c}}(z)] + \mathrm{i}[V^2(z) - \eta^2(z)]z/2 + \mathrm{i}\phi_0(z), \tag{10}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}z} = G\left(1 - \frac{1}{3}\,\eta^2\gamma_{\mathrm{a}}^2 - \gamma_{\mathrm{a}}^2V^2\right)\eta, \tag{11}$$

$$\frac{\mathrm{d}V}{\mathrm{d}z} = -\frac{2}{3}\,GV\gamma_{\mathrm{a}}^2\eta, \tag{12}$$

where $\tau_{\mathrm{c}}(z)$ is the coordinate of the centre of gravity of the pulse and ϕ_0 is the phase. To pass to the equation that is close to the initial model (1), we set $\gamma_{\mathrm{a}} = 0$. Then, system (5), (6) is transformed to the well-known nonlinear Schrödinger equation with amplification (absorption)

$$\mathrm{i}\,\frac{\partial\Psi}{\partial z} + \frac{1}{2}\,\frac{\partial^2\Psi}{\partial\tau^2} + |\Psi|^2\Psi = \frac{G}{2}\,P. \tag{13}$$

The perturbation theory for solitons [21] gives the solution of equation (13) in the form [1, 4]

$$\Psi(z,\tau) = \eta(z)\mathrm{sech}[\eta(z)\tau]\exp[\mathrm{i}\sigma(z)] + O(G),$$

$$\eta(z) = \Psi(0)\exp(2Gz), \tag{14}$$

$$\sigma(z) = \frac{\Psi^2(0)}{8G}[1 - \exp(4Gz)].$$

Note that solution (14) has been treated for many years (see, for example, book [4] and references therein) as a rather exact approximation for the description of amplification of solitons. However, our computer experiments showed that this is not the case. Recall that, as was already shown in paper [23], the distortions of the soliton shape in a linearly absorbing medium are caused by the appearance of the frequency modulation of the pulse, which substantially restricts the application of the apparatus of the perturbation theory. Let us show that a similar result is obtained for a linear amplification of solitons, and that there exist rather substantial qualitative and quantitative differences from the case corresponding to the propagation of a soliton in a linearly absorbing medium [23].

We performed the numerical study of the propagation and interaction of solitons in an active medium, by considering various relations between the parameters of the model (13). We found that there exists a limiting energy of the amplified soliton, when the results of the perturbation theory for solitons and expression (14) prove to be invalid at all. A soliton is not amplified as the whole in the regime of incoherent amplification: during the soliton amplification, its shape and spectrum substantially change.

During the soliton amplification, two characteristic features appear in its temporal structure: a narrow intense peak and a broad low-intensity pedestal to which an increasing fraction of the pulse energy transfers in the process of amplification. A detailed picture of the soliton amplification is shown in Figs 1 and 2, where the results obtained in the adiabatic and nonadiabatic amplification regimes are compared. The spectrum of the amplified soliton has a structure, which demonstrates the appearance of a substantial frequency modulation of radiation.

The processing of the results of numerical experiments suggests the existence of a maximum gain at which the soliton is no longer stable during amplification. This occurs when the soliton energy increases by a factor of e,

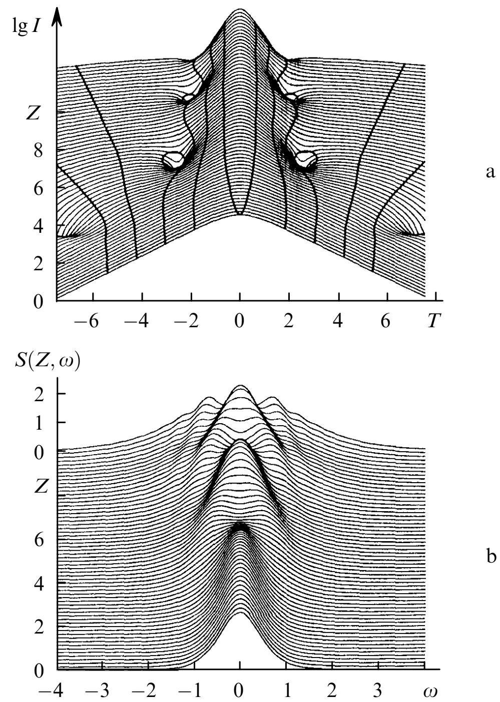

Figure 1. (a) Dependence of the shape $I(Z,T)$ of a soliton pulse (orthographic projection at a logarithmic scale, the upper view at an angle of 60°) and (b) the shape $S(Z,\omega)$ of its spectrum on the active-medium length Z upon adiabatic amplification of solitons and $G = 0.1$. Thick curves are geodesic equal-level lines for $\lg I = 0, -1, -2, -3, -4$.

