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Simple scheme for the astigmatic transformation of laser modes

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***Abstract*. A simple astigmatic scheme for obtaining focused Laguerre – Gaussian beams upon the π/2 conversion of Hermite – Gaussian radiation modes is described. A zone in the vicinity of the focal region of a lens is estimated where the beam satisfies the conditions for the capture and confinement of microparticles. It is shown that this optical scheme uses the fractional Fourier transform, whose application in the π/2 converter is demonstrated for the first time.**

Keywords: *Laguerre – Gaussian modes, astigmatic optics, focusing, fractional Fourier transform.*

1. The unique properties of Laguerre – Gaussian (LG) beams make them attractive for a variety of applications such as manipulations with microscopic biological objects and material particles [1, 2], capturing of atoms [3, 4], and control of their motion [5]. In most cases, LG beams should be focused into a spot with diameter of a few or tens of micrometers, depending on a specific aim. Such beams are usually obtained in two stages, by generating first the LG mode and then by focusing it into a proper spot.

At the first stage, the LG mode can be obtained in two ways. One of the methods is the use of computer holograms [6, 7], which can be in principle produced for almost any distribution of the initial beam. Another method is the transformation of the initial Hermite – Gaussian (HG) mode by means of a π/2 converter [8]. The second method is based on the similarity between the expansion of any LG mode in a series in the basis of HG modes

$$u_{nm}^{\mathrm{LG}}(x,y,z)=\sum_{k=0}^{N}\mathrm{i}^{k}b(n,m,k)u_{N-k,k}^{\mathrm{HG}}(x,y,z) \quad (1)$$

and the expansion of the HG mode rotated with respect to the Cartesian coordinate system through 45° ('diagonal' mode)

$$u_{nm}^{\mathrm{HG}}\left(\frac{x+y}{\sqrt{2}},\frac{x-y}{\sqrt{2}},z\right)=\sum_{k=0}^{N}b(n,m,k)u_{N-k,k}^{\mathrm{HG}}(x,y,z). \quad (2)$$

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The only difference between expressions (1) and (2) is that (1) contains the coefficient i^k, which is absent in (2) [8]. Therefore, to transform any HG mode of the $n+m$ order to the corresponding LG mode, it is necessary to introduce the phase delay π/2 between the terms in expansion (2). This can be performed by using an astigmatic π/2 converter, whose scheme is proposed in Ref. [8], or any of the converters described in Ref. [9]. The LG beam can be focused in a usual way. The only precaution that should be fulfilled is the accurate alignment of the beam because the LG beam can be substantially distorted by aberrations such as astigmatism and coma.

In this paper, we consider the method for obtaining focused LG beams, which combines the beam conversion from the HG mode and its focusing in a simple optical scheme. To explain the principle of the scheme proposed, we recall some fundamentals of the optics of Gaussian beams.

2. It is well known that the propagation of a Gaussian beam in space can be described by two simple relations, which give the dependences of the beam radius w and the curvature of its wave front ρ on the distance z [10]

$$w(z)=w_0\left(1+\tan^2\theta\right)^{1/2}, \quad (3)$$

$$\rho(z)=\frac{\tan\theta}{z_{\mathrm{R}}\left(1+\tan^2\theta\right)}, \quad (4)$$

where $\theta=\arctan(z/z_{\mathrm{R}})$ is the Gouy phase [10] of the beam for the coordinate z measured from the waist position [$w_0=w(z=0)$]; $z_{\mathrm{R}}=\pi w_0^2/\lambda$ is the Rayleigh length (or the waist length) of the beam (Fig. 1). At points corresponding to $\theta=\pm\pi/4$, the beam has identical sizes and the radii $\pm z_{\mathrm{R}}/2$ of the wave-front curvature of the opposite signs (maximal and minimal). After the propagation of the beam from these points to the point $\theta=\pi/2$, the difference of the accumulated Gouy phases (AGPs) for these points is

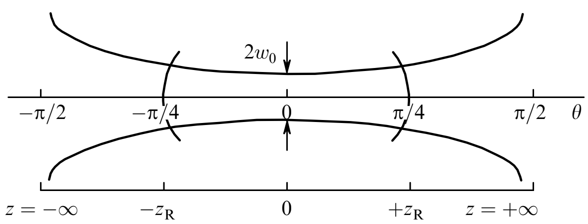

Figure 1. Dependence of the Gaussian beam profile on the Gouy phase $\theta=\arctan(z/z_{\mathrm{R}})$.

