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Properties of self-compensation for three-dimensional periodic phase distortions in active media of gas-flow lasers

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***Abstract*. A mathematical model for optimising the beam formation direction in an active medium with three-dimensional periodic perturbations is developed. This model is applicable to the refractive-index distributions that can be represented in a multiplicative form with separated variables. The analysis of the actual structure of perturbations that appear behind the original nozzle assembly of a cw chemical laser has shown that the optimal direction of the beam formation lies in the region of partial self-compensation for distortions, which ensures an acceptable wave-front quality of the formed radiation.**

Keywords: *optical quality of an active medium, periodic phase distortions, self-compensation for phase distortions.*

1. Introduction

Compensation for regular periodic inhomogeneities of various scales in gas-laser active media with a supersonic circulation of components is an urgent and complex problem. Among such lasers, gas-dynamic (GDLs) and cw chemical lasers (CLs) should be primarily mentioned. A necessary condition for their activation is the thermodynamically nonequilibrium cooling of the working gas mixture during its passage through a sequence of small-diameter supersonic nozzle – nozzle assembly.

The size of each nozzle is determined by physical conditions; therefore, the rated laser output power is achieved by using a nozzle assembly. In this case, a complex spatially inhomogeneous distribution of phase distortions appears as a result of the interaction of adjacent supersonic jets behind the nozzle-assembly edge in active media. It is obvious that the spatial structure of inhomogeneities is different for various nozzle-assembly designs. This affects not only the characteristics of the directivity of laser radiation but also the efficiency of using the correction of phase aberrations.

Numerous papers (e.g., Refs [1 – 5]) devoted to studying the structure of perturbations in active media of GDLs and cw CLs present the literature on this subject in more detail.

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However, their authors restricted themselves only to a statement of the integral perturbation structure and determination of the level of phase inhomogeneity along the optical-axis direction, without considering in detail the question of the effect of each perturbation component with its own spatial scale on the laser-beam divergence. Moreover, there is no simple and reliable mathematical technique that allows one to find the optimal conditions for efficient self-compensation for regular periodic inhomogeneities of various scales.

Therefore, the aim of this work was to develop a special mathematical apparatus for predicting the optical quality of active media with a spatial structure of periodic perturbations for any angular orientation of the optical axis relative to the characteristic directions in the nozzle structure of the assembly and to the downstream direction. The other aim was to test the efficiency of this mathematical model as applied to an active media formed by an actual nozzle assembly of a cw CL.

2. Mathematical model

Consider an active medium formed by a nozzle assembly that consists of a set of small supersonic nozzles. It is necessary to find an optimal direction of the optical beam-formation axis relative to the characteristic directions of the nozzle arrangement in the nozzle assembly. This direction must ensure the minimum wave-front distortions. Since we study only phase perturbations, a possible nonuniformity in the spatial gain distribution in active media can be neglected.

When describing the phase state of an active medium, the most convenient approximation of the three-dimensional (3D) refractive-index distribution is a function

$$n(x,y,z) = n_0 + n_x(x)n_y(y)n_z(z),$$

where n_0 is the volume-averaged value of the refractive index, and each spatial component of the refractive-index variation relative to the average value can be expanded into a Fourier series

$$n_t(t) = \sum_{m=-\infty}^{+\infty} C_m^t \, \mathrm{e}^{jmt}, \; t = x, y, z,$$

where x, y, and z are dimensionless coordinates and

$$C_m^t = \frac{1}{2\pi}\int_{-\pi}^{+\pi} n_t(\tau)\, \mathrm{e}^{jm\tau}\mathrm{d}\tau$$

are the Fourier coefficients.

An example of a 3D periodic structure of refractive-index perturbations in an active-medium flow is a medium formed by a nozzle assembly of a GDL with a hexagonal arrangement of supersonic nozzles (Fig. 1a). We consider a 3D oblique-angled basis with an angle ε between the x and z axes. If the radiation propagates along the z axis, then the y axis corresponds to the downstream direction along the flow behind the nozzle assembly. For the nozzle arrangement examined, $\varepsilon = \pi/3$.