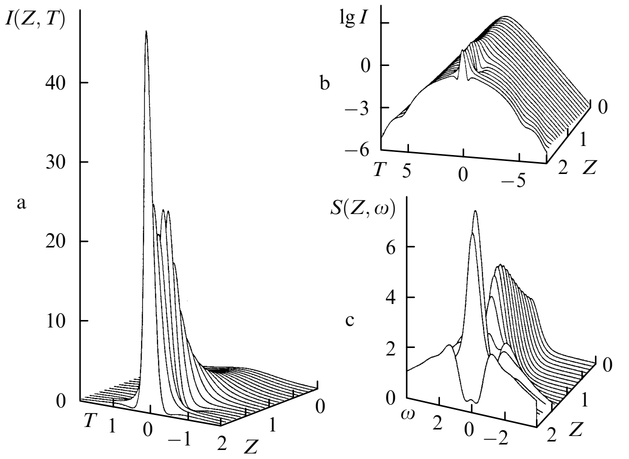

Figure 2. Dependences of the shape $I(Z, T)$ (a, b) of a soliton pulse and the shape $S(Z, \omega)$ (c) of the pulse spectrum on the active-medium length Z upon nonadiabatic amplification of solitons and $G = 1.0$.

irrespective of the way of the energy accumulation in the soliton [adiabatic (Fig. 1) or nonadiabatic (Fig. 2) amplification]. Detailed calculations performed in a broad range of soliton gains show that the structural instability of solitons is explained by the unbalance of the process of transformation of frequency modulation to amplitude modulation. All these features are well demonstrated at the logarithmic scale in Figs 1 and 2.

Therefore, the adiabatic and nonadiabatic regimes of incoherent amplification of solitons in an active medium with constant parameters are always accompanied by the appearance of the structural instability of solitons and the growth of the non-soliton component of the field during amplification. The region of applicability of the perturbation theory is determined by the inequality $G \ll 1$, which expresses in fact the condition of the adiabatic rearrangement of the soliton, which acquires new values of the amplitude and duration corresponding to its greater energy.

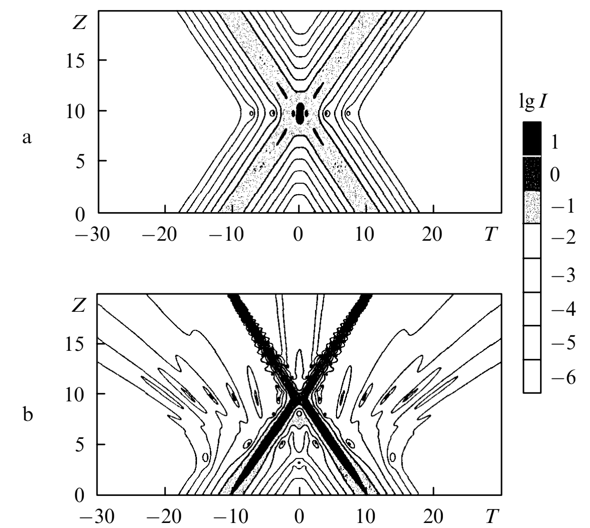

Figure 3. Interaction of two counterpropagating solitons (a) in the absence of amplification and (b) upon adiabatic amplification for the group-velocity detuning $\Delta V = 2.0$ and $G = 0.1$.

Due to the imperfect amplification of solitons and the growth of the non-soliton component of the field, the solitons cease to interact elastically with each other, as shown in Fig. 3, where the contour map (the equal-height lines) is presented for two interacting solitons moving toward each other. The calculations were performed both for ideal solitons, when the parameter $G = 0$, and linearly amplified solitons described by a simplest model (13).

