Quantum Electronics **33**(10) 849–855 (2003)

PACS numbers: 42.55.– f; 42.60.Mi; 42.60.Rn
DOI: 10.1070/QE2003v033n10ABEH002513

Features of the dynamics of lasers with a saturable absorber*

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***Abstract*.** **The dynamics of free-running lasers and lasers operating in the amplification regime is considered. It is shown that the stability of a wave propagating in the amplifier depends on the wave velocity. A regular wave propagating at the superluminal velocity in class A amplifiers can lose its stability, and its propagation becomes random. The appearance of the periodic or random pulsations of the amplitude in the laser-generator with a saturable absorber operating in the hard excitation regime leads to the self-switching off (quenching) of lasing despite continuous pumping.**

Keywords: *nonlinear dynamics, dynamic chaos, saturable absorber.*

1. Introduction: The dynamics of free-running lasers

In this introductory section, we consider briefly the known data on the laser dynamics, which will be required for the discussion of the results presented in the paper.

The dynamics of a single-mode laser is described by the following system of equations for the field E, the polarisation P of the active medium, and the difference N of the concentrations of particles at the upper and lower operating energy levels of the active medium

$$\frac{\mathrm{d}^2E}{\mathrm{d}t^2}+2\gamma\frac{\mathrm{d}E}{\mathrm{d}t}+\omega_{\mathrm{c}}^2E=-4\pi\frac{\mathrm{d}^2P}{\mathrm{d}t^2}, \tag{1a}$$

$$\frac{\mathrm{d}^2P}{\mathrm{d}t^2}+\frac{2}{\tau_2}\frac{\mathrm{d}P}{\mathrm{d}t}+\omega_{\mathrm{a}}^2P=2\omega\frac{\mu^2}{\hbar}NE, \tag{1b}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t}+\frac{1}{\tau_1}N=J+\frac{2}{\hbar\omega}\overline{E\frac{\mathrm{d}P}{\mathrm{d}t}}. \tag{1c}$$

Here, γ is the decay of the field in the resonator; ω_{c} is the resonator mode frequency; ω_{a} is the resonance frequency of the atomic transition; τ_1 and τ_2 are the relaxation times of the population of energy levels of the active medium and its polarisation, respectively; μ is the dipole moment of the resonance transition; and J is the pump intensity. The relaxation parameters in (1) are small compared to the frequencies of ω_{a} and ω_{c}, while the frequencies themselves are close to each other. For this reason, the dynamic parameters E and P are quasi-harmonic quantities varying at the frequency ω close to ω_{a} and ω_{c}. The amplitudes of these quantities changes, however, comparatively slowly. The bar over the product of the field by polarisation means the averaging over the period of their rapid variation. Equations (1) were written in this form for the first time in Ref. [1].

The symbolic notation in equation (1c) can be written explicitly by representing polarisation and the field in the form

$$E=A(t)\exp(-\mathrm{i}\omega t),\quad P=B(t)\exp(-\mathrm{i}\omega t), \tag{2}$$

where $A(t)$ and $B(t)$ are the slowly varying functions of time. Then, by neglecting the second time derivatives from 'slow' amplitudes, equations (1) are reduced to the system of equations

$$\begin{aligned}
&\frac{\mathrm{d}A}{\mathrm{d}t}+(\gamma+\mathrm{i}\Delta_{\mathrm{c}})A=\mathrm{i}2\pi\omega B,\quad \Delta_{\mathrm{c}}=\omega_{\mathrm{c}}-\omega,\\
&\frac{\mathrm{d}B}{\mathrm{d}t}+\left(\frac{1}{\tau_2}+\mathrm{i}\Delta_2\right)B=-\mathrm{i}\frac{\mu^2}{\hbar}NA,\quad \Delta_2=\omega_{\mathrm{a}}-\omega,\\
&\frac{\mathrm{d}N}{\mathrm{d}t}+\frac{1}{\tau_1}N=J+\frac{\mathrm{i}}{2\hbar}(AB^*-A^*B).
\end{aligned} \tag{3}$$

The derivation of equations (3) are described in more detail, for example, in papers [2–6]. We will assume below that the frequency of the corresponding mode of the resonator is exactly tuned to the resonance-transition frequency in the active medium, so that $\Delta_{\mathrm{c}}=\Delta_2=0$.

Let us write equations (2) in the dimensionless variables and parameters

$$\begin{aligned}
&\frac{\mathrm{d}X}{\mathrm{d}\tau}+\gamma\tau_2(X-rY)=0,\\
&\frac{\mathrm{d}Y}{\mathrm{d}\tau}+Y-XW=0,\\
&\frac{\mathrm{d}W}{\mathrm{d}\tau}+b(-1+W+XY)=0,
\end{aligned} \tag{4}$$

*Proofreading was made by A.G. Molchanov.

