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Optical solitons appearing during propagation of whispering-gallery waves

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***Abstract*. The properties of solitons appearing during the propagation of whispering-gallery waves in a homogeneous glass cylinder are considered. It is shown that such solitons can be used for the light frequency conversion.**

Keywords: *self-action, whispering-gallery waves, light frequency conversion, light propagation, optical solitons.*

1. Introduction

Whispering-gallery (WG) waves are mainly known as the resonator modes of spherical glass microresonators. Having a diameter about of hundred of micrometers, these resonators possess an extremely high Q factor, which can exceed 10^9 [1]. Similar waves can be excited in glass cylinders [2], which are called tunneling waves [3]. Theoretical WG waves have inevitable radiative losses [3]. However, these losses are negligible in most practical applications [1], when the cylinder diameter exceeds 10 μm. The record high Q factor of spherical microresonators of diameter a few tens of micrometers convincingly confirms this.

Unlike common waveguide modes in glass cylinders, which are used for the transfer of optical solitons, WG waves have a quite unusual property: their group velocity can be significantly reduced (down to zero) during the propagation of the waves in a cylinder with a gradually decreasing diameter. This makes it possible to obtain optical resonators for WG waves in barrel-shaped parts of a glass cylinder [4]. It turns out that to make a high Q resonator, it is necessary to increase the cylinder diameter only by 0.01 % in the region of length a few tens of micrometres. This can be also achieved by increasing by 0.01 % the refractive index of glass in the region of the same length instead of the cylinder diameter. When an acoustic pulse is propagating along the cylinder axis, the refractive index of glass inside the cylinder increases, so that the above-mentioned optical resonator can propagate along the cylinder axis at the acoustic-wave velocity $v_a \simeq 6000$ m s^{-1}. If the resonator contains optical radiation, this radiation propagates together with the resonator. During the movement of the resonator along the axis of the cylinder whose diameter gradually decreases (increases), the radiation in the resonator is compressed (extended), resulting in the increase (decrease) of its frequency [5, 6]. The change in the frequency of a WG wave under the action of an acoustic pulse was demonstrated experimentally in Ref. [7].

Very interesting effects appear in a glass cylinder in which a light pulse propagates as a part of the WG wave. Similarly to the appearance of optical solitons in a glass cylinder during the propagation of a usual intense light pulse, peculiar WG solitons can appear during the propagation of an intense WG pulse. Indeed, the refractive index inside the pulse increases due to the Kerr effect, as in a usual soliton. In this case, the pulse finds itself in a moving WG resonator formed by the pulse itself. Such a self-confined light pulse, or a WG soliton, has very interesting properties, in particular, its propagation velocity can be quite small. In this case, except virtually inertialess Kerr nonlinearity, which is responsible for the appearance of usual solitons, the inertial nonlinearity associated with the appearance of the electrostriction pressure inside the light pulse has time to reveal itself in full measure. This nonlinearity in glass is known to be of the same order of magnitude as the Kerr nonlinearity [8]. In this paper, we consider the relation between these acoustic and optical effects.

2. Properties of the propagation of WG waves in a dielectric cylinder with diameter and refractive index varying along its axis

A WG wave propagating in an infinite dielectric cylinder can be treated as a part of a plane wave multiply scattering from the internal surface of the cylinder due to total internal reflection and rotating in such a way around its axis (Fig. 1). It is known that if the waveguide diameter changes sufficiently slowly along the waveguide axis, then the mode of the wave propagating in the waveguide is preserved, i.e., the waveguide mode is an adiabatic invariant. This means that the integers r and a, which characterise the waveguide mode and are equal to the number of radial and azimuthal variations, do not change. In this case, the transverse component $k_\perp$ of the wave vector changes inversely proportional to the waveguide diameter $D(x)$, i.e.,

$$k_\perp(x)D(x) = \text{const}. \quad (1)$$

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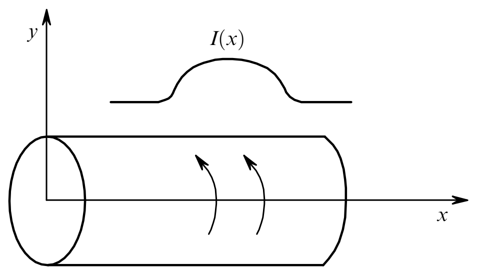

Figure 1. Direction of the coordinate axes during the propagation of a WG soliton in a glass cylinder.

