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On the spectral – spatial instability of a light wave in a medium with cubic nonlinearity

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***Abstract*. Based on the analysis of frequency-nondegenerate four-photon parametric scattering, the spectral-angular dependences of the increments of perturbing modes are obtained in the field of an intense light wave propagating in a medium with cubic nonlinearity.**

Keywords: *four-photon parametric scattering, spectral – spatial instability*.

The stability of a high-power light wave with respect to small angular perturbations in a nonlinear medium was first considered by Bespalov and Talanov [1]. By now an extensive literature has appeared devoted to the study of the spatial instability of light waves in various nonlinear media, including media with cubic nonlinearity (see Ref. [2] and references therein). The spatial instability was analysed in these papers assuming that the frequency ω of small angular perturbations coincides with the frequency ω_0 of the high-power wave. The spatial instability of a plane wave can be described in this case as frequency-degenerate four-photon parametric scattering (FPS), which enhances weak angular perturbations in some regions of angles Θ, whose values are determined by the radiation intensity [3, 4].

In real situations, small perturbations (noise) can have a rather broad spatial and frequency spectrum, and in the general case the problem of instability of a high-power light wave in a nonlinear medium should take the non-degeneracy of parametric interaction into account. Because of this, it is reasonable to analyse the spectral – spatial instability by considering frequency-nondegenerate FPS, which is one of the mechanisms of frequency-angular diffusion of radiation in a nonlinear medium. In this paper, we study, in the linear approximation in the high-power wave intensity, frequency-nondegenerate FPS in a self-focusing medium with cubic nonlinearity. We calculated the intensities of weak perturbing modes and used them to describe the development of the spectral-angular instability of a high-power wave.

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In the case of frequency-nondegenerate FPS, the nonlinear polarisation $\mathcal{P}_{\rm n}$ of a medium in the field of parametrically interacting waves

$$E = \mathcal{E}_0 e^{-i(\omega_0 t - \boldsymbol{k}_0 \boldsymbol{r})} + \sum_{\pm} \mathcal{E}_{\pm} e^{-i(\omega_{\pm} t - \boldsymbol{k}_{\pm} \boldsymbol{r})} \quad (1)$$

can be conveniently written in the form

$$\mathcal{P}_{\rm n} = \chi^{(3)} |E|^2 E \equiv \chi_{\rm n} E, \quad (2)$$

where $\mathcal{E}_0$ is the amplitude of a high-power wave subjected to instability; $\mathcal{E}_{\pm}$ are the amplitudes of parametrically coupled perturbing waves ($|\mathcal{E}_{\pm}| \ll |\mathcal{E}_0|$); $\boldsymbol{k}_0$ and $\boldsymbol{k}_{\pm}$ are the linear wave vectors; ω_0 and $\omega_{\pm}$ are the frequencies of the interacting waves satisfying the relation $2\omega_0 = \omega_+ + \omega$; and $\chi^{(3)}$ is the cubic susceptibility of the medium. We will find nonlinear polarisation using the known dynamic equation [2]

$$t_0 \frac{\partial \chi_{\rm n}}{\partial t} + \chi_{\rm n} = \chi^{(3)} \{ |\mathcal{E}_0|^2 + [\mathcal{E}_0 \mathcal{E}_+^* e^{-i(\Omega t - \boldsymbol{q}_+ \boldsymbol{r})}$$

$$+ \mathcal{E}_0 \mathcal{E}_-^* e^{i(\Omega t + \boldsymbol{q}_- \boldsymbol{r})} + \text{c. c.}] \}, \quad (3)$$

where t_0 is the relaxation time of nonlinearity; $\boldsymbol{q}_{\pm} = \boldsymbol{k}_0 - \boldsymbol{k}_{\pm}$; $\Omega = \omega_0 - \omega_+$; and $|\Omega| \ll \omega_0$. In the steady-state regime (for $t \gg t_0$), by substituting the solution of equation (3) into (2), we obtain

$$\mathcal{P}_{\rm n} = \chi^{(3)} \Bigg\{ |\mathcal{E}_0|^2 \mathcal{E}_0 e^{-i(\omega_0 t - \boldsymbol{k}_0 \boldsymbol{r})}$$

$$+ \sum_{\pm} \left[\left(1 + \frac{1}{1 \pm i\delta} \right) |\mathcal{E}_0|^2 \mathcal{E}_{\pm} e^{i\boldsymbol{k}_{\pm} \boldsymbol{r}} + \frac{\mathcal{E}_0^2 \mathcal{E}_{\mp}^*}{1 \pm i\delta} e^{i\boldsymbol{\varDelta}_{\pm} \boldsymbol{r}} \right] e^{-i\omega_{\pm} t} \Bigg\}, \quad (4)$$

where $\delta = \Omega t_0$ and $\boldsymbol{\varDelta}_{\pm} = 2\boldsymbol{k}_0 - \boldsymbol{k}_{\pm}$.