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Received 14 April 2003
Kvantovaya Elektronika **33** (10) 849–855 (2003)
Translated by M.N. Sapozhnikov

where

$$\tau = \frac{t}{\tau_2};\ b = \frac{\tau_2}{\tau_1};\ r = \frac{2\pi\omega\mu^2 J\tau_1\tau_2}{\hbar\gamma};\ X = \sqrt{\sigma}A; \qquad (5)$$

$$\sigma = \frac{\mu^2\tau_1\tau_2}{\hbar^2};\ Y = \frac{P}{\sqrt{b}\mu J\tau_1};\ W = \frac{N}{J\tau_1}.$$

System of equations (4) has three time parameters: $\gamma\tau_2$, $\gamma\tau_2 r$, and b. Depending on their magnitude, lasers are divided into three dynamic classes. Class A includes lasers in which all the three parameters have comparable magnitudes. For class B lasers, the relation $\gamma\tau_2,\ \gamma\tau_2 r \ll 1,\ b < \gamma\tau_2$ is valid. If $\gamma\tau_2,\ \gamma\tau_2 r \ll 1$ and $b \approx 1$, the lasers belong to class C. The dynamics of class A lasers is most interesting, although real lasers mainly belong to class B. It follows from equations (4) that in the regime of stationary lasing with a constant amplitude,

$$X_{\rm st} = \pm(r-1)^{1/2},\ Y_{\rm st} = \pm\left(\frac{r-1}{r\gamma\tau_2}\right)^{1/2},\ W_{\rm st} = \frac{1}{r\gamma\tau_2}. \qquad (6)$$

The question arises: What is the physical meaning of the negative values of the amplitudes of the field and polarisation? Two opposite signs in expressions (6) mean that the laser has two different operating regimes in which the field phases (polarisations) differ by π [6]. These two regimes cannot be distinguished experimentally. However, the existence of two stationary states is manifested in lasing regimes with the pulsating amplitude, which is discussed below.

Regime (6) loses stability if the excitation parameter r exceeds the critical value r^* [7–9]:

$$r > r^* \equiv \frac{\gamma\tau_2(\gamma\tau_2 + b + 3)}{\gamma\tau_2 - b - 1}. \qquad (7)$$

The polarisation of the active medium plays a decisive role in the origin of the instability described by expression (7). Therefore, this instability can be called coherent instability. The instability of this type is absent if r^* is negative, i.e., if $\gamma\tau_2 < 1 + b$. For this reason, this instability is absent in class B lasers.

When condition (7) is fulfilled, laser radiation exhibits non-periodic pulsations [7–9], which were called the dynamic chaos. An example of such pulsations is shown in Fig.1, and the phase portrait of these pulsations is presented in Fig. 2. The phase portrait clearly demonstrates the existence of two regimes shifted in phase by π. The amplitude of laser radiation pulsates around one of the equilibrium states. When the amplitude vanishes, the laser finds itself in the region of attraction by another equilibrium state. The amplitude of laser radiation begins to pulsate around this equilibrium state, by abruptly changing its phase by π. This phase jump in not accompanied by any 'energy catastrophe' because the phase abruptly changes in the state with the zero amplitude.

All the theoretical results are in remarkable agreement with experimental data [10]. However, it is not simple to build class A lasers because most of the active media have a very broad gain band in the visible range compared to the spectral width of the resonator mode. In this connection, we point out here Ref. [11] in which pulsating regimes were observed for the first time in class A lasers.

Note that the character of pulsating regimes depends substantially on the parameter b, i.e., on the ratio of the relaxation times of polarisation and the number of particles. As the parameter b is decreased, the rest of the parameters being invariable, pulsations become more regular, which is clearly shown in Figs 3 and 4. The smaller b, the greater value of r is required to produce non-regular pulsations.

An amplifier, in which radiation freely propagates in the active medium in the absence of a resonator, has an

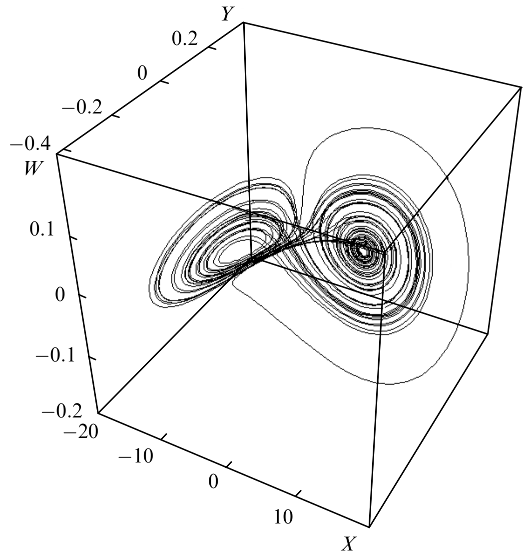

Figure 2. Phase portrait of the laser corresponding to the parameters in Fig. 1.

