OPTICAL SOLITONS

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Fibreoptic communication lines with distributed Raman amplification: Numerical simulation

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Abstract. **The properties of optical solitons in variable-dispersion fibreoptic communication lines in which distributed Raman amplification of optical signals is used are studied by numerical simulation. It is shown that solitons can serve as carriers of information in communication systems with a data transmission rate exceeding 10 Gbit s^{-1}.**

Keywords: *soliton, dispersion, nonlinearity, optical fibre, fibreoptic communication lines, Raman amplification.*

The rapid progress in the investigations of optical communication lines is associated with the growth of the Internet and enhanced demand on telecommunication services, the need to attain the highest data transfer rates in the existing fibreoptic communication lines, and with the economical feasibility of modernisation of such systems. Following the development of optical amplifiers, losses in optical fibres are no longer treated as the main factor limiting the efficiency of fibre communication systems. The broadbandness (product of the signal transfer rate and the communication range) of fibreoptic communication lines is limited at present by dispersive and nonlinear distortions of signals. At the modern stage, soliton communication lines with a variable dispersion are seen as the most likely candidates for developing long ultrafast lines (with a data transmission rate exceeding 40 Gbit s^{-1} in a single frequency channel) because a dynamic balance exists between the group-velocity dispersion and self-phase modulation during propagation of solitons (see, for example, Refs [1 – 4]). Such systems are based on the application of different types of fibres with chromatic dispersions of opposite signs limiting the dispersive broadening of a pulse.

In such systems, dispersion-managed solitons (DM-solitons) are used as information carriers [5 – 14]. Here, we deal with a stable localised solution rather than a conventional (fundamental) soliton. A periodic variation in the system dispersion makes it possible to increase the soliton amplitude compared to the amplitude of an analogous pulse in systems with a constant dispersion and, consequently, enhance the signal-to-noise power ratio [11]. An increase in this ratio leads to a considerable decrease in statistical errors arising during optical transmission of information. In addition, unlike fundamental optical solitons whose steady propagation (i.e., compensation for the dispersion and nonlinearity effects) requires an anomalous chromatic dispersion, DM-solitons can be transmitted stably along a line with a zero or even negative mean chromatic dispersion. A low mean chromatic dispersion allows a suppression of the Gordon – Haus effect [15], as was demonstrated experimentally [6] for the first time in systems with a lumped amplification. Similar results obtained by numerical simulation are presented in Refs [10, 11]. The high local dispersion of a line weakens the interaction between adjacent channels due to four-wave mixing [11].

One of the most significant features of DM-solitons is their phase modulation, which leads to a rapid phase change between adjacent solitons and a weakening of interaction between them. Thus, the use of DM-solitons opens basically new avenues for increasing the transmission capacity of fibreoptic communication lines.

Until recently, erbium-doped fibre amplifiers (EDFAs) were used for amplifying optical signals in fibre communication lines [16]. They were called lumped amplifiers because the distance over which a signal is amplified (a few tens of metres) is much smaller than the separation between the amplifiers (tens of kilometres). The use of dispersion-controlled erbium-doped optical fibre amplifiers made it possible to develop commercial fibre communication lines with wavelength-division multiplexing (WDM systems) with a data transmission rate up to 10 Gbits s^{-1} per channel for a separation of about 80 km between the amplifiers.

The transmission ability of long WDM systems can be increased significantly by broadening the optical frequency band. However, the frequency band of conventional EDFAs covers the wavelength range 1530 – 1565 nm and severely hampers the broadening of the optical frequency band of transmission lines and hence the number of transmitting channels and the overall data transmission rate. Another significant drawback of such amplifiers is the nonuniformity of the gain in the working wavelength range. As a result, the power in one of the channels begins to exceed the power in another channel in the long-range WDM systems, thus increasing the error in data transmission.

The restrictions inherent in EDFAs imposed by the application can be eliminated to a considerable extent by using distributed Raman amplifiers [17]. The operation of

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such amplifiers is based on the application of stimulated Raman scattering that ensures the small-signal gain by transforming a part of the energy of a powerful pump wave.

The basic advantages of Raman amplifiers over EDFAs as follows [18]. Their amplification band can be broadened by increasing the number of pump waves. They have a relatively low noise level and hence a higher signal-to-noise ratio. The gain of Raman amplifiers can be equalised over a wide frequency range by regulating the input pump power. Finally, the fibre itself can be used as the active medium in them.

