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On the possibility of observing polarisation of vacuum in a magnetic field

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***Abstract*. The possibility of detecting birefringence induced by a magnetic field in vacuum is analysed. It is shown that the state of the art of the laser measuring technique allows one to hope for setting up successful experiments.**

Keywords: *nonlinear electrodynamics, polarisation of vacuum, precision measurements.*

The nonlinear electrodynamics of vacuum [1] has been treated for a long time as an abstract theoretical model. However the state of the art of laser technique opens up the way to set up the experiments for the discovery of effects predicted by nonlinear electrodynamics.

A number of various experiments aimed at the discovery and study of the effects of nonlinear electrodynamics have been already discussed in the literature [2–11]. The polarisation of vacuum in a strong magnetic field is one of the interesting effects predicted by nonlinear electrodynamics [12]. Although the nonlinear corrections to Maxwell's equations in electric and magnetic fields achievable in terrestrial laboratories are very small and it is very difficult to observe the effects produced by them, nevertheless the modern level of the laser measuring technique allows one to hope to discover these effects.

As shown in Ref. [1], the first two terms in the expansion of the nonlinear Lagrangian of the electromagnetic field in vacuum in small parameters $(\boldsymbol{B}^2 - \boldsymbol{E}^2)/B_0^2$ and $(\boldsymbol{BE})/B_0^2$ have in the Gaussian system of units the form

$$\mathscr{L} = -\frac{1}{8\pi}(\boldsymbol{B}^2 - \boldsymbol{E}^2) + \frac{\alpha}{360\pi^2 B_0^2}[(\boldsymbol{B}^2 - \boldsymbol{E}^2)^2 + 7(\boldsymbol{BE})^2], \quad (1)$$

where $B_0 = m^2c^3/e\hbar = 4.41 \times 10^{13}$ G; $\alpha = e^2/\hbar c \simeq 1/137$ is the fine structure constant; e is the electron charge; m is the electron mass; and c is the speed of light.

The equations of the electromagnetic field in nonlinear electrodynamics, obtained from this Lagrangian, are similar to the equations of macroscopic electrodynamics

$$\mathrm{rot}\boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}, \quad \mathrm{div}\boldsymbol{D} = 0,$$
$$\mathrm{rot}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \quad \mathrm{div}\boldsymbol{B} = 0. \quad (2)$$

However, the equations relating the vectors $\boldsymbol{D}$ and $\boldsymbol{H}$ with the vectors $\boldsymbol{B}$ and $\boldsymbol{E}$ differ from the equations of linear electrodynamics and have the form

$$\boldsymbol{D} = 4\pi\frac{\partial \mathscr{L}}{\partial \boldsymbol{E}} = \boldsymbol{E} + \frac{\alpha}{45\pi B_0^2}[2(\boldsymbol{E}^2 - \boldsymbol{B}^2)\boldsymbol{E} + 7(\boldsymbol{BE})\boldsymbol{B}],$$
$$\boldsymbol{H} = -4\pi\frac{\partial \mathscr{L}}{\partial \boldsymbol{B}} = \boldsymbol{B} + \frac{\alpha}{45\pi B_0^2}[2(\boldsymbol{E}^2 - \boldsymbol{B}^2)\boldsymbol{B} - 7(\boldsymbol{BE})\boldsymbol{E}]. \quad (3)$$

One can see that equations (3) are nonlinear, i.e., the principle of superposition for electromagnetic fields is violated. As a result, the polarisation of vacuum is possible. In the pseudo-Riemannian space with the metric tensor g_0^{ik}, a weak electromagnetic wave $\boldsymbol{E}_{\rm las}$ propagates along the geodesic lines of some efficient pseudo-Riemannian space–time [13, 14], whose metric tensor g^{ik} depends on the polarisation of this wave, the metric tensor g_0^{ik} of the initial space–time, and on the tensor F_{ik} of the external electromagnetic field [15]. The expression for g^{ik} has the form

$$g^{ik} = g_0^{ik} + \frac{7\alpha}{45\pi B_0^2}F^{in}F_{nk} \quad (4)$$

for a slow normal mode (when the polarisation plane of the wave $\boldsymbol{E}_{\rm las}||\boldsymbol{B}$) and

$$g^{ik} = g_0^{ik} + \frac{4\alpha}{45\pi B_0^2}F^{in}F_{nk} \quad (5)$$

for the normal (fast) mode orthogonal to the slow mode (if $\boldsymbol{E}_{\rm las}\perp\boldsymbol{B}$).