3. Singular amplification of frequency-modulated solitons

As mentioned above, the accepted opinion that ideal Schrödinger solitons cannot be formed in an active medium upon incoherent amplification was subjected to the first blows in papers [9, 10]. The authors of these papers showed that there exists a unique possibility of amplifying a soliton as whole, if the gain is a singular hyperbolic function of the active medium length, while

the soliton phase at the entrance to the medium is a parabolic function of time. The interaction of such frequency-modulated solitons becomes completely elastic when soliton phases and the gain are made self-consistent. In the following papers [12, 15], a regular method was found for searching solutions in the form of frequency-modulated solitons. It was shown that, in the case under study, there exist not one but four soliton solutions corresponding to hyperbolically amplified or decayed clear and dark solitons for positive or negative initial gains (4). Strictly speaking, the solutions presented in [9, 10] are one of the particular cases of the application of the second theorem, which was proved in paper [12], where the Wronskian of two constant functions: the dispersion $[D(Z) = 1]$ and nonlinearity $[R(Z) = 1]$ is exactly zero $[W(R, D) = 0]$.

We will show that the singular amplification of frequency-modulated solitons can be described by an equivalent model without singularities. Let us make a change of variables in (1):

$$\Psi = a(Z)u, \quad a(Z) = \exp\int_0^Z \Gamma(Z')\mathrm{d}Z',$$

$$Z' = \int_0^Z a^2(Z'')\mathrm{d}Z''.$$

In this case, the Schrödinger equation will have the form

$$\mathrm{i}\,\frac{\partial u}{\partial Z'} + \frac{D_{\mathrm{opt}}(Z)}{2}\,\frac{\partial^2 u}{\partial T^2} + \alpha|u|^2 u = 0, \tag{15}$$

where

$$D_{\mathrm{opt}}(Z) = \frac{D_2(0)}{a^2_{\mathrm{opt}}(\Gamma_{\mathrm{opt}}(Z))}$$

is the optimal dispersion function. By using the relations

$$\Gamma_{\mathrm{opt}}(Z) = \frac{1}{2(C - Z)}, \quad a^2_{\mathrm{opt}}(Z) = \frac{C}{C - Z},$$

we obtain

$$D_{\rm opt}(Z) = D_2(0)\left(1 - \frac{Z}{C}\right), \quad Z' = -C \ln \frac{C - Z}{C},$$

$$Z = C\left[1 - \exp\left(-\frac{Z'}{C}\right)\right],$$

$$D_{\rm opt}(Z) = D_2(0)\exp\left(-\frac{Z'}{C}\right).$$

After simple transformations, equation (15) takes the form

$$\mathrm{i}\,\frac{\partial u}{\partial Z'} + \frac{1}{2}\,D_2(0)\exp\left(-\frac{Z'}{C}\right)\frac{\partial^2 u}{\partial T^2} + \alpha|u|^2 u = 0. \qquad (16)$$

Therefore, model (1) is transformed to model (15) in new variables, and its solutions (2) and (3) for clear and dark solitons are transformed to the solutions

$$u_{\rm c}(Z', T') = -\eta\alpha^{-1/2}\exp[\Gamma(0)Z']$$
$$\times\,\mathrm{sech}\{\eta T'\exp[2\Gamma(0)Z']\}\exp\{-\mathrm{i}T'\,2\Gamma(0)\exp[2\Gamma(0)Z']$$
$$-\mathrm{i}\frac{1}{2}\,\eta^2 Z'\exp[2\Gamma(0)Z']\}, \qquad (17)$$

$$u_{\rm d}(Z', T') = \eta\alpha^{-1/2}\exp[\Gamma(0)Z']$$
$$\times\,\tanh\{\eta T'\exp[2\Gamma(0)Z']\}\exp\{\mathrm{i}T'^2\Gamma(0)\exp[2\Gamma(0)Z']$$
$$-\mathrm{i}\eta^2 Z'\exp[2\Gamma(0)Z']\}. \qquad (18)$$

These chirped soliton solutions differ in two respects from canonical solitons of the model of nonlinear Schrödinger equation without amplification or absorption. First, solitons (2), (3), and (17), (18) are frequency-modulated pulses, whose amplitude, duration, and frequency modulation are described by the same function of Z – a hyperbolic function of the amplifying-medium length. Second, solitons (2) and (3) are the singular solutions of equation (1) and have a meaning only for $Z < C$. When the amplifying medium length approaches the singularity $Z = C$, the soliton amplitude increases infinitely and its duration decreases. Nevertheless, solutions (2) and (3) are of certain practical interest, because the active-medium length can be always restricted by the condition that the singularity is not achieved [10].