Until recently, the main drawback of Raman amplifiers was their low efficiency, which necessitated the use of continuous high-power pumping ($\sim$ 1 W). However, highly efficient Raman fibre lasers generating almost at any wavelength in the range 1.2 – 1.5 μm have been developed in recent years [19, 20], as well as an amplifier of this type based on fibres with a high concentration of germanium and with low optical losses [21].

In view of the above, it is quite urgent to study systems with distributed amplification. A large number of publications in recent years have been devoted to theoretical and experimental investigations of fibreoptic communication lines with distributed Raman amplification [22 – 30]. In this work, we have obtained numerically the soliton solutions for fibreoptic communication lines with distributed Raman amplification. Individual sections of such lines consist of alternating segments of a standard single-mode fibre and a dispersion-compensated fibre (Fig. 1). Table 1 shows the parameters of fibres used in the calculations.

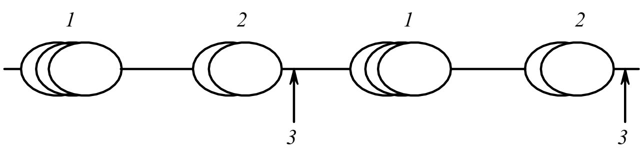

Figure 1. A section of a fibreoptic communication line with distributed Raman amplification: (*1*) standard single-mode fibre; (*2*) fibre for dispersion compensation; (*3*) reverse pumping.

To describe the dynamics of optical pulses in fibreoptic communication lines, we use the nonlinear Schrödinger equation for the complex envelope $\Psi(Z,T)$ of the electromagnetic field [31]:

$$\mathrm{i}\frac{\partial\Psi}{\partial Z}-\frac{1}{2}\beta_2(Z)\frac{\partial^2\Psi}{\partial T^2}+\sigma(Z)|\Psi|^2\Psi=\mathrm{i}G(Z)\Psi. \quad (1)$$

Here, Z is the distance in kilometres over which a soliton propagates; T is the time in picoseconds; $|\Psi|^2$ is the power of an optical pulse in watts; $\beta_2(Z)$ is the group-velocity dispersion coefficient; $\sigma(Z)=2\pi n_2(Z)/[\lambda_0 A_{\mathrm{eff}}(Z)]$ is the nonlinearity coefficient; n_2 is the nonlinear refractive index; λ_0 is the carrier wavelength; $A_{\mathrm{eff}}(Z)$ is the effective fibre area;

$$G(Z)=-\gamma(Z)+\gamma_{\mathrm{r}}(Z) \quad (2)$$

describes the optical losses in a fibre and the distributed Raman amplification; $\gamma_{\mathrm{r}}(Z)=g_0\times\ \exp[-2\gamma_{\mathrm{p}}(L-Z)]$; L is the length of one section of the line; $\gamma(Z)=0.05\times\ln(10)\alpha$; $\gamma_{\mathrm{p}}(Z)=0.05\ln(10)\alpha_{\mathrm{p}}$; and α, α_{p} are pulse attenuation coefficients at the carrier wavelength and at the wavelength of the return pump wave; g_0 is the input pump power (chosen in such a way that the signal power is completely restored at the end of each section).

We make the following change of variables in Eqn (1):

$$\Psi(Z,T)=A(Z,T)\exp\left[\int_z G(z')\mathrm{d}z'\right].$$

In this case, we go over to dimensionless variables $z=Z/L$ (L is in kilometres); $t=T/t_0$, where t_0 is the characteristic time in picoseconds; and $|\Psi|^2=P_0|A|^2$, where P_0 is the characteristic power in watts. In this case, the equation for the function $A(z,t)$ satisfies the nonlinear Schrödinger equation with periodic coefficients:

$$\mathrm{i}A_z+d(z)A_{tt}+c(z)|A|^2A=0. \quad (3)$$

Here

$$d(z)=-\frac{L\beta_2(z)}{2t_0^2};\ c(z)=P_0L\sigma(z)\exp\left[2\int_z G(z')dz'\right].$$

Using the averaging procedure described in Ref. [32], we can construct numerically the soliton solution of Eqn (3) for arbitrary initial conditions.