Note that $k_\perp(x)$ is independent of the refractive index $n(x)$ and the light-wave frequency ω and is determined only by the mode type and diameter of the cross section at the point x. The longitudinal component $k_\parallel$ of the wave vector is usually called the propagation constant and is equal to the projection of the wave vector on the waveguide axis. The longitudinal and transverse components are related by the expression

$$k_\parallel^2 + k_\perp^2 = k^2. \tag{2}$$

As the cross-section diameter decreases, the component $k_\perp$ increases, however, the modulus of the wave vector k remains invariable if the light wave frequency does not change. In this case, as follows from (2), the component $k_\parallel$ decreases. If the component $k_\perp$ becomes equal to k for some radius $r_1(x_1)$ of the cross section, then the propagation constant vanishes. The point $x = x_1$ is called the return point. The WG wave cannot propagate in the region where $r_1(x) < r_1(x_1)$, and, hence, $k_\parallel^2 < 0$. Such regions are called beyond regions.

Unlike a sphere, the cross section in the resonators considered here decreases rather slowly. As mentioned above, such resonators were also studied experimentally [4]. Consider a real situation, when a change in the refractive index $\Delta n(x)$ is very small ($\Delta n/n$ does not exceed 10^{-4}). Taking into account that $k = \omega n/c$ and using expressions (1) and (2), we obtain the expression for the propagation constant

$$k_\parallel(x,\omega)^2 \simeq \left[\frac{\omega n(x)}{c}\right]^2 - k_\perp^2(x)$$

$$= \left[\frac{\omega n(0)}{c}\right]^2 \left\{\left[\frac{n(x)}{n(0)}\right]^2 - \left[\frac{k_\perp(0)D(0)}{D(x)}\right]^2\right\}, \tag{3}$$

where the functions $n(x)$ and $D(x)$ describe changes in the refractive index and the waveguide diameter along the x axis coinciding with the waveguide axis. For the case $\Delta D/D \ll 1$, $\Delta n/n \ll 1$, and $\Delta\omega/\omega \ll 1$, where $\Delta D(x) = D(x) - D(0)$, $\Delta n(x) = n(x) - n(0)$, and $\Delta\omega = \omega - \omega_0$, expression (3) can be rewritten as

$$k_\parallel(x,\omega) = \left\{k_\parallel^2(0,\omega_0) + 2k^2(\omega_0) \times \left[\frac{\Delta D(x)}{D(0)} + \frac{\Delta n(x)}{n(0)} + \frac{\Delta\omega}{\omega}\right]\right\}^{1/2}. \tag{4}$$

One can see from (4) that the propagation constant depends in the same way on the relative changes in the diameter $\Delta D(x)/D(0)$, the refractive index $\Delta n(x)/n(0)$, and light frequency $\Delta\omega/\omega$. By introducing the notation $\Delta k_\parallel(x,\omega) = k_\parallel(x,\omega) - k_\parallel(0,\omega_0)$, we obtain in the case

$$k_\parallel^2(0,\omega_0) \ll 2k^2(\omega_0)\left[\frac{\Delta D(x)}{D(0)} + \frac{\Delta n(x)}{n(0)} + \frac{\Delta\omega}{\omega}\right]$$

the relation

$$\frac{\Delta k_\parallel(x,\omega)}{k_\parallel(0,\omega_0)} = \frac{k^2(\omega_0)}{k_\parallel^2(\omega_0)}\left[\frac{\Delta D(x)}{D(0)} + \frac{\Delta n(x)}{n(0)} + \frac{\Delta\omega}{\omega}\right]. \tag{5}$$

Therefore, the relative change in the propagation constant proves to be larger by a factor of $k^2(\omega_0)/k_\parallel^2(0,\omega_0)$ than the relative changes in the initial factors. For a distinctly pronounced WG wave, for which $k(\omega_0) \gg k_\parallel(0,\omega_0)$, we can substantially change the propagation constant by slightly varying the waveguide diameter, its refractive index or the light frequency.