A linear law of the frequency modulation of the pulse at the entrance to the active medium determines all the subsequent regime of pulse compression it the medium. Fig. 4a shows computer simulations of the amplification of a frequency-modulated soliton. The compression of this soliton does not change the law of frequency modulation only its value being changed according to expression (2). On the contrary, the amplification of a usual (without frequency modulation) soliton is accompanied by the appearance of nonlinear frequency modulation, which is described by the envelope of the amplified pulse shown in Fig. 4b.

At present, a variety of methods for light modulation

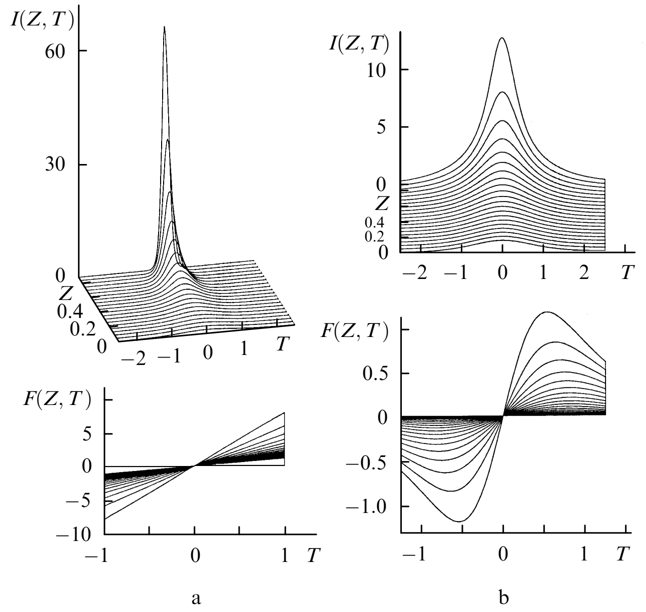

Figure 4. Amplification dynamics of an optical soliton in a medium with hyperbolically increasing gain: (a) an amplified frequency-modulated soliton of equation (1), whose envelope $I(Z,T)$ and frequency modulation $F(Z,T)$ are calculated for $G = 1.0$ and the active-medium length restricted by the condition of approaching a singularity by the value $Z - C = 0.125$ and (b) a change in the envelope and the appearance of the frequency modulation of the pulse upon amplification in an inhomogeneous medium in the absence of initial modulation of the pulse for $G = 0$.

[24, 25] and different fibreoptic systems have been developed, in which, as stated in paper [26], virtually any dispersion profile can be realised. In this connection, the main difficulty of the experimental detection of frequency-modulated solitons (2) is the necessity of producing linear frequency modulation of radiation at the entrance to a medium.

Let us assume that the initial phase modulation of a pulse is produced due to phase cross modulation and has the form

$$\Phi(Z = 0, \tau) = A\exp\left[-\left(\frac{\tau}{\tau_{\rm mod}}\right)^{2n}\right]. \qquad (19)$$

According to (19), the frequency depends linearly on time only at the central part of the pulse (2). The numerical experiment allows one to study in detail the dependence of the degree of pulse compression on the magnitude of frequency modulation at the entrance to the medium. By varying the duration of the control super-Gaussian pulse (19), which propagates at the different wavelength, we can expand the region of a linear frequency scan in the soliton pulse. As expected, the soliton is compressed efficiently only in the part of the initial pulse where the time dependence of the frequency slightly differs from a linear law.