In most cases of practical importance, the shape of the real DM-soliton in the power region is close to Gaussian. Using this circumstance, Gabitov and Turitsyn [33] used the variational approximation to obtain a system of two ordinary differential equations for describing the dynamics of the main characteristics of a Gaussian pulse over a compensation period, namely, the duration of a pulse and the phase modulation parameter. The system of equations has the form

$$\frac{\mathrm{d}T_0}{\mathrm{d}z}=4d(z)M(z), \quad (4)$$

$$\frac{\mathrm{d}M}{\mathrm{d}z}=\frac{d(z)}{T_0^3}-\frac{c(z)N^2}{T_0^2}. \quad (5)$$

Here, the variable $T_0(z)$ is the optical pulse duration; $M(z)$ is its phase modulation parameter, and N^2 is the parameter proportional to the pulse energy.

Later, such a system of equations was also obtained [34] for the mean square angular momenta assuming a parabolic law of its phase variation:

Table 1.

Fibre	Distance/km	Dispersion D at $\lambda=1553$ nm /ps km^{-1} nm^{-2}	$\frac{\mathrm{d}D}{\mathrm{d}\lambda}$/ps km^{-1} nm^{-2}	Losses/dB km^{-1}	$n_2/10^{-20}$ m^2 W^{-1}	A_{eff}/μm
Standard single-mode fibre	102	+16.4	+0.06	0.21	2.6	80
Dispersion-compensated fibre	17.425	−96	−0.18	0.5	2.6	26

$$T_0(z) = \left(\frac{\int t^2 |A|^2 \mathrm{d}t}{\int |A|^2 \mathrm{d}t} \right)^{1/2}, \tag{6}$$

$$M(z) T_0(z) = \frac{\mathrm{i}}{4} \frac{\int t (A A_t^* - A^* A_t) \mathrm{d}t}{\int t^2 |A|^2 \mathrm{d}t}. \tag{7}$$

The periodic solutions of the system of equations (4) and (5) describe quite accurately the main characteristics of optical solitons (peak power, phase modulation parameter and duration) for a number of specific configurations of fibreoptic communication lines. A comparison of solutions of the system of equations (4) and (5) with solutions of Eqn (3) of the DM-type solitons will be made below.

We write the function $G(z)$ in the dimensionless variables:

$$G_1 = -\gamma_1 + g_0 \exp[-2\gamma_{\mathrm{p}1}(l_1 - z) - 2\gamma_{\mathrm{p}2}(1 - l_1)], \quad 0 < z < l_1, \tag{8}$$

$$G_2 = -\gamma_2 + g_0 \exp[-2\gamma_{\mathrm{p}2}(1 - z)], \quad l_1 < z < 1, \tag{9}$$

where l_1 is the length of a segment of the standard single-mode fibre; $\gamma_{\mathrm{p}2}$, $\gamma_{\mathrm{p}2}$ are the attenuation factors of the return pump wave for various regions of the fibre. The condition

$$\int_0^1 G(z) \mathrm{d}z = 0 \tag{10}$$

leads to the input pump power

$$g_0 = 2[\gamma_2 - (\gamma_2 - \gamma_1) l_1] \left\{ \frac{1}{\gamma_{\mathrm{p}2}} + \left(\frac{1}{\gamma_{\mathrm{p}1}} - \frac{1}{\gamma_{\mathrm{p}2}} \right) \times \exp[-2\gamma_{\mathrm{p}2}(1 - l_1)] - \frac{1}{\gamma_{\mathrm{p}1}} \exp\{-2[\gamma_{\mathrm{p}1} l_1 + \gamma_{\mathrm{p}2}(1 - l_1)]\} \right\}^{-1}. \tag{11}$$

Consider first the results of numerical solution of the system of equations (4) and (5). The periodicity conditions

$$T_0(0) = T_0(1), \quad M(0) = M(1) \tag{12}$$

make it possible to formulate the boundary value problem for this system. Because only two conditions (12) are imposed for determining three independent parameters $T_0(0)$, $M(0)$, N^2, one of these can be treated as free. The algorithm for solving the system of equations (4) and (5) is based on a combination of the fourth-order Runge–Kutta method and the Newton method [35] for determining the other two independent parameters, namely, $M(0)$ and N^2.