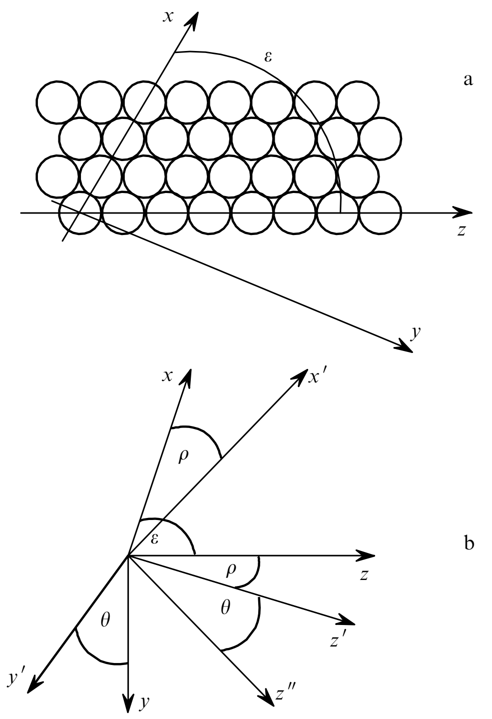

Figure 1. (a) Schematic of a cellular structure filled with axially symmetric nozzles of the nozzle assembly and (b) transformation of a coordinate system for determining the optimal direction of radiation extraction.

An efficient self-compensation for phase inhomogeneities in an active medium consists in a search for an optimal direction of radiation propagation through a volume with periodic refractive-index perturbations. For this purpose, we introduce two angles (ρ and θ) of spatial reorientation of the z optical axis into the z'' axis (Fig. 1b). After rotations about the y and x' axes through angles ρ and θ, respectively, we have the following transformation of coordinates:

$$x = (y' \sin\theta - z'' \cos\theta) \sin\rho + x' \cos\rho,$$

$$y = y' \cos\theta - z'' \sin\theta,$$

$$z = (y' \sin\theta + z'' \cos\theta) \cos\rho - x' \sin\rho.$$

Because the x and z axes are not orthogonal, the final expression for the function of spatial radiation propagation direction is as follows:

$$n(x', y', z'') = n_0$$

$$+\sum_m \sum_n \sum_k C_m^x C_n^y C_k^z \exp(jA_{mk}x') \exp(jB_{mnk}y') \exp(jC_{mnk}z''),$$

where the coefficients depending on the indices m, n and k have the form

$$A_{mk} = (m \operatorname{cosec}\varepsilon - k \cot\varepsilon) \cos\rho - k \sin\rho,$$

$$B_{mnk} = (m \operatorname{cosec}\varepsilon - k \cot\varepsilon) \sin\rho \sin\theta$$

$$+ n \cos\theta + k \cos\rho \sin\theta,$$

$$C_{mnk} = (m \operatorname{cosec}\varepsilon - k \cot\varepsilon) \sin\rho \cos\theta$$

$$- n \sin\theta + k \cos\rho \cos\theta.$$

Integrating the function $n(x', y', z'')$ along the z''optical axis, we obtain a two-dimensional real optical-path (phase-incursion) function

$$F(x', y') = \int_0^{L_z} n(x', y', z'') \mathrm{d}z''$$

$$= L_z \sum_m \sum_n \sum_k C_m^x C_n^y C_k^z \exp(jA_{mk}x') \exp(jB_{mnk}y')$$

$$\times \left[-\operatorname{sinc}\left(\frac{L_z}{\pi} C_{mnk}\right) + j \operatorname{sinc}\left(\frac{L_z}{2\pi} C_{mnk}\right) \sin\left(\frac{L_z}{2} C_{mnk}\right) \right],$$

where $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$ and L_z is the geometrical path of radiation propagation in the active medium along the z'' axis. During the integration, we omit the constant phase-incursion component $n_0 L_z$, which introduces no contribution to the active-medium optical quality along the z'' axis for its arbitrary orientation in space due to an obvious independence of n_0 of ρ and θ.