Typical results of computer simulations are presented in Figs 5 and 6. One can see that the pulse self-compression as a whole cannot occur. This fact is clearly demonstrated by the dependence of the pulse shape on the gain length in Fig. 6. The self-compression and compression occur only in the upper part of the pulse, whereas the pedestal duration is

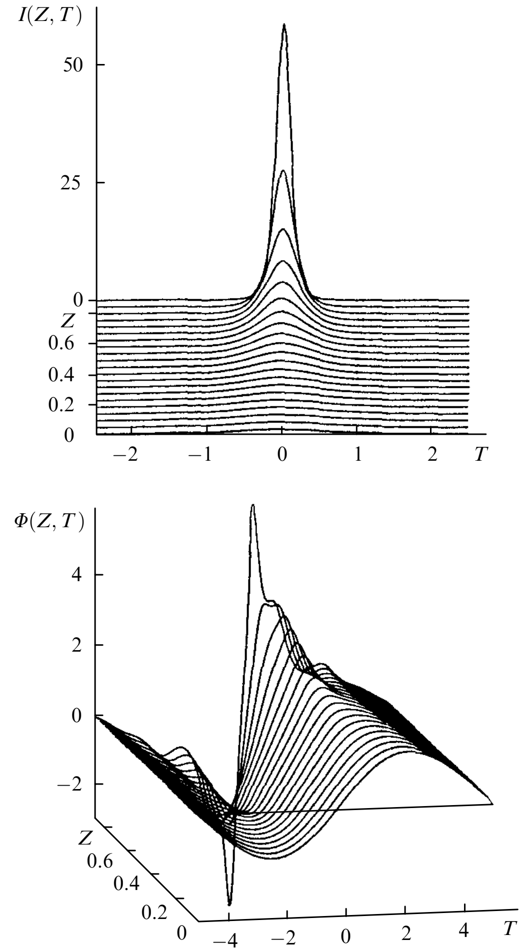

Figure 5. Amplification of a phase-modulated soliton $I(Z, T)$ in an active medium with the hyperbolic gain in the case when the initial phase modulation $\Phi(Z, T)$ is produced due to phase cross modulation by the pulse at the nonresonance wavelength (19) with duration $\tau_{\mathrm{mod}} = 5.0$, amplitude $A = 1.0$, and parameter $n = 1$.

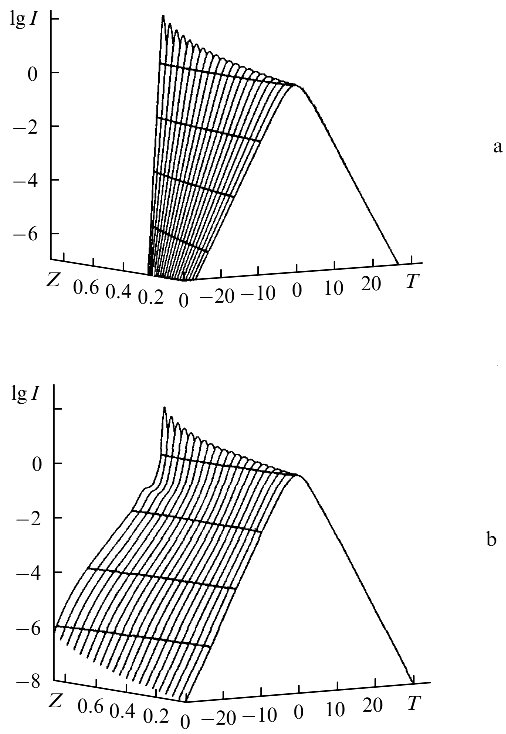

Figure 6. Comparison of the relative energy redistribution during amplification of solitons for (a) a perfect parabolic profile of the initial phase in (2) and (b) for the initial phase specified by the temporal profile of a control pulse (19).

not changed. Let us compare these results with Fig. 4, where an exact solution is presented at the same scale, which, on the contrary, demonstrate the compression of the frequency-modulated soliton as a whole.

Consider now how solitons can be perfectly amplified in a real experiment. It was shown already in paper [27] that two-photon absorption (and Raman amplification) can change the shape of a soliton pulse is such a way that its amplitude (and hence, duration) will vary as

$$\eta^2(z) = \frac{\eta^2(0)}{1 \pm (8/3)\eta^2(0)z}. \tag{20}$$

It is obvious that, if we take the minus sign in the dominator in (20), we obtain a singular function coinciding with expression (4) for the gain. Therefore, the answer to the above-formulated question is obvious – its is necessary to create a two-photon fibreoptic amplifier.

To be consistent to the end, we will study the properties of interaction of frequency-modulated solitons during their amplification. The aim of our computer experiments was to elucidate the possibility of the existence of a static soliton attractor, which causes the inelastic interaction and merging of solitons[1]. Note that inelastic scattering is a characteristic feature of non-integrable systems.