Let us assume that the linearly polarised light wave $\boldsymbol{E}_{\rm las}$ ($\boldsymbol{E}_{\rm las}||\boldsymbol{H}$) with the wave vector $\boldsymbol{k}$ is propagating through a region with the characteristic size L in a strong homogeneous magnetic field, and $\boldsymbol{B}||\boldsymbol{z}$ and $\boldsymbol{k}\perp\boldsymbol{B}$. In this case, the electromagnetic wave acquires an additional path difference with respect to the wave propagated along the same path in the absence of the magnetic field [16]:

$$\Delta L = \frac{7\alpha B^2 L}{90\pi B_0^2}. \quad (6)$$

It is assumed here that the amplitudes of external fields are

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much greater than the amplitude of the laser radiation field, while the frequencies of external fields are much smaller than the light-wave frequency.

Let us estimate the value of ΔL. It is rather difficult to produce superstrong stationary magnetic fields in a relatively large volume with the characteristic size of the order of a few centimetres. Nevertheless, a magnetic field of the order of 3×10^5 G can be produced in the region with a linear size of 5 cm [17]. By substituting these values into (6), we find $\Delta L \simeq 0.9 \times 10^{-19}$ cm.

The direct measurement of such low values is quite problematic. However, the situation can be drastically changed if we use the method applied for detecting gravitational waves in the LIGO project [18]. This method is based on a multiple increase in the path difference of probe waves in a two-beam Michelson interferometer included into high-Q Fabry–Perot interferometers. In this case, the sensitivity of the measuring scheme increases by a factor of $2N$, where

$$N = \frac{\pi(\tau_0 R)^{1/2}}{1 - \tau_0 R}; \tag{7}$$

$1 - \tau_0$ is the losses in the interferometer and R is the effective reflection coefficient of the interferometer mirrors.

The same method can be used for detecting the birefringence of vacuum. The principle scheme of the experiment is shown in Fig. 1. Linearly polarised radiation from a highly stable single-frequency laser is directed to the Michelson interferometer, with two high-Q Fabry–Perot interferometers of length L_0 placed in its arms. Inside one of the interferometers, a strong magnetic field is produced, whose direction coincides with the polarisation vector of the light-wave field. The high Q factor of the interferometers provides the $2N$-fold propagation of the light wave through the interferometer base. As a result, the path difference between the two light beams increases by a factor of $2N$.

Let us estimate the possibility of measuring such low values using the detection system developed for the LIGO project. It is known that a laser–interferometric detector of gravitational waves has been already built in the USA [18].

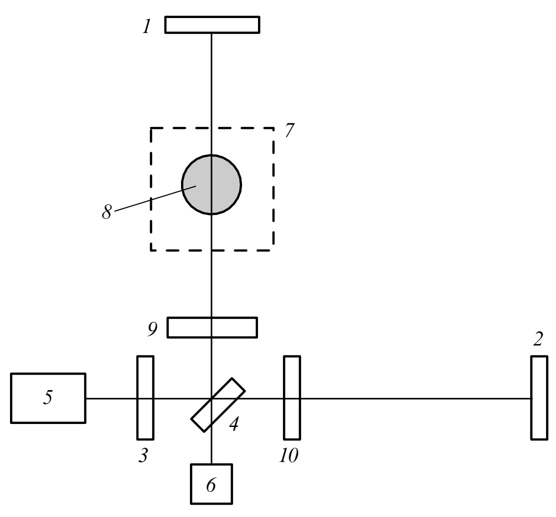

Figure 1. Principal scheme of the experiment: (*1–3*) Michelson interferometer mirrors; (*4*) beamsplitter; (*5*) laser; (*6*) photodetector; (*7*) vacuum chamber; (*8*) region where a magnetic field is produced; (*1, 9* and *2, 10*) mirrors forming Fabry–Perot interferometers.

It consists of a Michelson interferometer with high-Q Fabry–Perot interferometers in each of its arms. Laser radiation in these interferometers propagates in tubes evacuated down to 10^{-9} Torr. This device is capable already at present of detecting the optical path difference $\Delta L = 1.2 \times 10^{-14}$ cm for $N = 30$. It is anticipated that the sensitivity will be further increased by more than two orders of magnitude in the near future [18] by using high-quality dielectric mirrors. The possibility of fabricating interference coatings with the reflectivity $R \simeq 0.99999$ was demonstrated in Ref. [19], which means that an interferometer with $N \simeq 2 \times 10^5$ can be built.

In our case (detection of the birefringence of vacuum), the path difference between the two light beams produced at the output of the Michelson interferometer is

$$\Delta L = 2\frac{7\alpha B^2 LN}{90\pi B_0^2}. \tag{8}$$

By using the values of parameters in this expression presented above, we obtain $\Delta L \simeq 2 \times 10^{-14}$ cm. This means that we can hope to detect the birefringence of vacuum in a magnetic field.