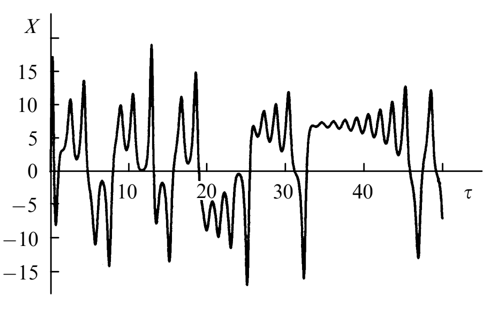

Figure 1. Amplitude X of output laser radiation as a function of τ for $r > r^*$ and $\gamma\tau_2 = 3$, $b = 0.3$, and $r = 50$.

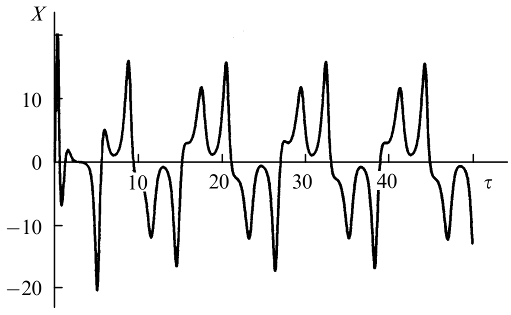

Figure 3. Complicated periodic pulsations of the field amplitude in a laser for $\gamma\tau_2 = 3$, $b = 0.1$, and $r = 50$.

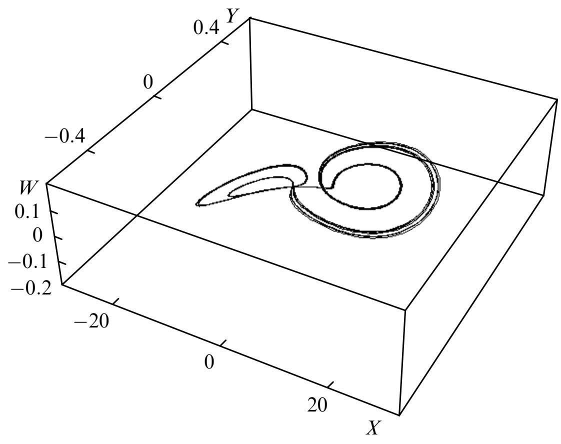

Figure 4. Phase portrait of the laser corresponding to the parameters in Fig. 3.

interesting dynamic property. In this case, equations (3) are transformed to the system of partial differential equations

$$\frac{\partial A}{\partial t} + c\frac{\partial A}{\partial z} + \gamma A = \mathrm{i}2\pi\omega B,$$
$$\frac{\partial B}{\partial t} + \frac{1}{\tau_2}B = -\mathrm{i}\frac{\mu^2}{\hbar}NA, \tag{8}$$
$$\frac{\partial N}{\partial t} + \frac{1}{\tau_1}N = J + \frac{\mathrm{i}}{2\hbar}(AB^* - A^*B).$$

The experiments and theory show [12, 13] that the stationary wave $A(t - z/u)$ can appear in the amplifier, which propagates in the amplifying medium at the velocity u exceeding the speed of light in vacuum. Equations (8) for the stationary wave take the form

$$\frac{\mathrm{d}A}{\mathrm{d}\xi}\left(1 - \frac{c}{u}\right) + \gamma A = \mathrm{i}2\pi\omega B, \quad \xi = t - \frac{z}{u},$$
$$\frac{\mathrm{d}B}{\mathrm{d}\xi} + \frac{1}{\tau_2}B = -\mathrm{i}\frac{\mu^2}{\hbar}NA, \tag{9}$$
$$\frac{\mathrm{d}N}{\mathrm{d}\xi} + \frac{1}{\tau_1}N = J + \frac{\mathrm{i}}{2\hbar}(AB^* - A^*B).$$

The system of equations (3) for $\Delta_{\mathrm{c}} = \Delta_2 = 0$ is isomorphous to system (9). Therefore, when the amplifier parameters satisfy the condition [14–16]

$$r > r^* \equiv \frac{\tilde{\gamma}\tau_2(\tilde{\gamma}\tau_2 + b + 3)}{\tilde{\gamma}\tau_2 - b - 1}, \quad \tilde{\gamma} = \gamma\frac{u}{u - c}, \tag{10}$$

the stationary regime becomes unstable and should pass to a chaotic regime. In the case of an amplifier, a new parameter appears – the velocity of pulse propagation, which gives an additional possibility to control the lasing regime. When the pulse velocity is close to the speed of light, but is somewhat greater, the condition $\tilde{\gamma}\tau_2 > 1 + b$ required for producing the chaotic regime can be achieve more easily.