Consider, for example, the situation when solitons are amplified by a factor of 100 (strongly nonadiabatic regime). The different group velocities of solitons mean that the solitons were initially subjected to WDM. Let us assume that the spectra of initial pulses do not overlap. If solitons are amplified in a system with constant parameters, then the energy distribution substantially changes in time (Fig. 7a). In this case, the interaction between solitons is inelastic. If, however, solitons are amplified in a system with the inhomogeneous gain, their interaction becomes completely elastic (Fig. 7b). Let us assume now that the spectra of initial solitons are partially overlapped (Fig. 8). The interaction of solitons in the inhomogeneous amplifying medium is still elastic (Figs 8a and 8b). For comparison, Fig. 8c shows the interaction between ideal solitons in the absence of amplification.

[1] Segev and Stegeman assert erroneously in their review [40] that Gatz and Herrman [41] were the first to discover the inelastic scattering of solitons in the model of nonlinear Schrödinger equation with saturation. Note that actually the effect of inelastic scattering of Schrödinger solitons, resulting in their merging to a pulse and called the soliton attractor in the model with saturation, was first discovered by Zakharov and co-workers [42]. Nikonova and Serkin [43] were probably the first to apply the concept of the soliton attractor to the problems of nonlinear optics. They used this concept to the model of nonlinear Schrödinger equation with saturation and calculated the inelastic scattering of Schrödinger solitons and the conditions of merging of their bound states, which appear at the intensities of solitons pulses when the higher terms in the expansion of the nonlinear polarisability should be taken into account [4].

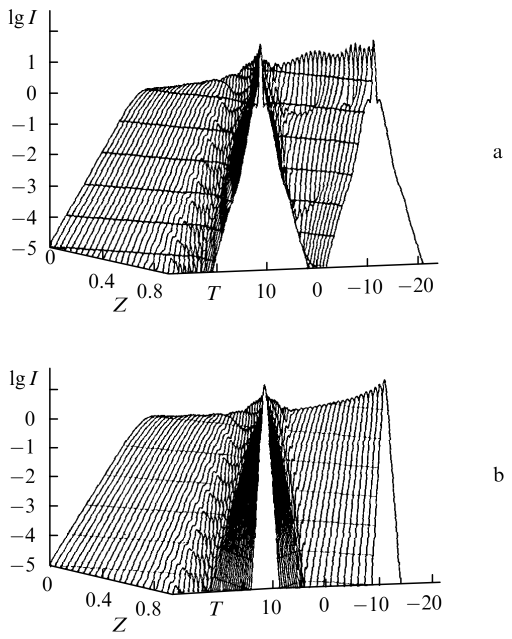

Figure 7. Comparison of the dynamics of interaction of solitons for a strongly nonadiabatic amplification regime (amplification by a factor of 100) in an active medium with (a) the gain that is constant over the medium length and (b) a hyperbolically increasing gain for the non-overlapped spectra of interacting solitons and $\Delta V = 10.0$.

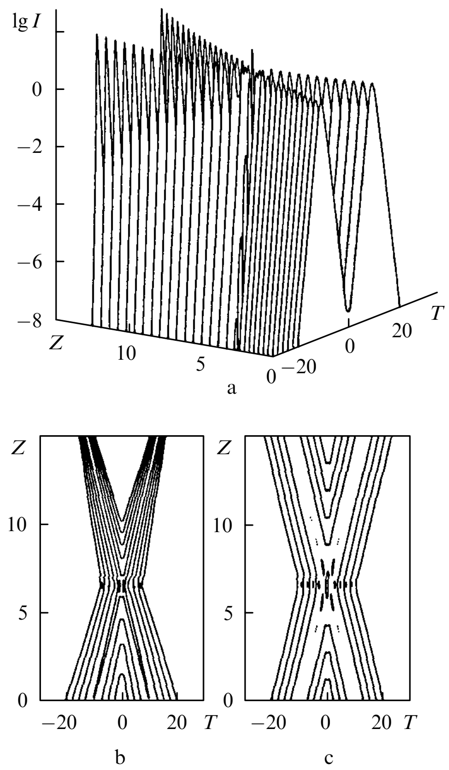

Figure 8. (a, b) Interaction of hyperbolically amplified solitons and (c) ideal solitons with strongly overlapped spectra in the absence of amplification for $\Delta V = 1.5$.