It is clear that, up to an accuracy of a constant factor, the function $F(x', y')$ describes the radiation wave front after its passage through an active medium with a refractive-index distribution $n(x', y', z'')$. An important fact is that the presence of a large number of wave-front perturbation periods in the $L_x \times L_y$ aperture is initially implied; i.e., the wave front is free of optical-wedge-type distortions. Consequently, the multiple integral of the square of this function over the area of an aperture of a given size represents the spread D of the wave-front phase incursion with respect to the average level. As is known, this quantity is a criterion for the active-medium optical quality and the laser-beam divergence related to it. Note that D is a function of angles ρ and θ:

$$D(\rho, \theta) = \int_{-L_x/2}^{+L_x/2} \int_{-L_y/2}^{+L_y/2} F^2(x', y') \, \mathrm{d}y' \mathrm{d}x'$$

$$= L_x L_y L_z^2 \sum_{m_1} \sum_{m_2} \sum_{n_1} \sum_{n_2} \sum_{k_1} \sum_{k_2} C_{m_1}^x C_{m_2}^x C_{n_1}^y C_{n_2}^x C_{k_1}^x C_{k_2}^x$$

$$\times \operatorname{sinc}\left[\frac{L_x}{2\pi}(A_{m_1k_1} + A_{m_2k_2})\right] \operatorname{sinc}\left[\frac{L_y}{2\pi}(B_{m_1n_1k_1} + B_{m_2n_2k_2})\right]$$

$$\times \left[-\operatorname{sinc}\left(\frac{L_z}{\pi} C_{m_1n_1k_1}\right) + j \operatorname{sinc}\left(\frac{L_z}{2\pi} C_{m_1n_1k_1}\right) \right.$$

$$\times \sin\left(\frac{L_z}{2} C_{m_1 n_1 k_1}\right)\Big]\left[-\mathrm{sinc}\left(\frac{L_z}{\pi} C_{m_2 n_2 k_2}\right)\right.$$

$$\left.+j\,\mathrm{sinc}\left(\frac{L_z}{2\pi} C_{m_2 n_2 k_2}\right) \sin\left(\frac{L_z}{2} C_{m_2 n_2 k_2}\right)\right];$$

where D and the optical path are real functions.

As an example, we examine three functions of the spatial refractive-index distribution: $n(x, y, z) = \sin(x)$, $n(x, y, z) = \sin(x)\sin(y)$, and $n(x, y, z) = \sin(x)\sin(y)\sin(z)$. For simplicity, $n_0 = 0$ is assumed. If $L_x = L_y = 10\pi$, $L_z = 20\pi$ and $\varepsilon = \pi/3$ (the nozzle arrangement shown in Fig. 1a), the dependence of lg$D$ on the combination of angles ρ and θ looks like that shown in Fig. 2. Test calculations have confirmed the well-known experimental fact [4] that, in the conventional orientation of the rows in the nozzle assembly relative to the radiation-extraction direction ($\rho = 0$ and $\theta = 0$), an unfavourable phase spread is usually formed due to a resonant accumulation of wave-front perturbations; i.e., the active medium has a low optical quality for this radiation-propagation direction.