$\Delta\theta = \pi/2$, which is determined by the difference in the initial coordinates on the axis θ. Note that, starting from different spatial points, at the point $\theta = \pi/2$ (or $z = \infty$) the beam becomes infinite in both cases. The beam propagation to infinity or the far zone can be replaced by its propagation to the focal region of the lens, which leads to the same result.

To use this property of the Gaussian beam for the transformation of the HG mode to the LG mode, it is necessary to have at some fixed spatial point the diagonal mode (2) with a saddle-shaped wave front, i.e., the wave front whose curvatures over axes x and y are equal in magnitude and have opposite signs. According to the above reasoning, the HG mode, modified in this way and then focused, should appear in the focal plane of the lens with the distribution corresponding to the LG radiation mode. To verify the validity of our reasoning, we consider the operation of the optical scheme shown in Fig. 2.

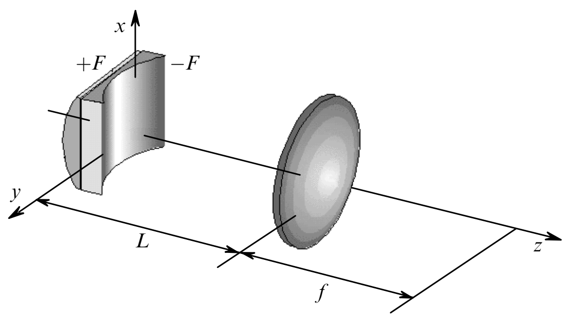

Figure 2. Simple optical scheme for the transformation of the HG mode to the focused LG mode.

The scheme in Fig. 2 consists of two combined orthogonal cylindrical lenses with the power $\pm 1/F$ (optical quadrupole [11]) and a spherical lens with the focal distance f separated from cylindrical lenses by the distance L. It can be shown that the value of L does not affect in our case the AGP difference and can be excluded from the consideration for simplicity. One can easily understand this, taking into account that, according to the above reasoning, the initial beam should propagate to 'infinity'. The *ABCD* matrices describing the propagation of the beam from the entrance to the optical quadrupole to the focal plane of the spherical lens in planes xz and yz have the form

$$T_{xz} = \begin{pmatrix} -f/F & f \\ -1/f - 1/F & 1 \end{pmatrix}, \quad T_{yz} = \begin{pmatrix} f/F & f \\ -1/f + 1/F & 1 \end{pmatrix}. \tag{5}$$

The calculation of AGPs in these planes, using the relation [12]

$$\tan\theta = \frac{\lambda B}{(A + B\rho)\pi w^2}, \tag{6}$$

where A and B are the elements of the ABCD matrix, and assuming that the wave front of the initial HG beam with $w = w_0$ is plane ($\rho = 0$), gives

$$\tan\theta_{xz} = \frac{-F\lambda}{\pi w_0^2}, \quad \tan\theta_{yz} = \frac{F\lambda}{\pi w_0^2}. \tag{7}$$

According to (7), the difference $\Delta\theta = \theta_{yz} - \theta_{xz} = \pi/2$ can be achieved only when $w_0 = (F\lambda/\pi)^{1/2}$, and in this case it is independent of the focal distance f.

By introducing the additional distance L or a beam expander, for example, a Kepler telescope between the elements of the scheme in Fig. 2, one can vary the size of the spot produced by the focused LG mode. A system with a beam expander is described by matrices

$$T'_{xz} = \frac{1}{M}\begin{pmatrix} -f/F & f \\ -M^2/f - 1/F & 1 \end{pmatrix}, \qquad T'_{yz} = \frac{1}{M}\begin{pmatrix} f/F & f \\ -M^2/f + 1/F & 1 \end{pmatrix}, \tag{8}$$

where M is the expansion coefficient, and we assume that the optical quadrupole and spherical lens are located in conjugated planes in front and behind the beam expander. According to (8), the beam radius $w_f = f(\lambda/\pi F)^{1/2} M^{-1}$ in the focal plane of the spherical lens is the same in both planes xz and yz. However, the curvatures of the wave front are different:

$$\rho_{xz} = \frac{1}{f}\left(1 + \frac{M^2 F}{2f}\right), \quad \rho_{yz} = \frac{1}{f}\left(1 - \frac{M^2 F}{2f}\right). \tag{9}$$