The superluminal pulse velocity is determined by the expression [14]

$$u = \frac{c}{1 - \gamma\tau_0[r\tau_1/(\tau_1 + \tau_2) - 1]}, \tag{11}$$

where τ_0 is the characteristic rise time of the leading edge in the linear part of the laser pulse entering the amplifying medium. For $\tau_1 \gg \tau_2$, expression (11) transforms to the formula for the class B lasers, which was obtained in Refs [12, 13]. The superluminal pulse velocity can be controlled by varying the time τ_1. Unfortunately, no examples of the propagation of radiation in the class A amplifier have been reported in the literature so far. This problem is rather complicated for calculations because the system of partial differential equations (8) should be solved. A detailed study of this problem requires time-consuming computer calculations compared to the study of the system (1).

The examples considered above can give the impression that, when condition (7) is fulfilled, the lasing regime with the constant field amplitude passes to the pulsating regime. However, the lasing dynamics within the framework of model (4) is much more complicated. The matter is that the instability condition (7) is obtained upon studying the stationary regime by the Lyapunov method, i.e., by the method of linearisation of equations. A global numerical study shows that the phase space of system (4) begins to change when the excitation parameter r is smaller than r^*. In particular, it is found that the pulsating regime and the constant-amplitude regime can coexist at the same values of r. The development of one or another regime depend on the initial conditions. This is discussed in more detail in Ref. [6] and references therein.

2. Dynamics of a laser with a saturable absorber

The dynamics of a laser with a saturable absorber (SA) attracts the attention of researchers beginning from papers [17–19]. However, these studies concerned mainly class B lasers. Below, we will consider the general case of class A lasers.

The model of a laser with an inertialess SA is obtained from the system of equations (3) by the replacement $\gamma \to \gamma[1 + \alpha/(1 + \sigma\eta A^2)]$, where α is the relative absorption coefficient of the SA; η is the saturation coefficient of the SA divided by the laser saturation parameter σ. The system of equations for the laser with a SA in the dimensional variables has the form

$$\frac{\mathrm{d}X}{\mathrm{d}\tau} + \gamma\tau_2\left(1 + \frac{\alpha}{1 + \eta X^2}\right)X - \gamma\tau_2 rY = 0,$$
$$\frac{\mathrm{d}Y}{\mathrm{d}\tau} + Y - XW = 0, \tag{12}$$
$$\frac{\mathrm{d}W}{\mathrm{d}\tau} + b(-1 + W + XY) = 0.$$

The system of equations (12) contains five parameters. A detailed study of the lasing regimes of such a laser in the five-dimensional space is very time consuming. Because of this, we will study this system depending on the excitation parameter r, giving specific values to the rest of the parameters.

In the stationary lasing regime, the field amplitude in the resonator is determined from the equation

$$\left(1+\frac{\alpha}{1+\eta X^2}\right)X^2=\frac{r}{1+X^2}\,X^2, \tag{13}$$

while the corresponding population difference and polarisation are found from the expressions

$$W=\frac{1}{r}\left(1+\frac{\alpha}{1+\eta X^2}\right),\quad Y=XW. \tag{14}$$

It is obvious that the zero and positive values of X^2 have the physical meaning. The zero solution always exists*. The nonzero solutions of equation (13) are shown in Fig. 5 ($X_{\rm b}^2$ and $X_{\rm s}^2$ are the greater and smaller of these solutions, respectively). In the region $r_{\rm th}<r<r_1$, three stationary values of the intensity exist: zero (zero regime) and two nonzero. The nonzero values at the point $r_{\rm th}$ becomes identical. This point corresponds to the lasing threshold. It follows from the general concepts of the theory of nonlinear oscillations that the nonzero regime with a lower intensity is always unstable [20]. In this sense, it can be called absolute unstable. The zero regime in this region of the values of the parameter r is stable, and lasing develops beginning from a finite radiation intensity (a start signal is required to trigger the laser).

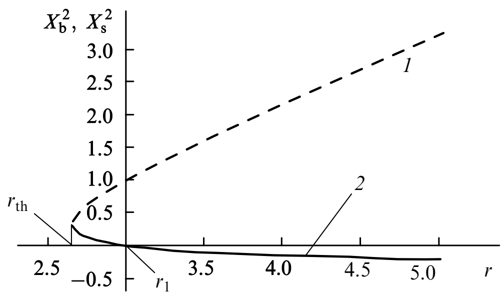

Figure 5. Dependences of $X_{\rm b}^2$ (*1*) and $X_{\rm s}^2$ (*2*) on the excitation parameter r for the laser with a saturable absorber for $\alpha=2$ and $\eta=20$.