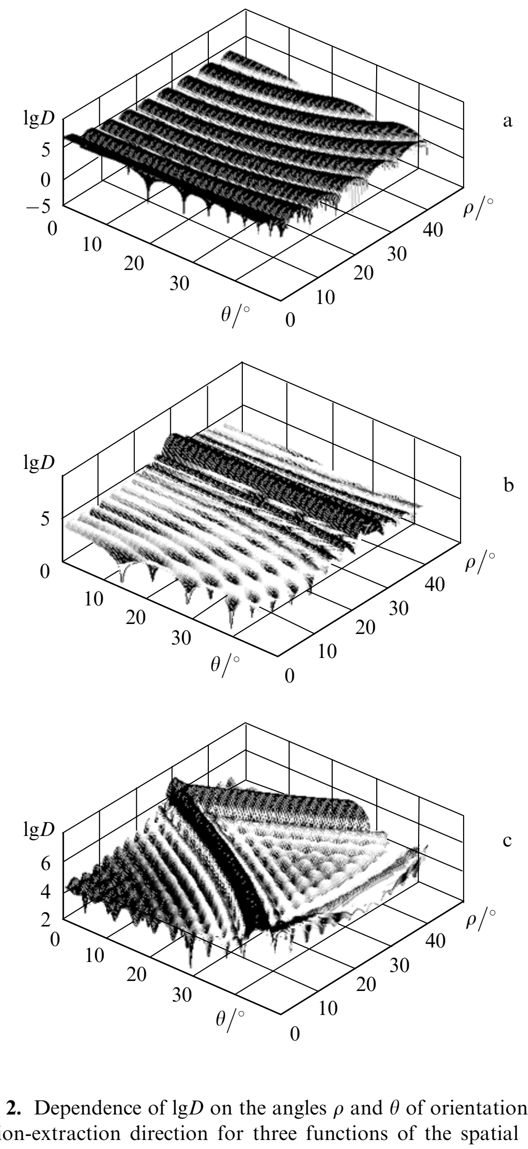

Figure 2. Dependence of lg$D$ on the angles ρ and θ of orientation of the radiation-extraction direction for three functions of the spatial refractive-index distribution: (a) $n(x, y, z) = \sin(x)$, (b) $n(x, y, z) = \sin(x)\sin(y)$, and (c) $n(x, y, z) = \sin(x)\sin(y)\sin(z)$.

By changing the optical-axis orientation of the beam extraction, it is possible to significantly reduce integral perturbations on the probing length Lz. During simulations, it has been established that the wave-front spread can be several orders of magnitude lower than for the conventional passage of radiation ($\rho = 0$ and $\theta = 0$).

The more complex the structure of the spatial refractive-index distribution $n(x, y, z)$, the more complex the dependence of the wave-front spread on the combination of angles $\rho = 0$ and $\theta = 0$. However, certain combinations of angles that ensure an acceptable spread in the radiation wave front upon averaging regular periodic perturbations, may always take place. This circumstance is obvious from the example considered above (Fig. 2).

In reality, when the length of the nozzle assembly of a large-size laser in the radiation-propagation direction is large, an optimal averaging of perturbations is achieved when the optical axis has a very small tilt relative to the characteristic direction in the arrangement of perturbing elements in the nozzle assembly (for the case of a single-pass laser amplifier). For a laser resonator with spherical mirrors, in which an oblique passage of rays in the active medium is initially realised, a partial self-compensation for phase distortions of almost all spatial scales always takes place.

3. Analysis of the results of the experimental study of the cw-CL active medium

The model considered was used in a search for the optimal conditions for implementing the self-compensation mechanism for radiation propagating in perturbations of complex structure formed by an actual nozzle assembly of a cw CL [5, 6]. The nozzle assembly schematically shown in Fig. 3 has several perturbing elements; therefore, the wave-front distortions are of essentially multiscale character [5, 7]. In addition to a search for the optimal radiation-extraction direction, an important task is to study the effect of self-compensation for a phase perturbation of one spatial scale on wave-front perturbations of other scales.

The period of arrangement of the nozzle-assembly blades containing nozzles for supplying a fuel is $P = 6$ mm, the period of arrangement of the nozzles for supplying the secondary fuel is $P_2 = 1$ mm, and the period of arrangement of technological channels in each nozzle for supplying an