3. Self-focusing of a WG wave along a cylinder axis

Consider first the effects related to the self-action of a WG wave, which cause a change in the wave parameters along the x axis of an infinite dielectric cylinder with quadratic nonlinearity (Fig. 1). Such a cylinder can be treated as a waveguide inside which the WG wave propagates. It is known that the propagation of a wave in any waveguide is described by the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} + k_\parallel^2 u = 0, \tag{6}$$

where

$$\lambda = \frac{2\pi}{k}, \quad \lambda_w = \frac{2\pi}{k_\parallel} \tag{7}$$

are the wavelengths of light in glass and along the waveguide, respectively. In the general case, if the refractive index n depends on x, the parameters k and $k_\parallel$ also depend on x. However, $k_\perp$ is determined only by the waveguide cross section (cylinder diameter) and is independent of x. The field distribution in the waveguide cross section for any mode is independent of λ_w and coincides with the field distribution of this mode in the case of transverse resonance [9], when the field is invariable along the waveguide axis, i.e., $\lambda_w = \infty$.

Consider an infinite WG wave rotating in a cylinder, with the wave vector perpendicular to the cylinder axis, i.e., $\lambda_w = \infty$. Let us confine now this wave by the region $-\eta^{-1}/2 < x < \eta^{-1}/2$, so that the width of a rotating circular belt is η^{-1}. In this case, the refractive index in this region increases by $\Delta n = n_2 E^2$ due to the optical Kerr effect, where E is the amplitude of the light wave field and n_2 is a constant characterising the nonlinearity of the Kerr medium. It is known that a region with an increased refractive index can play the role of a waveguide. In the case under study, such a waveguide is closed, i.e., represents a circular resonator of width η^{-1}. The intensity distribution of the light wave along the resonator width differs from a rectangular one. For this reason, the distribution of the increment of the refractive index caused by the Kerr effect also differs from a rectangular distribution and $\Delta n(x) = n_2 E^2(x)$. Therefore, it is necessary to find the profile of the refractive index $\Delta n(x)$ for which the propagating light wave has the intensity profile $I(x)$ providing the formation of the profile $\Delta n(x) = n_2 E^2(x)$.

Such a problem is considered mathematically in the analysis of formation and propagation of a two-dimensional soliton along the z axis, in which the field along the coordinate y is constant [10]. Taking into account the diffraction divergence and self-focusing in the Kerr medium, the propagation of a two-dimensional wave along the z axis is described by the equation [10]

$$-2\mathrm{i}k_0 \frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial x^2} + 2k_0^2 \frac{n_2}{n_0} |u^2| u = 0, \tag{8}$$

where $k_0 = \omega n_0/c$ and n_0 is the refractive index of the medium for low-intensity waves. The solution of equation (8) is the wave [10]

$$u(x,z) = \left(\frac{n_0}{k_0^2 n_2}\right)^{1/2} \eta \frac{\exp[-\mathrm{i}\eta^2 z/(2k_0)]}{\cosh(\eta x)}. \tag{9}$$

The propagation constant of this wave along the z axis is

$$k_z = k_0\left(1 + \frac{\eta^2}{2k_0^2}\right), \tag{10}$$

from which it follows that $k_z^2 = k_0^2 + \eta^2$ for $\eta^2 \ll k_0^2$, i.e., k_z depends on the beam width η^{-1}. In addition, the perpendicular component of the wave vector appears, which can be found from equation (8). By substituting instead of $\partial u/\partial z$ the quantity $-\mathrm{i}\eta^2 u/(2k_0)$ obtained by differentiating (9), we have $\partial^2 u/\partial x^2|_{x=0} = -\eta^2 u$. On the other hand, by definition

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=0} = -k_x^2(0)u.$$

By comparing these expressions, we obtain

$$k_x(0) = \eta. \tag{11}$$

The modulus of the wave vector $k(x)$ of the wave, which generally depends on the refractive index of the medium where the wave propagates, is determined by the expression

$$k(x) = k_0\left[1 + \frac{\Delta n(x)}{n}\right]. \tag{12}$$

On the other hand, from the general relation

$$k^2(x) = k_x^2(x) + k_z^2(x), \tag{13}$$

taking into account (10) and (11), we have

$$k^2(0) = k_x^2(0) + k_z^2(0) = k_0^2\left(1 + 2\frac{\eta^2}{k_0^2}\right). \tag{14}$$