When $r>r_1$, the nonzero solution with a lower intensity becomes negative, and only one stationary value of the intensity exists. The zero regime in this region is unstable, so that the excitation regime of the laser becomes soft. The laser can be triggered by an arbitrarily small perturbation of the field or polarisation. The study of the stability of the lasing regime with the constant radiation intensity $X_{\rm b}^2$ leads to the following characteristic equation

$$s^3+As^2+Bs+C=0, \tag{15}$$

in which

$$A=1+b+\gamma\tau_2\left(1+\frac{\alpha}{1+\eta X_{\rm b}^2}-2\alpha\,\frac{\eta X_{\rm b}^2}{1+\eta X_{\rm b}^2}\right),$$

$$B=\gamma\tau_2 b\left(1+\frac{\alpha}{1+\eta X_{\rm b}^2}\right)+b\left(1+X_{\rm b}^2\right)-$$
$$-\gamma\tau_2\,\frac{2\alpha\eta X_{\rm b}^2}{\left(1+\eta X_{\rm b}^2\right)^2}\,(1+b), \tag{16}$$
$$C=2\gamma\tau_2 bX_{\rm b}^2\left(1+\frac{\alpha}{1+\eta X_{\rm b}^2}\right)$$
$$-\gamma\tau_2 b\,\frac{2\alpha\eta}{\left(1+\eta X_{\rm b}^2\right)^2}\,\left(1+X_{\rm b}^2\right).$$

It follows from the Gurvits theorem [21] that the stationary regime with the constant output amplitude becomes unstable if at least one of the coefficients (16) or the determinant $D\equiv AB-C$ becomes negative. Note that, for $\alpha=0$, the inequality $D<0$ is equivalent to condition (7).

The value of D proves to be decisive for the determination of the stability of lasing regimes. Figures 6 and 7 show the values of the determinant D and $X_{\rm s}^2$ for two different sets of parameters. As mentioned above, the positive value of $X_{\rm s}^2$ corresponds to the hard regime of excitation of oscillations in the laser. The curves in Figs 6 and 7 demonstrate different dynamic regimes of the laser with a SA. For example, the parameters can be chosen so that the laser will not have operating regimes with the constant radiation intensity at all (Fig. 7).

Let us study the type of pulsations appearing in the region $r>r^*$. The character of lasing regimes in this region

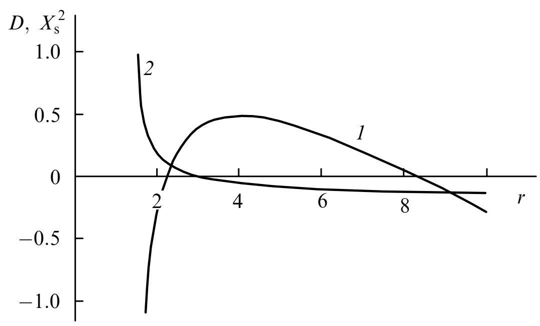

Figure 6. Determinant D (*1*) and $X_{\rm s}^2$ (*2*) as functions of r for $\alpha=2$, $\eta=30$, $b=0.1$, and $\gamma\tau_2=3$.

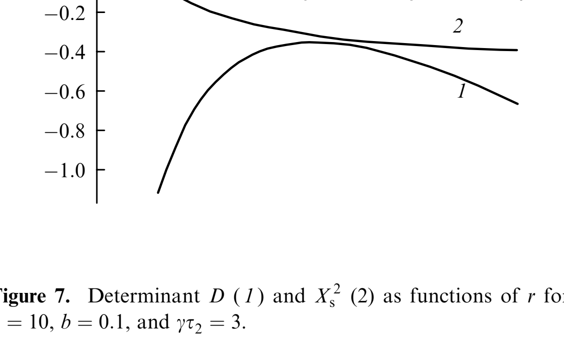

Figure 7. Determinant D (*1*) and $X_{\rm s}^2$ (2) as functions of r for $\alpha=2$, $\eta=10$, $b=0.1$, and $\gamma\tau_2=3$.

* This statement is true if spontaneous emission into one resonator mode is neglected.