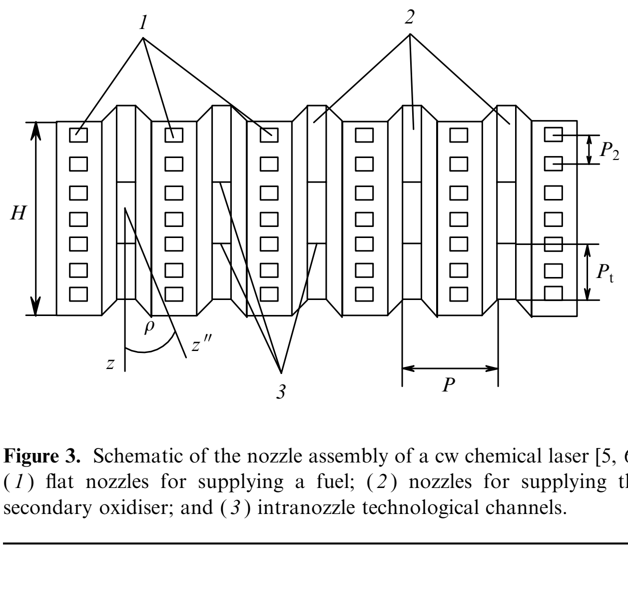

Figure 3. Schematic of the nozzle assembly of a cw chemical laser [5, 6]: (*1*) flat nozzles for supplying a fuel; (*2*) nozzles for supplying the secondary oxidiser; and (*3*) intranozzle technological channels.

oxidiser is $P_t = 21$ mm. The active-medium flow interferometry [5, 7] behind the nozzle assembly allowed the intensity (the modulation amplitude) of each detected periodic perturbation to be determined. Processing the interferograms has shown that the maximum intensity corresponds to a perturbation with a period P that appears in the direction perpendicular to the edges of flat nozzles. The spread in the wave-front phase incursion for 3-μm radiation on a 220-mm path, which is equal to two passes along the nozzle-assembly width or the height $H = 110$ mm of the nozzle blade, is 0.55 rad^2. Such a high perturbation intensity is associated with the shock waves forming in the flow as a result of an interaction of neighbouring supersonic oxidiser jets. A small-scale inhomogeneity on the same geometric path appearing in the perpendicular direction due to a discrete supply of the secondary fuel at $P_2 = 1$ mm has a wave-front spread of 0.006 rad^2. The spread in the same direction due to the effect of technological channels was 0.022 rad^2.

For a height H of the nozzle blade, the self-compensation for a distortion with a period P requires an orientation of the radiation-extraction direction relative to the blades' edges at a minimum sufficient angle $\rho_{\text{opt}} = \arctan(P/H) = 3.12°$ with respect to the most unfavourable direction coinciding with the z axis; for the periods P_2 and P_t, these angles are 89.48° and 79°, respectively. An important fact is that the direction of radiation propagation at an angle of 90° is most unfavourable for distortions with the periods P_2 and P_t.

A dependence of the phase-incursion spread D for each separate periodic distortion on the angle $\rho = 0 - 35°$ of the radiation-extraction direction relative to the blades' edges was obtained for an aperture $L_x = L_y = 50$ mm (Fig. 4). The absence of more complete experimental data did not allow us to analyse D as a function of angle θ between the optical axis and the downstream direction; therefore, $\theta = 0$ was assumed in the simulation.

At $\rho = 0$, spatial distortions with the periods P_2 and P_t are averaged and integrally contribute to the distortion amplitude with the period P. As the angle ρ increases, distortions with the periods P_2 and P_t begin to manifest themselves independently, which is indicated by ascending curves (*2*) and (*3*) (Fig. 4). At the same time, all the curves tend to a periodic decrease in the spread value to its minimum level for each distortion at certain angle ρ values, which directly confirms the self-compensation effect for each of these periodic aberrations.

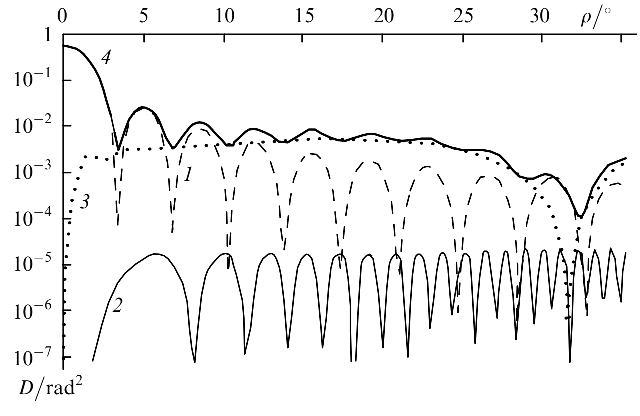

Figure 4. Dispersion of regular periodic distortions as a function of the angle ρ of orientation of the direction of probing the cw-CL active medium: (*1, 2, 3*) distortions with periods P, P_2, and P_t, respectively, and (*4*) superposition of distortions.