The solution of the system of equations (4) and (5) is presented in Fig. 2 as the dependence of the maximum power of a Guassian pulse on its FWHM duration (for $z = 0$). While constructing the soliton solution, the initial approximation (for $z = 0$) was chosen in the form of a Gaussian pulse

$$A(0, t) = N \left[\frac{2\sqrt{2}}{T_0(0)} \right]^{1/2} \exp\left[-\frac{t^2}{2T_0^2(0)} + \frac{\mathrm{i} M(0) t^2}{T_0(0)} \right]. \tag{13}$$

In this formula, the parameters N, $T_0(0)$ and $M(0)$ were obtained from a solution of the boundary-value problem (4), (5). After this, the averaging procedure described in Ref. [32] was used to construct solutions of the DM-soliton type for the initial configuration of fibreoptic communication lines. The numerical solution of the nonlinear Schrödinger equation (3), describing the evolution of an optical pulse, was carried out by the method of operator exponent [31]. A solution of the DM-soliton type was assumed to be constructed if the difference between its characteristics at the beginning and the end of a periodic section did not exceed 0.001 %. The dependence of the maximum power of DM-solitons on their duration is also shown in Fig. 2 (points). One can see that the solutions of the system of equations (4) and (5) are in fairly good agreement with the soliton solutions described by Eqn (3).

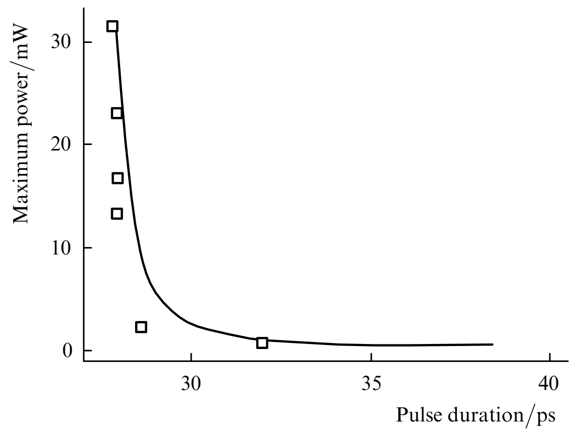

Figure 2. Dependence of the maximum pulse power on its duration. The solid curve is the solution of the system of Eqns (4) and (5), and the points are the soliton solutions of Eqn (3).

Fig. 3 shows the distribution of the DM-soliton power at distances Z that are multiples of the length L of one section of the fibreoptic communication line. One can see that the main parameters of the constructed soliton solutions vary significantly over the compensation period. This is confirmed by the dynamics of the main soliton parameters presented in Fig. 4 for a single period, namely, the maximum power, duration and the phase modulation parameter. Figure 5 shows the dynamics of interaction of two optical

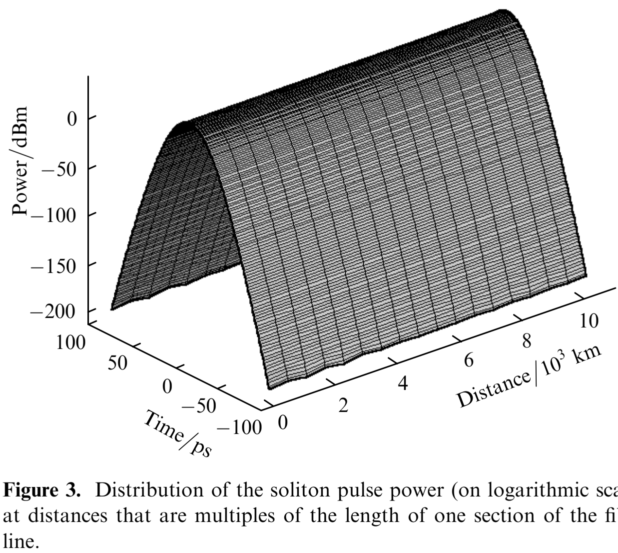

Figure 3. Distribution of the soliton pulse power (on logarithmic scale) at distances that are multiples of the length of one section of the fibre line.