The experiment on the detection of the birefringence of vacuum has an advantage over the experiment on the detection of gravitational waves because the modulation method can be used in the former case. To detect the nonlinear polarisation of vacuum reliably, the detected signal is modulated at the known frequency. This can be accomplished in several ways. First of all, a magnet can be periodically displaced at the frequency ~ 0.1 Hz with the amplitude of a few centimetres. A higher-frequency modulation can be achieved by rotating the polarisation plane of laser radiation.

Finally, it should be remembered that pulsed magnetic fields can be used, whose strength under laboratory conditions can achieve a few megagauss [17]. Although the repetition rate of these pulses is low, the quadratic dependence of the detected effect on the magnetic field strength attracts attention to this method. When pulsed magnetic fields are used, the condition $NL/c \ll \tau$ (where τ is the duration of a magnetic-field pulse) should be satisfied. Obviously, this condition can be easily achieved.

Therefore, by using the modern laser equipment and the system for measuring the optical-path difference in the interferometer arms developed for the LIGO project, we can hope to observe nonlinear effects in the interaction of laser radiation with an external magnetic field, predicted by quantum electrodynamics.

The main difficulty, which should be overcome to detect the birefringence of vacuum, is probably the production of a 'real' vacuum, i.e., the evacuation of a volume down to an extremely low pressure. Otherwise, the Cotton–Mouton effect in a residual gas will mask the effect caused by the birefringence of vacuum in a magnetic field. However, the estimates (performed assuming that the change ΔL caused by the Cotton–Mouton effect in the residual gas is smaller than 10 % of the change ΔL due to the birefringence of vacuum) show that the required pressure of residual gasses should not exceed $10^{-9} - 10^{-10}$ Torr [20], which is quite achievable.

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References

1. Heisenberg W., Euler H.Z. *Phys.*, **26**, 714 (1936).
2. Narozhnyi N.B. *Zh. Eksp. Teor. Fiz.*, **55**, 714 (1968).
3. Aleksandrov E.B., Ansel'm A.A., Moskalev A.N. *Zh. Eksp. Teor. Fiz.*, **89**, 1181 (1985).
4. Rozanov N.N. *Zh. Eksp. Teor. Fiz.*, **103**, 1996 (1993).
5. Denisov V.I. *Phys. Rev. D*, **61**, 036004 (2000).
6. Denisov V.I. *J. Opt. A*, **2**, 372 (2000).
7. Denisov V.I., Denisova I.P. *Opt. Spektrosk.*, **90**, 329 (2001).
8. Ginzburg V.L., Tsytovich V.N. *Zh. Eksp. Teor. Fiz.*, **74**, 1621 (1973).
9. Denisov V.I., Denisova I.P. *Opt. Spektrosk.*, **90**, 1023 (2001).
10. Denisov V.I., Kravtsov N.V., Lariontsev E.G., Zubrilo A.A., Pinchuk V.B. *Vestn. Mosk. Univ., Fiz. Astron.*, **5**, 51 (2000).
11. Burke D.L., Feld R.C., Horton-Smith G., Spencer J.E., Walz D., Berridge S.C., Bugg W.M., Shmakov K., Weidemann A.W., Bula C., McDonald K.T., Prebyes E.J., Bamber C., Boege S.J., Koffas T., Kotseroglou T., Melissinos A.C., Meyerhofer D.D., Reis D.A., Ragg W. *Phys. Rev. Lett.*, **79** (9), 1626 (1997).
12. Ginzburg V.L. *O fizike i astrofizike* (On Physics and Astrophysics) (Moscow: Byuro-kvantum, 1995).
13. Denisov V.I., Denisova I.P. *Dokl. Ross. Akad. Nauk*, **378**, 463 (2001).
14. Denisov V.I., Denisova I.P., Krivchenkov I.V. *Zh. Eksp. Teor. Fiz.*, **122**, 227 (2002).
15. Landau L.D., Lifshits E.M. *Teoriya polya* (Theory of Field) (Moscow: Nauka, 1988).
16. Berestetskii V.B., Lifshits E.M., Pitaevskii L.P. *Kvantovaya elektrodinamika* (Quantum Electrodynamics) (Moscow: Nauka, 1989) p. 650.
17. Herlach F. (Ed.) *Strong and Superstrong Magnetic Fields and Their Applications* (Springer, 1985; Moscow: Mir, 1988).
18. Braginskii V.B. *Usp. Fiz. Nauk*, **179**, 743 (2000).
19. Rempe G., Thopson R., Kimble H.J. *Opt. Lett.*, **17**, 363 (1992).
20. Rizzo C., Rizzo A., Bishop D.M. *Int. Rev. Phys. Chem.*, **16**, 81 (1997).