Moreover, for $M^2 F \geqslant 2f$, they have the same magnitudes but the opposite signs. This results in the variation in the beam size, the curvature of the wave front, and the AGP near the focal plane of the lens, which in turn restricts the region where the beam can be treated as a pure LG mode. The values of three parameters mentioned above calculated in the planes xz and yz, as well as the AGP difference are shown in Fig. 3 (the calculations were performed for $F = 85$ cm, $f = 2$ cm, $M = 5$, and $\lambda = 0.63$ μm). The results of numerical calculation of the propagation of the diagonal HG mode u_{01}^{HG} in the vicinity of the focal plane of the spherical lens (see Fig. 2) performed using the Fresnel program [13] are presented in Fig. 4. It follows from Figs 3a and 4 that the size of a zone, in which the beam ellipticity $d_{\max}/d_{\min}$ does not exceed 1.3 (if the beam diameter is measured by the method of second moments [14]) and the intensity 'barrier' around the axial point decreases less than by 6.9 %*, is approximately $\pm 0.14 z_{\mathrm{r}}$ ($z_{\mathrm{r}} = \pi w_f^2/\lambda$) near the focal plane of the lens.

One can see from Fig 3c that, for $z/z_{\mathrm{r}} = 0$, the AGP in planes xz and yz is equal to $3\pi/4$ and $\pi/4$, respectively. In terms of fractional Fourier transforms (FTs) [15], this is similar to one and a half FT of this beam in the xz plane and half FT in the orthogonal yz plane, i.e., we have FTs of the orders $a = 3/2$ ($\mathscr{F}^a[u_{nm}^{\mathrm{GH}}(x,y)]_x$) and $b = 1/2$ ($\mathscr{F}^b[u_{nm}^{\mathrm{GH}}(x,y)]_y$). Therefore, the scheme in Fig. 2 is nothing but a $\pi/2$ converter based on fractional FTs. An additional Kepler beam expander placed in front of the spherical lens supplements two other FTs to each of the planes xz and yz. Because of this, in principle, the AGP curves in Fig. 3c should be displaced upward by unity along the vertical axis. This does not change, however, the principle of the optical scheme, although neither expression (5) nor (8) coincide

*The variation in the maximal intensity in a circular distribution is characterised by the difference $1 - I_{\max}(\varphi)/I_{\max}^{\mathrm{abs}}$, where $I_{\max}^{\mathrm{abs}}$ is the absolute maximum in the beam distribution and φ is the polar angle.

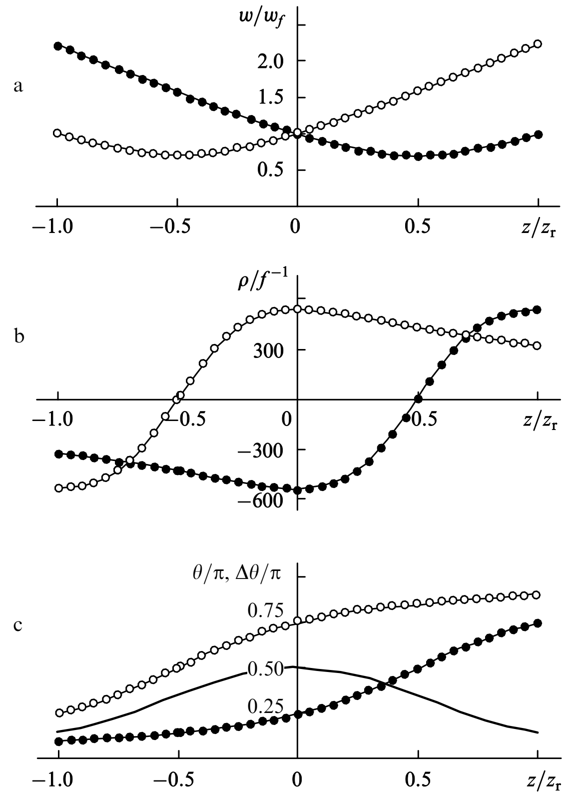

Figure 3. Variations of the normalised beam diameter w/w_f (a), the wave-front curvature ρ/f^{-1} (b), and AGP θ/π (c) near the focal plane of the focusing lens in planes xz (○) and yz (●). The solid curve is the AGP difference $\Delta\theta/\pi$. The normalised coordinate $z/z_r = 0$ corresponds to the focal plane of a lens with the focal distance f.