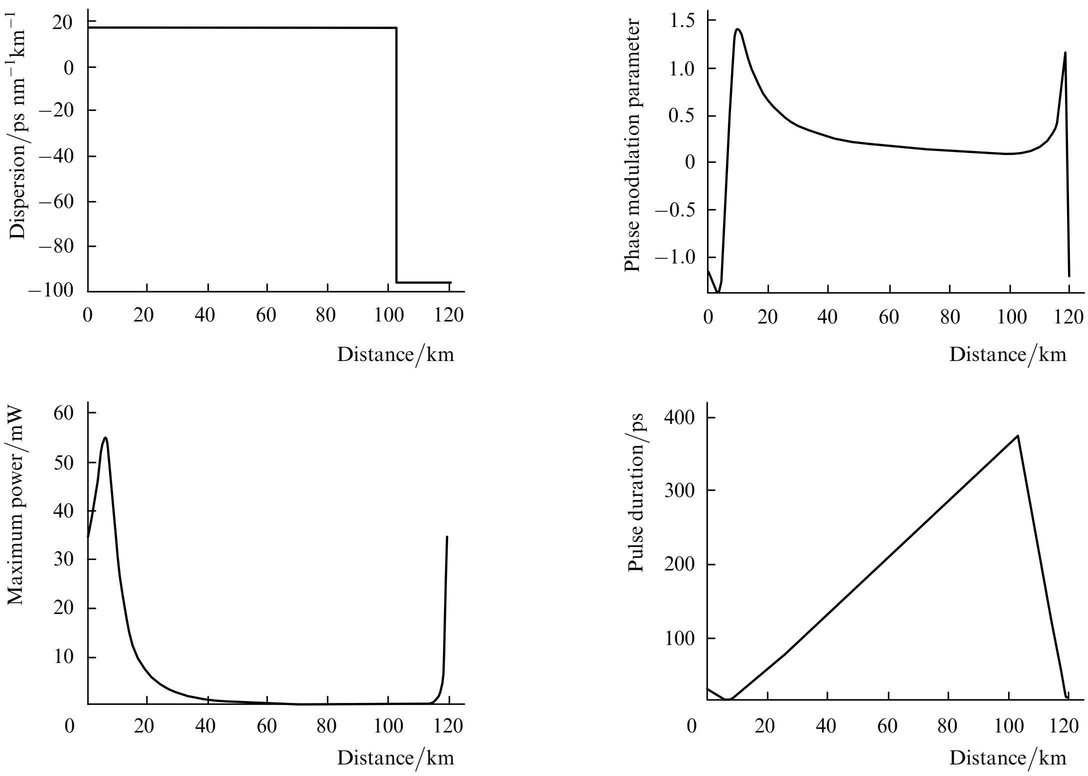

Figure 4. Dynamics of the main parameters of a soliton pulse over one dispersion compensation period.

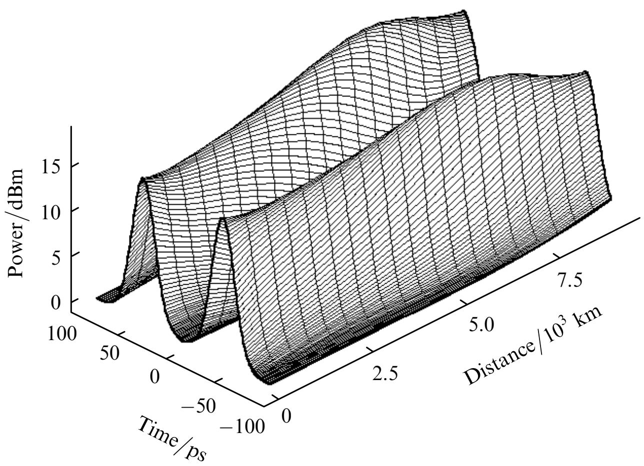

Figure 5. Dynamics of interaction of two soliton pulses initially displaced relative to each other by 100 ps.

DM-solitons (initially displaced relative to each other by 100 ps) over a distance of over 10000 km with a data transmission rate (capacity of the line) 10 Gbit s^{-1}.

Let us summarise the results of the above investigations. The method of mathematical simulation was used to study the propagation of DM-solitons along a fibreoptic communication line with distributed Raman amplification. Individual sections of this line are formed by a standard single-mode fibre and a fibre for compensating dispersion. Soliton solutions are constructed for the nonlinear Schrödinger equation with periodic coefficients and compared with the solutions obtained by using the Gabitov–Turitsyn approach. It is shown that soliton data transmission with a capacity exceeding 10 Gbit s^{-1} in a single frequency channel is possible across the oceans for such a configuration.

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