By comparing (12) and (14), we obtain

$$\frac{\Delta n(0)}{n} = \frac{\eta^2}{k_0^2}. \tag{15}$$

The component k_z remains invariable during propagation along the x axis, whereas the component k_x decreases. Indeed, expression (10), taking into account that $\Delta n(x)/n \sim u^2(x)$ and expressions (14), (15), can be written in the form

$$k^2(x) = k_0^2\left(1 + \frac{2\eta^2}{k_0^2}\cosh^{-2} x\right). \tag{16}$$

Because $k_z = k_0[1 + \eta^2/(2k_0^2)]$, we have $k_x^2(x) = \eta^2 \times (2\cosh^{-2} x - 1)$. It follows from this that $k_x^2(x_0) = 0$ for $x = x_0$ and $k_x^2(x) < 0$ for $x > x_0$. The points $x = \pm x_0$ are return points, and according to the WKB method [9], the condition

$$\int_{-x_0}^{0} k_x(x)\mathrm{d}x = \frac{\pi}{2} \tag{17}$$

should be fulfilled for the phase integral. Between points $-x_0$ and x_0, we have $k_\perp^2 > 0$, and the wave can propagate. For $|x| > x_0$, we have $k_x^2(x_0) < 0$, and the wave decreases exponentially without changing its phase. However, after reflection from the return point, the wave acquires the additional phase shift equal to $\pi/2$.

It is known that two-dimensional solitons are characterised by the so-called area integral

$$J = \left(\frac{n_0}{n_2}\right)^{1/2} \int_{-\infty}^{\infty} |u(x)|\mathrm{d}x. \tag{18}$$

It was shown [10] that $J = 2\pi$ in the soliton and the integral is independent of the initial function $u(x)$ for $z = 0$. If a beam is not a soliton for $z = 0$, but its area integral satisfies the condition $\pi < J < 3\pi$, then the beam becomes eventually a soliton with $J = 2\pi$. The nonequilibrium beam propagating in a waveguide produced by the beam itself acquires eventually the profile at which the field distribution in the waveguide proves to be exactly the one required for the formation of this waveguide. This profile is stable. All the beams that weakly differ from the stable beam, i.e., the beams satisfying the above condition, acquire this profile.

Let us now use the known results, obtained for two-dimensional spatial solitons propagating in an infinite Kerr medium, for the case under study. We will pass to this case in several steps. At the first stage, we change the conditions of propagation of a spatial soliton. Instead of an infinite wave along the y axis, we consider the propagation of a fragment of this wave in an infinite planar waveguide, which is formed by two flat perfect mirrors oriented perpendicular to the y axis, assuming that the strength of the light wave field is parallel to the y axis. In this case, the field distribution between the mirrors coincides with its distribution in an infinite medium and, hence, the profile of the wave amplitude along the x axis corresponds to the two-dimensional soliton considered. At the second stage, we roll up the planar waveguide to a cylindrical layer so that the cylinder axis would be parallel to the x axis. Then, the wave begins to rotate in the cylindrical layer along the cylinder axis. At the third stage, we fill the entire cylinder with glass and remove the internal mirror. In this case, the wave is restricted along the radius toward the centre not by the mirror but due to the waveguide curvature. At the fourth stage, we remove the external mirror. In this case, the wave is confined along the radius in the direction from the centre due to total internal reflection from the glass – air interface. At each of the stages, the conditions of the confinement of the wave in the direction perpendicular to the x axis were changed. However, the conditions of the wave self-confinement along the x axis were invariable. In all the cases, the intense wave increases the refractive index inside itself, thereby producing a waveguide that confines the diffraction divergence of the wave.

Propagating in the cylinder in the azimuthal direction, the WG wave returns to the site where it has already been. In this case, the phase shift after the wave passage around the cylinder should be multiple of 2π, i.e., the condition

$$\pi D_0 k_z = 2\pi a, \text{ или } D_0 k_z/2 = a \qquad (19)$$

should be fulfilled, where a is an integer equal to the azimuthal index and D_0 is the cylinder diameter. Therefore, solitons with the zero group velocity along the cylinder axis x (a wave that is symmetric with respect to $x = 0$ is considered) and the specified intensity (i.e., at fixed values of η and k_z) can exist only at certain frequencies of light. The inevitable decrease in k_z caused by the reduction of n during the wave decay is compensated by the increase in k_z due to the increase in the carrier frequency of the WG soliton with decreasing n. It is known that during the propagation of light in a medium with the refractive index decreasing in time, the light frequency increases [11]. One can easily see that the relative change in the carrier frequency for the WG soliton is equal to the relative change in the refractive index with decreasing light intensity.