Note that the problem of soliton amplification in a spatially inhomogeneous system is well known to experimenters for a long time [1, 4, 17, 18]. Moreover, during the energy exchange between nonlinear waves, the situation that is very close to this considered here is always realised. Indeed, we can consider for example, spatially inhomogeneous energy transfer during the intracavity generation of ultrashort laser pulses and of solitons of stimulated Raman scattering upon cascade frequency conversion in optical fibres [1, 4, 17–22, 28]. The inhomogeneous gain, close to the required hyperbolic gain, can be also obtained, if a pump pulse, for example, during Raman amplification of solitons represents a multi-soliton pulse, which itself undergoes avalanche-like self-compression during propagation in a medium at the initial stage of energy transfer to the Stokes pulse. As shown in [29], the degree of compression of a multi-soliton pulse is a hyperbolic function of the number of solitons in the pulse, while an increase in the peak power with the fibre length is described by a function that is close to the required hyperbolic function. The required gain can be also obtained by using special tapered fibres, which were recently fabricated (see review [6] and papers [30, 31]).

Upon self-compression of a soliton in an active medium in the case of femtosecond pulses, it is necessary to take into account the higher approximations of the dispersion theory and the Raman frequency shift [32, 33] appearing due to Raman self-scattering [17]. We will consider elsewhere the optimal amplification of femtosecond solitons, when one of the main limiting factors is the Raman self-conversion of the soliton frequency.

4. Conclusions

We have studied in this paper the new possibilities of amplification of optical solitons. We have shown that the most important condition determining the principal possibility of amplification and compression of a frequency-modulated soliton in an amplifying medium is the maintaining of a linear frequency modulation during time intervals exceeding the initial soliton duration. The optimal function of the soliton gain can be obtained in a two-photon fibre amplifier.

The ideal amplification of optical solitons is possible because, in the case of the inhomogeneous gain, the conditions are produced for the changing of a chirped soliton in such a way that an amplified soliton completely retains its unique properties of elastic interaction with similar solitons. We will show in our next paper that a finite width of the gain band of an amplifying medium does not prevent the formation of ultrashort optical solitons (which is a new fact). We will show that a complete analogy exists with the problems of linear amplification of ultrashort pulses, which have been considered already in paper [3]. Concerning periodic problems and the so-called soliton Bloch waves [34], we can assert that they are completely analogous to the problems of generation of ultrashort pulses during the intracavity self-focusing of radiation [35, 36]. We intend to consider this old problem using the space-time analogy, which allows us to generalise easily the obtained results to the case of optimal intracavity self-focusing of radiation. As noted in review [36], the studies initiated in a pioneering paper [35] have anticipated the advent of lasers of the type of a Kerr self-mode-locked Ti : Al_2O_3 laser.

Therefore, the problem of optimal (in our terminology, ideal) amplification of solitons proves to be closely related to various problems of generation of ultrashort laser pulses and the development of new communication systems [1–6].

These problems are still far away from their complete solution. The model of nonlinear Schrödinger equation cannot answer the question about the limiting duration of solitons in an active medium. This problem can be solved by the methods based on a direct numerical solution of the Maxwell system of equations in a nonlinear amplifying medium [37].

By analysing the references presented below (which are not complete due to the limited scope of the paper), we find a very important circumstance, in our opinion, which was pointed out in recent reviews [5, 6]. Although the problem of generation of ultrashort laser pulses has already a thirty-year history [3], it is now that interest in old problems was rekindled. This is especially clear demonstrated by the development of high-speed communication lines and the generation of a femtosecond supercontinuum in optical fibres (see, for example, reviews [2, 5, 6] and references therein, and paper [38]). The fabrication of new fibres with special dispersion characteristics [30] and tapered-waist fibres and photonic crystals [31] stimulated the further development of the theory.

Note that the problem of optimal amplification of solitons is important from the point of view of general physics because a soliton is one of the fundamental natural objects. This nonlinear object is being studied almost in all fields of modern science, and it is difficult not to agree with a hypothesis put forward in paper [39] that the soliton paradigm will serve as a unifying basis for the further development of science.

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