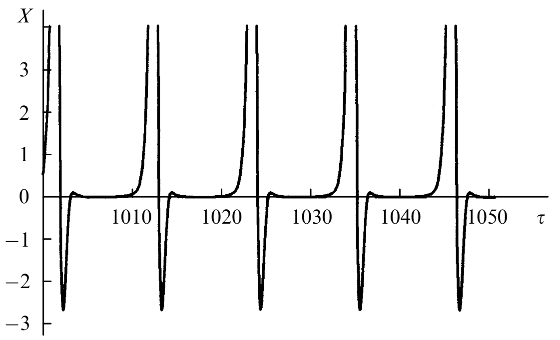

Figure 8. Amplitude X of output laser radiation as a function of τ for $\alpha = 2$, $r = 15$, $\eta = 20$, $b = 0.1$, and $\gamma\tau_2 = 3$.

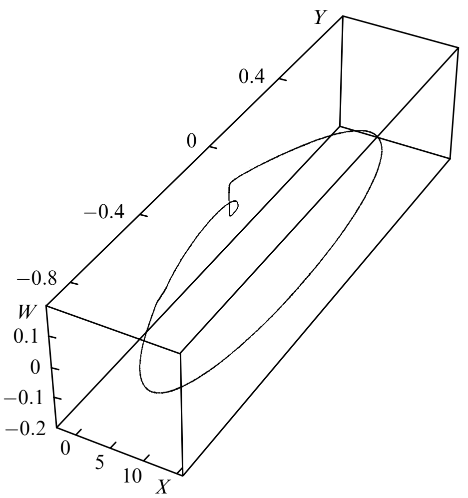

Figure 9. Phase portrait of the laser corresponding to the parameters in Fig. 8.

significantly depends on the parameter b. Figures 8 and 9 show one of the possible regimes. We see that the amplitude pulsations are complicated but periodical. Let us now increase the value of b by five times, keeping the rest of the parameters invariable. As a result, the regime of periodic pulsations passes to the regime of dynamic chaos (Fig. 10). Therefore, the property that the lower b, the greater excitation parameter is required to produce dynamic chaos in the laser, is also inherent in lasers with a SA.

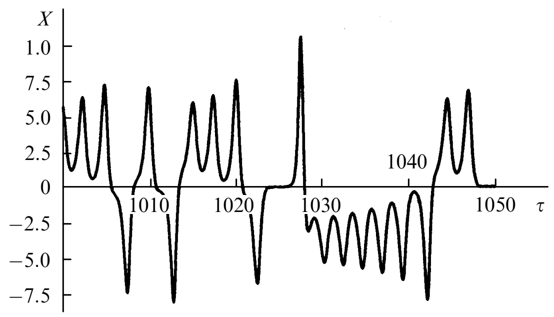

Figure 10. Pulsations in the laser for $\alpha = 2$, $r = 15$, $\eta = 20$, $b = 0.5$, and $\gamma\tau_2 = 3$ (cf. pulsations in Fig. 8).

Let us select the laser parameters to provide the hard excitation regime (Fig. 11). Then, the development of stationary lasing will be accompanied by pulsations of laser radiation. Let us now increase the pump intensity, remaining in the hard excitation region. Fig. 12 shows on of the typical lasing regimes in this case. One can see that chaotic pulsations are interrupted within some time after the onset of lasing and lasing ceases despite continuous pumping. This situation appears paradoxical because an increase in the pump intensity over the lasing threshold resulted finally in the quenching of lasing. This effect can be explained in the following way [22]. In the chaotic regime in a free-running laser (Fig. 1), the radiation field and polarisation pass repeatedly through zero, but this does not quench lasing because the polarisation and field are shifted in phase and do not pass through zero simultaneously. However, generation in a laser with a SA can be quenched not only when the field and polarisation vanish simultaneously but also when the absolute values of their amplitudes become simultaneously lower than the required starting values. In the chaotic regime, this occurs sooner or later, resulting in the quenching of lasing.

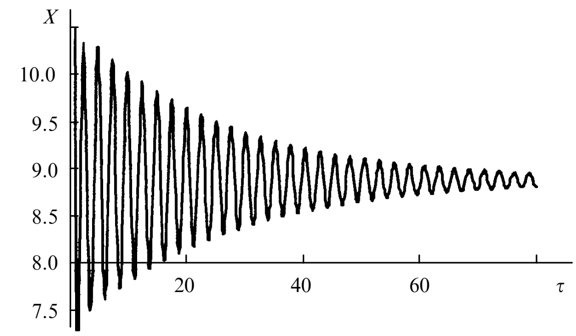

Figure 11. Development of the lasing regime with the constant amplitude for $\alpha = 20$, $r = 10$, $\eta = 500$, $b = 0.5$, and $\gamma\tau_2 = 3$.