We assume that the unstable high-power wave with the field amplitude $\mathcal{E}_0$ propagates along the z axis. Then, by using expression (4), the system of truncated wave equations for the amplitudes of the interacting modes can be written in the form

$$\frac{d\mathcal{E}_+}{dz} = i\gamma_0 \left(\frac{2 + i\delta}{1 + i\delta} |\hat{\mathcal{E}}_0|^2 \mathcal{E}_+ + \frac{\hat{\mathcal{E}}_0^2 \mathcal{E}_-^*}{1 + i\delta} e^{i\varDelta z} \right), \quad (5)$$

$$\frac{d\mathcal{E}_-^*}{dz} = -i\gamma_0 \left(\frac{2 + i\delta}{1 + i\delta} |\hat{\mathcal{E}}_0|^2 \mathcal{E}_-^* + \frac{\hat{\mathcal{E}}_0^{*2} \mathcal{E}_+}{1 + i\delta} e^{-i\varDelta z} \right), \quad (6)$$

where $\hat{\mathcal{E}}_0 = \mathcal{E}_0 \exp\{i\gamma_0 |\mathcal{E}_0|^2 z\}$ is the amplitude of the high-power wave in the medium; $\gamma_0 = 2\pi k_0 \chi^{(3)}/n_0^2$; n_0 is the refractive index of the medium; $\Delta = k_0 \Theta^2$ is the linear phase mismatch; and $\Theta \ll \pi/2$ is the angle between the wave vectors $\boldsymbol{k}_0$ and $\boldsymbol{k}_\pm$.

From the solutions of the system of equations (5) and (6) with symmetric boundary conditions $\mathcal{E}_\pm(z=0) = \mathcal{E}_s$

$$\mathcal{E}_+(z) = \frac{\mathcal{E}_s}{2\Gamma} e^{i(\Delta/2 + I_0)z} \left[\left(\lambda_+ - i\frac{I_0}{1+i\delta} \right) e^{-\Gamma z} - \left(\lambda_- - i\frac{I_0}{1+i\delta} \right) e^{\Gamma z} \right], \quad (7)$$

$$\mathcal{E}_-^*(z) = \frac{\mathcal{E}_s}{2\Gamma} e^{-i(\Delta/2 + I_0)z} \left[\left(\lambda_+ - i\frac{I_0}{1+i\delta} \right) e^{\Gamma z} - \left(\lambda_- - i\frac{I_0}{1+i\delta} \right) e^{-\Gamma z} \right] \quad (8)$$

we find the intensities $I_\pm = |\mathcal{E}_\pm|^2$ of the perturbing modes

$$I_\pm(z) = \frac{I_s}{4|\Gamma|^2} \times \left| \left(\lambda_+ - i\frac{I_0}{1+i\delta} \right) e^{\mp\Gamma z} - \left(\lambda_- - i\frac{I_0}{1+i\delta} \right) e^{\pm\Gamma z} \right|^2, \quad (9)$$

where

$$\lambda_\pm = i\left(\frac{\Delta}{2} - \frac{I_0}{1+i\delta} \right) \pm \Gamma; \quad (10)$$

$$\Gamma = \left[\Delta\left(\frac{I_0}{1+\delta^2} - \frac{\Delta}{4} \right) - i\frac{2\delta I_0}{1+\delta^2}\left(\Delta - \frac{I_0}{1+\delta^2} \right) \right]^{1/2}; \quad (11)$$

$I_s = |\mathcal{E}_s|^2$; $I_0 = \gamma_0 |\hat{\mathcal{E}}_0|^2$. Note that the parameter $I_0 < 0$ in defocusing media ($\chi^{(3)} < 0$). It can be easily shown that the relation $I_+(\delta) = I_-(-\delta)$ is satisfied in the case of symmetric boundary conditions under study.