The resulting curve (*4*) (the integral distortion) also has pronounced minimum spread D values. However, they slightly differ from the complete self-compensation condition (i.e., $D \to 0$) that takes place in each of curves (*1*)–(*3*). Nevertheless, the wave-front phase spread in each local minimum in curve (*4*) remains many times lower than its most unfavourable value at $\rho = 0$.

In other words, the complete self-compensation for the most intense phase distortion with the period P occurs at $\rho_{\text{opt}} = 3.12°$. Although other periodic distortions manifest themselves at such an angle of radiation propagation, their effect is quite tolerable. The insignificance of this effect on spatial periodic distortions is directly confirmed by a tendency to a change in the dependence of the Strehl number Sh on the angle ρ (Fig. 5). This curve is plotted for a single-pass amplifier with a nozzle assembly consisting of nine modules [6] with a 1-m total length along the beam.

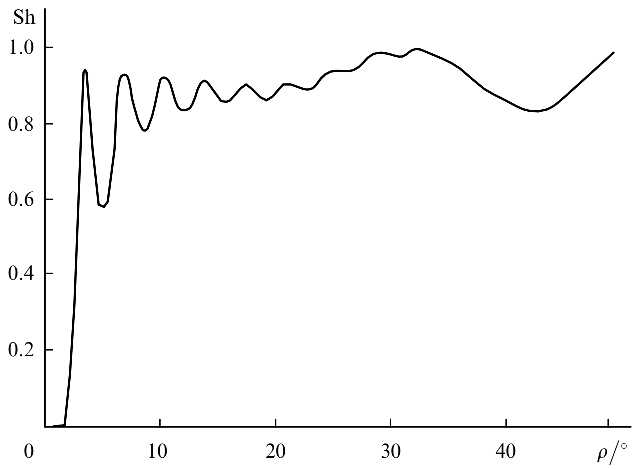

Figure 5. Strehl number as a function of the angle ρ of orientation of the radiation-extraction direction relative to the 'blade' edges.

Using the rules for converting the spread of regular wave-front distortions when scaling the medium for light propagation [7], it has been obtained that, at $\rho_{\text{opt}} = 3.12°$ for a modular nozzle assembly, the Strehl number is Sh ~ 0.95, testifying to a very high optical quality of the active medium for this direction. However, an abrupt degradation of the active-medium optical quality, which is displayed as a decrease in Sh almost down to a limiting minimum value of ~ 0.6, is observed already at $\rho \sim 1.5\rho_{\text{opt}} = 4.68°$. This means that the attainment of the maximum Sh value requires an extremely precise adjustment of the optical axis to the angle ρ_{opt}.

In practice, proceeding from the behaviour of the curve in Fig. 5, in order to ensure an acceptable optical homogeneity of the active medium without too stringent requirements to the adjustment, it is expedient to orient the optical axis at an angle of $15° < \rho < 30°$, which guarantees the condition Sh > 0.8. The most acceptable compromise is observed precisely within this angular range, which manifests itself as a partial self-compensation for distortions of various spatial scales.

4. Conclusions

The model developed makes it possible to reliably predict the optimal spatial orientation of the beam-extraction axis in a gas-flow laser relative to both the structure of the nozzle-assembly arrangement and the downstream direction. The latter ensures minimum wave-front aberrations of radiation transmitted through the active medium. The necessary conditions for applying this model are the possibilities of representing the refractive-index function in a multiplicative form with separating variables and expanding each of its spatial components into a Fourier series.

This mathematical apparatus was used to analyse perturbations in the active medium behind the edge of an actual nozzle assembly of a cw chemical laser. This study confirms the efficiency of the self-compensation mechanism for correcting spatial phase inhomogeneities of different scales in the active medium. It occurred that a partial self-compensation for phase distortions is an optimal mechanism for obtaining an active medium of an acceptable optical quality.

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