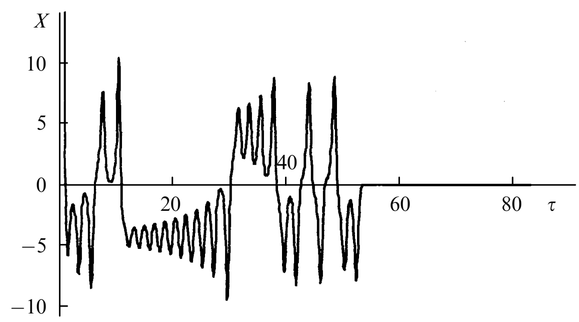

Figure 12. Quenching of lasing in the regime of dynamic chaos for $\alpha = 20$, $r = 20$, $\eta = 500$, $b = 0.5$, and $\gamma\tau_2 = 3$.

Figure 13 shows the phase portrait of a laser with a saturable absorber operating in the hard excitation regime. One can see that the phase trajectory enters the region of a strange attractor and, after remaining there for some time, goes away to the region of the zero amplitudes of the field and polarisation. Due to the chaotic character of the motion in the region of the strange attractor, the residence time of the phase trajectory in this region should depend randomly on the initial conditions. This is indeed the case. Table 1 presents the dependence of the typical duration τ_p of the chaotic pulse on the initial amplitude $X(0)$. Polarisation and the difference of concentrations of particles on the upper and lower levels at the initial instant were assumed zero. The intervals of lasing between its appearance and quenching were such that it was difficult to find any regularity in the interval variation with changing the initial field amplitude.

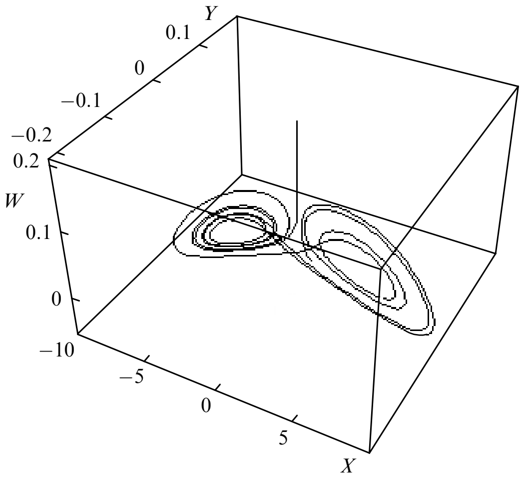

Figure 13. Phase portrait of the laser corresponding to the quenching of lasing (cf. Fig. 12).

Table 1.

$\gamma\tau_p$	$X(0)$	$\gamma\tau_p$	$X(0)$	$\gamma\tau_p$	$X(0)$	$\gamma\tau_p$	$X(0)$	$\gamma\tau_p$	$X(0)$
21.5	1.0	62.5	3.0	8.7	5.0	35.5	7.0	18.7	9.0
4.7	2.0	50.0	4.0	19.0	6.0	16.8	8.0	22.3	10.0

Note: $\alpha = 20$, $r = 20$, $\eta = 500$, $\gamma\tau_2 = 3$, $Y(0) = 0$, $W(0) = 0$.

Let us take the following laser parameters: $\alpha = 2$, $\eta = 3$, $\gamma\tau_2 = 10^{-4}$, and $b = 10^{-5}$. The last two values are typical of class B lasers. Figure 14 shows the dependences of X_s^2 and the determinant D on the excitation parameter r^*. One can see that the determinant D for class B lasers is negative (and the lasing regime with a constant amplitude is unstable) only when the excitation parameter is comparatively small. In this region, in the case of soft excitation, the pulsating regime is established, which can be called the relaxation regime, following the notation used in Ref. [20] (Fig. 15).

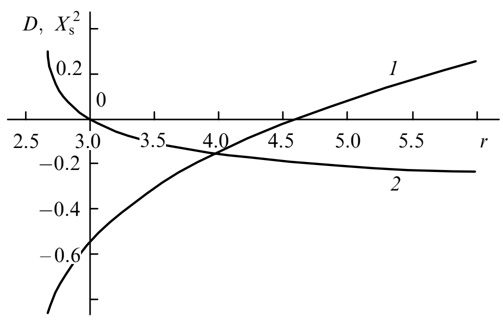

Figure 14. Determinant D (*1*) and X_s^2 (*2*) as functions of r for class B lasers for $\alpha = 2$, $\eta = 10$, $b = 10^{-5}$, and $\gamma\tau_2 = 10^{-4}$.