and an optical quadrupole*, which is similar to that placed at the input. The distance between all the three elements should be equal to $F/\sqrt{2}$, as shown in Fig. 5. The spherical lens and the second optical quadrupole return the beam diameter and its wave-front curvature to their initial values. In this case, the required phase delay $\pi/2$ is accumulated between the normal components of the diagonal HG mode. As a result, the undistorted LG mode appears at the output of the optical system, which propagates in a free space without any distortions and can be focused with an additional lens to a spot of the required size. The ABCD matrices in the corresponding planes of the scheme in Fig. 5 have the form

$$T''_{xz} = \begin{pmatrix} \cos\dfrac{3\pi}{4} & F\sin\dfrac{3\pi}{4} \\ -\dfrac{1}{F}\sin\dfrac{3\pi}{4} & \cos\dfrac{3\pi}{4} \end{pmatrix},$$
$$T''_{yz} = \begin{pmatrix} \cos\dfrac{\pi}{4} & F\sin\dfrac{\pi}{4} \\ -\dfrac{1}{F}\sin\dfrac{\pi}{4} & \cos\dfrac{\pi}{4} \end{pmatrix}. \quad (11)$$

4. Therefore, the transformation of the HG mode to the LG mode simultaneously focused to a fine spot has been demonstrated. According to numerical calculations, the interval on the optical axis of the focusing lens, in which microparticles can be captured and confined, is approximately $\pm 0.14 z_r$ in the scheme considered, provided that some ellipticity of the beam ($d_{max}/d_{min} = 1.3$) and the decrease in the intensity 'barrier' around the axial point by 6.9 % are admissible. This scheme is similar to a $\pi/2$

$-0.135z_r$	$-0.07z_r$	0	$+0.07z_r$	$+0.14z_r$
93.13 %	98.23 %	99.95 %	98.2 %	93.1 %

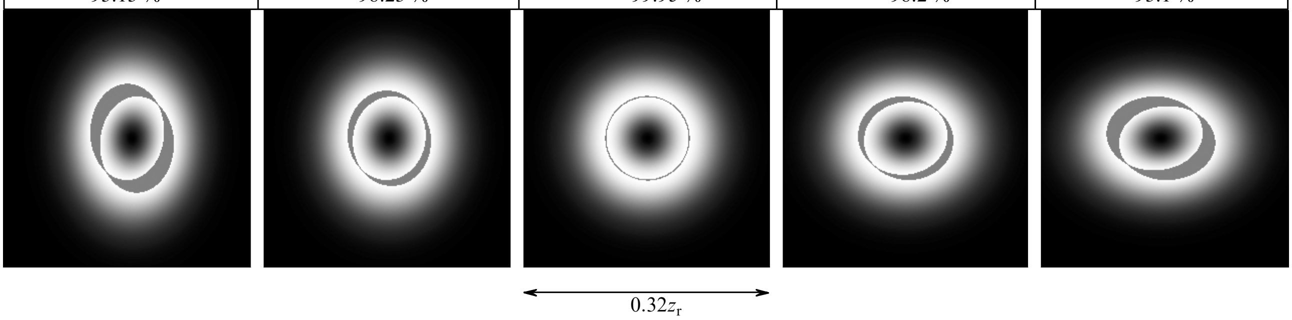

Figure 4. Changes in the LG mode intensity near the focal plane of a lens with the focal distance f. The numbers in the upper line over the figures are distances from the focal plane; the numbers in the lower line are equidensity levels (in percent of the maximum intensity) representing the boundaries of the marked zones.

with the canonical form of the *ABCD* matrix of the fractional FT ($a = 2\theta/\pi$), which should have the form [16]

$$\begin{pmatrix} \cos\theta & f'\sin\theta \\ -\sin\theta/f' & \cos\theta \end{pmatrix}, \quad (10)$$

where f' is the scale of the fractional FT.

3. The scheme in Fig. 2 can be transformed to a 'perfect' $\pi/2$ converter based on the fractional FT by replacing the spherical lens with the focal distance f by two other elements: a spherical lens with the focal distance $F/\sqrt{2}$

converter based on the fractional FTs of orders 3/2 and 1/2. The scheme can be transformed to obtain a pure LG radiation mode by retaining the FT order.

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* An optical quadrupole can be replaced by a lens with the flat and partially toroidal surfaces.

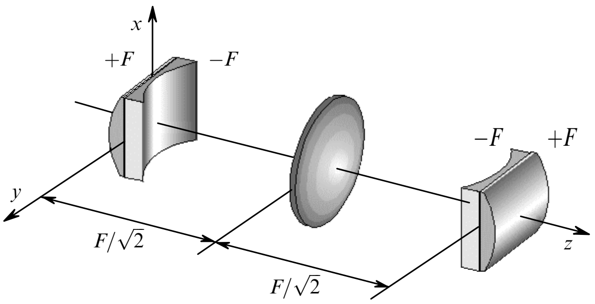

Figure 5. Optical scheme of the $\pi/2$ mode converter based on the fractional FTs of orders 3/2 (the xz plane) and 1/2 (the yz plane).

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