It follows from equations (9)–(11) that upon frequency-degenerate FPS ($\delta = 0$) in self-focusing media($\chi^{(3)} > 0$), the angle Θ corresponding to the maximum increment of parametric amplification of perturbing components is determined by the expression [3]

$$\Theta_0 = (2I_0/k_0)^{1/2}. \quad (12)$$

In defocusing media ($\chi^{(3)} < 0$), additions to the wave vectors of the interacting waves are negative, and spatial phase matching cannot be achieved at any values of Θ, so that no efficient amplification of spatial perturbations occurs. It follows also from (9) that the parametric coupling between interacting waves becomes insignificant in the region of large angles $\Theta > \Theta_0$, and the expression for $I_\pm(z)$ takes the form

$$I_\pm(z) = I_s \exp\left(\pm 2I_0 \frac{\delta}{1+\delta^2} z \right). \quad (13)$$

Figure 1 shows the dependences $I_\pm$ of the output intensities of weak waves on the frequency detuning δ for a given length $z = 2$ cm of a nonlinear medium and $I_0 = 2\ \text{cm}^{-1}$, calculated from (9) for $I_s = 1$ and different values of $\Delta = k_0\Theta^2$. These dependences allow us to estimate the spectral-angular characteristics of the gain of perturbing components. One can see from Fig. 1 that, for $\Theta = \Theta_0$ ($\Delta = 4$), the maximum gain is achieved for the frequency-degenerate FSP components ($\delta = 0$). In this case, the profile of the gain line is not symmetric with respect to $\delta = 0$ and has a broader wing in the region of positive detunings ($\delta > 0$). As the angle Θ increases, the maximum of the spectral density of the perturbing modes at the output from the medium shifts to the point $\delta \simeq 1$ and their gain increases considerably. Such a transformation of the gain profile is caused by the weakening of parametric coupling between the waves and the enhancement of cross interaction processes. In particular, in the region of parameters under study and for $\Delta = 12$, the maximum gain corresponds to $\delta = 1$ and is caused by two-wave mixing – the induced scattering of a high-power wave by the travelling wave of the refractive index of the medium [5]. In this case, the gain upon two-wave mixing exceeds the increment of the parametric gain of weak perturbing modes upon quasi-degenerate FPS.

Therefore, the spectral–spatial distributions of the intensity of perturbing modes in the field of an intense light wave propagated through a nonlinear medium represent a 'cone' structure, with the spectrum of frequencies close to ω_0, i.e., $|\omega_\pm - \omega_0| t_0 < 1$, concentrated in its core. As the cone angle Θ increases, the maximum of the spectral density of the perturbing components shifts to the point $\delta = 1$ and the spatial distribution of the spectrum becomes diffuse. The results obtained in the paper clearly demonstrate the development of the spectral–spatial instability of intense light waves in cubic focusing media.

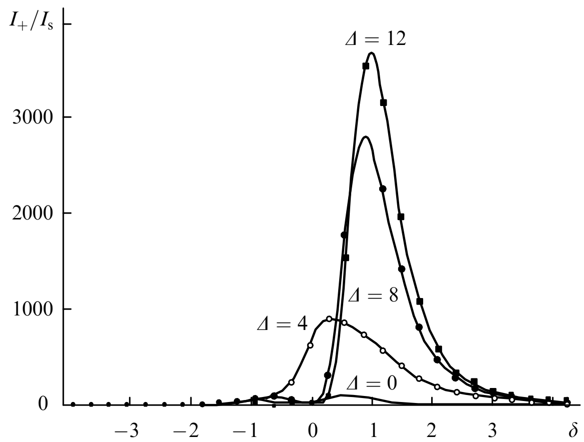

Figure 1. Frequency dependences of the output intensity of weak perturbations for different angular perturbing modes.

References

1. Bespalov V.I., Talanov V.I. *Pis'ma Zh. Eksp. Teor. Fiz.*, **3**, 471 (1966).
2. Akhmanov S.A., Vysloukh V.A., Chirkin A.S. *Optika femtosekundnykh lazernykh impul'sov* (Optics of Femtosecond Laser Pulses) (Moscow: Nauka, 1988).
3. Starunov V.S., Fabelinskii I.L. *Usp. Fiz. Nauk*, **98**, 441 (1969).
4. Zel'dovich B.Ya. *Kratk. Soobshch. Fiz. FIAN*, (5), 20 (1970).
5. Afanasev A.A., Tolkacheva E.G., Tredicce J., Volkov V.V. *Phys. Rev. A*, **60**, 2375 (1999).