Consider now the case when the group velocity of the WG wave is nonzero. In the two-dimensional soliton analogue, this corresponds to the propagation of the wave at some angle θ to the z axis. Then, expression (9) takes the form [10]

$$u(x,z) = \left(\frac{n_0}{k_0^2 n_2}\right)^{1/2} \times \eta \, \frac{\exp\left[-\mathrm{i}\left(\eta^2 - \xi^2\right)z/(2k_0) + \mathrm{i}\xi x\right]}{\cosh(\eta x)}, \qquad (20)$$

where $\xi = \theta k_0$. In this case, the azimuthal component k_z of the propagation constant decreases by $\xi^2/(2k_0)$ compared to that in the case $\theta = 0$, and condition (19) can be satisfied for any frequency by a proper choice of ξ^2 (or of the angle θ determining the group velocity $c\theta/n$). As a result, we obtain that, unlike free space, where the two-dimensional wave can propagate at any angle θ, the WG wave can propagate in a cylinder only at certain angles. To each of the angles, its own value of a and own waveguide mode correspond. In fact, we have a soliton, which can propagate at different velocities.

The similarity between the equations describing the propagation of solitons in a cylindrical optical fibre and self-focusing of two-dimensional beams in a nonlinear medium was pointed out by many authors. In this case, the action of dispersion in the optical fibre resulting in the broadening of the transmitted light pulse is similar to the action of diffraction during the propagation of a spatially confined beam in free space, which causes the broadening of the beam. For the case under study, these types of solitons differ from each other by angles θ.

We can analyse the properties of WG solitons in a different way by using the methods for studying the properties of usual solitons in optical fibres. It is known that a soliton is formed due to the action of two competing processes: nonlinear compression, which is proportional to the light intensity, and the dispersion spread, which is proportional to the group-velocity dispersion. On passing from a usual soliton to a WG soliton, the light intensity in the soliton increases inversely proportionally to the group velocity $v_g = (c/n)(k_\parallel/k)$. On the other hand, the group-velocity dispersion

$$\frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{c}{n}\frac{k_\parallel}{k}\right) \simeq \frac{n}{c}\frac{k}{k_\parallel}$$

is also inversely proportional to the group velocity. By comparing these expressions, we can see the balance between the competing processes is retained on passage from a usual soliton to a WG soliton.

4. Properties of WG solitons with the group velocity comparable to the sound speed in glass

Because the velocity of the WG soliton is relatively small, the electrostriction effect produced by the appearance of an additional pressure inside the light wave has time to reveal itself in full measure along with almost inertialess Kerr effect. Although the electrostriction effect itself is virtually inertialess, its action on the propagation of light occurs with a delay. The electrostriction pressure increases the glass density, resulting in the increase in the refractive index n. The time required for increasing n is determined by the expression $\tau \simeq l/v_a$, where l is the size of the region where the electrostriction pressure appears.

Consider a WG soliton propagating with the initial velocity v_g smaller than v_a in a cylinder with a gradually increasing diameter. When the diameter D increases, the longitudinal component $k_\parallel$ increases according to (5), and the relation $v_g > v_a$ becomes valid at some value of D. The acoustic pulse cannot propagate independently at the velocity v_g. On the other hand, the WG soliton also cannot propagate independently, having separated from the acoustic pulse, because the latter is involved in the formation of a region with the increased refractive index, which restricts the WG soliton. As a result, the velocity of the WG soliton does not exceed v_a. The energy of the WG soliton decreases [5, 11] and a part of its energy is transferred to the acoustic pulse. The decrease in the WG soliton energy is accompanied by a decrease in its frequency and, as follows from (5), by a decrease in the propagation constant $k_\parallel$, which in turn results in a decrease in the group velocity of the WG soliton. It is only necessary to provide not too strong change in the cylinder diameter to ensure that the change in the soliton frequency had time to follow the change in the cylinder diameter. As shown in Ref. [5], during the propagation of a soliton in a cylinder with a gradually increasing diameter, the shape of the soliton becomes asymmetric. The radiation proves to be concentrated in the part of the moving resonator where the refractive index increases in time.

If the WG soliton propagates initially along the cylinder at the velocity $\upsilon_g > \upsilon_a$, it loses a part of energy containing in the acoustic wave produced by it. The smaller υ_g, the greater part of the energy is lost because there is more time for excitation of the acoustic wave at each part of the cylinder. In this case, the WG soliton leaves after itself the acoustic wave with the energy that exceeds the energy of a usual soliton by a factor of $(c/n)^2/\upsilon_g^2$. As a result, the velocity, at which the frequency of light in the WG soliton decreases, increases by the same factor. When the frequency decreases, $k_\parallel$ and the group velocity υ_g also decrease. This facilitates a further decrease in the WG soliton energy and its group velocity. As a result, the soliton velocity becomes equal to the sound speed and the soliton transforms to a WG resonator filled with light and moving at the sound speed. The properties of such WG resonators filled with light are considered in Refs [5, 6, 11].