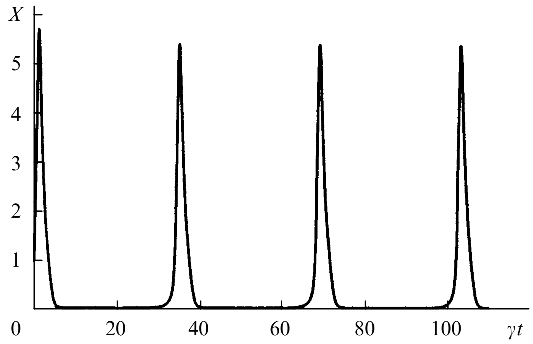

Figure 15. Relaxation pulsations in class B lasers in the region of soft excitation of oscillations for $\alpha = 2$, $r = 4$, $\eta = 3$, $b = 10^{-5}$, and $\gamma\tau_2 = 10^{-4}$.

What will happen with class B lasers in the hard excitation regime? Figure 16 gives the answer to this question: the laser, emitting a pulse according to the initial conditions, cannot generate in the stationary regime at all. This is quite clear in view of the above discussion. Class B lasers, even in the stable stationary regime with a constant amplitude, pass to the stationary regime through a sequence of decaying intensity pulsations. If the intensity of pulsations at the minimum is very low (lower than the initial intensity), then lasing ceases after the first pulsation.

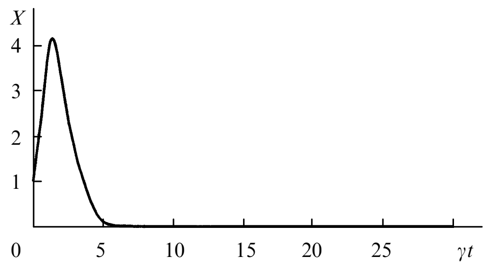

Figure 16. Quenching of lasing in class B lasers in the region of hard excitation of oscillations for $\alpha = 2$, $r = 2.95$, $\eta = 3$, $b = 10^{-5}$, and $\gamma\tau_2 = 10^{-4}$.

3. Conclusions

Although the dynamics of masers and lasers have been studied for more than forty years, many questions remain to be answered.

It is obvious that the structure of the phase space in the laser model with a SA (12) should be no less complicated than that in model (4). The global study of the former model requires time-consuming calculations and, hopefully, will be performed in the near future. We present here only one example of the dependence of the lasing regime on the initial conditions. As shown in Fig. 17a, when the initial population inversion is maximal, lasing with the constant amplitude develops. If the initial population inversion is zero, lasing ceases after several pulsations (Fig. 17b).

Note also another property of lasing for the laser parameters corresponding to Fig. 17. During the development of lasing, the laser field changes its phase by π compared to the initial-signal phase. We saw that this

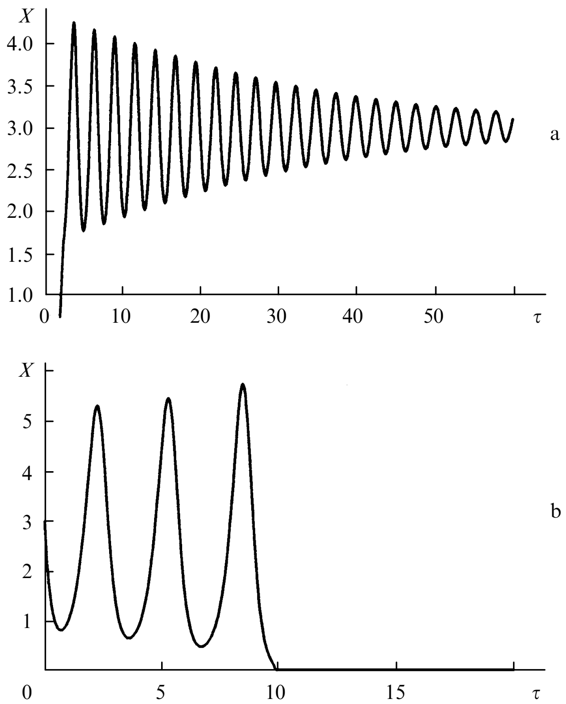

Figure 17. Dependences of the stationary lasing regime on the initial conditions for $X(0) = -3$, $W(0) = 1$ (a) and $X(0) = 3$, $W(0) = 0$ (b) and $\alpha = 20$, $r = 10$, $\eta = 50$, $b = 0.5$, and $\gamma\tau_2 = 3$.

does not always occur and depends, in particular, on the excitation parameter. It is undoubtedly interesting to study numerically in detail the propagation of a pulse in an amplifying medium, for example, the transformation of a regular superluminal pulse to a chaotic pulse. Unfortunately, this was not accomplished yet.

Although it is difficult to obtain in laser experiments the numerous regimes corresponding to model (12), many of them can be observed in nonlinear systems of different nature. In particular, the author does not abandon the attempt to understand whether is it possible to provide the self-defence of nuclear reactors based on the quenching of generation?

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