5. On the possible involvement of WG solitons in the supercontinuum generation

The properties of WG solitons can be use to explain processes going on in the generation of a supercontinuum in a tapered optical fibre [12–14]. High-power short light pulses were coupled into a usual optical fibre whose diameter gradually decreased from $d_0 = 125$ μm down to $d_2 = 2$ μm (d_0 and d_2 are the initial and final diameters). The length of the fibre part with a minimal diameter was about of 90 mm. The conditions of the supercontinuum generation are similar to those for the generation of WG solitons. Indeed, in both cases, intense light pulses are used, which are subjected to self-action effects.

A supercontinuum was generated using a single-mode fibre with a gradually decreasing diameter. For some core diameter, which decreases proportionally to the external diameter of the fibre, the core is no longer capable of holding radiation. The radiation leaves the core and is confined by the cladding–air interface. Therefore, the light pulse begins to propagate under the same conditions as the WG soliton considered above. Let the fibre diameter at which this occurs be d_1, where $d_0 > d_1 > d_2$. It is difficult to imagine that the waveguide symmetry with respect to its axis is completely preserved with decreasing its diameter. In this case, radiation emerging from the fibre core will inevitably acquire the azimuthal component, which is inversely proportional to the fibre diameter. Therefore, when the fibre diameter decreases significantly, for example, by a factor of 30 (from $d_1 = 60$ μm down to $d_2 = 2$ μm), an intense WG wave can appear, whose group velocity is much smaller than the speed of light. Due to effects considered above, such a wave can produce WG solitons, whose wavelength can strongly decrease during their propagation to the fibre region with a decreasing diameter. After the propagation through the region with a minimal diameter, the wavelength of such WG solitons begins to increase.

If we consider the generation of a continuum by a periodic train of light pulses, we should take into account that each WG soliton can be subjected to the action of previous acoustic pulses. For example, we can assume that there exist some reflected acoustic pulses. Then, the process occurs against the background of additional acoustic waves (resembling sea waves). In this case, an acoustic compression pulse, accompanying the WG soliton, loses its confining properties during propagation through a counterpropagating acoustic extension pulse, and radiation inside the WG soliton is no longer confined by this pulse and, hence, changes its wavelength. Such a destruction of the WG soliton can occur at any stage of the light frequency conversion. As a result, the emission spectrum at the fibre output will contain components with frequencies that are both higher and lower than the incident light frequency. The surprising thing is that the width of this spectrum exceeds two octaves. Without assuming that WG waves with a much lower group velocity appear, it is difficult to imagine that the light frequency can be changed twice for the time ~ 0.5 ns, during which a usual light wave propagates by 90 mm. For this to occur, the rate of a change in the relative refractive index should be $(\mathrm{d}n/\mathrm{d}t)/n$ s^{-1}. If we assume that the ultimate admissible change is $\Delta n/n = 10^{-4}$, then it should takes place for 50 fs. This time is substantially shorter than the time constant of Kerr nonlinearity.

The validity of this explanation of the supercontinuum generation can be tested in two additional experiments. In the first experiment, instead of a periodic train of pulses with a pulse repetition rate of 76 MHz, a pulse train with a much lower repetition rate of 1 kHz is used [12]. In this case, perturbations caused by a previous pulse have time to decay before the appearance of the next pulse. Then, the spectrum of the output signal can be substantially shifted, but its width should be much lower than two octaves because the conditions for the destruction of WG solitons disappear. In the second experiment, only the tapering part of a fibre with a minimal diameter is used. In this case, the blue shift of the output radiation frequency should be substantially greater that the red shift.

Of course, WG solitons can propagate not only in homogeneous glass cylinders but also in fibres with a core. However, in this case the core diameter should be somewhat larger than in usual single-mode fibres. In addition, the parameter showing how abruptly the refractive index changes on passing from the fibre core to its cladding is also very important. These questions require special studies.

Therefore, WG solitons, unlike usual solitons, possess a number of interesting properties related to the fact that their velocity can change from zero to the velocity of a usual soliton. This property allows one to use WG solitons for the light wavelength conversion.

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