PACS numbers: 42.65.Re; 42.25.Hz; 42.70.Nq
DOI: 10.1070/QE2003v033n11ABEH002543

Transformation of pulses with the help of thin-layer interference structures

Yu.A. Bobrovnikov, P.N. Gorokhov, A.V. Kozar'

Abstract. **The propagation of phase-modulated optical pulses through thin-layer interference antireflection structures is studied. An analytic expression relating the parameters of the incident and reflected pulses is obtained. The time dependence of the phase modulation of the incident pulse was obtained using this expression together with experimental data. The splitting of the pulse after its reflection from the interference structure into two pulses with different spectra allows the use of these pulses in compressors to obtain ultrashort pulses with different carrier frequencies.**

Keywords: *thin-layer structures, ultrashort pulses.*

Thin-layer interference structures attract the attention of researchers first of all because they allow one to control directly the amplitude – phase characteristics of incident ultrashort pulses. It is sufficient to mention chirping mirrors used to compress phase-modulated (PM) pulses [1, 2]. One-dimensional photonic crystals, which can be also classified with thin-layer periodic structures, can be used for this purpose as well [3, 4]. The phase of optical pulses in these papers was controlled with the help of multilayer structures, which have unique dispersion properties.

The authors of Ref. [5] studied the interaction of varying-amplitude waves with antireflection thin-layer structures, which allow one to control the incident-wave amplitude. The analytic expression obtained in this paper for antireflection periodic structures of some types can be used to analyse the time dependence of the amplitude of transform-limited pulses. The solution of a similar problem for such structures in the case of the waves with time-dependent amplitude and phase is undoubtedly of interest. Such a problem appears, for example, upon the interaction of ultrashort PM pulses with periodic structures.

In this paper, we analyse the interaction of such pulses with a broad class of antireflection structures, which are known in the literature as thin-layer interference matching devices (TIMDs). The theory of TIMDs (their amplitude – spectral, structural, and invariant properties) was developed in detail by one of the authors in Refs [6 – 8]. A TIMD is a multilayer periodic dielectric structure consisting of alternating layers with low and high refractive indices and having a total optical thickness $D < \lambda/4$ (where λ is the wavelength of monochromatic radiation for which the TIMD serves as an antireflection element). Irrespective of the composition and optical properties of its layers, a TIMD is equivalent in the vicinity of the long-wavelength transmission peak to an antireflection quarter-wave film with the refractive index $n = (n_0 n_s)^{1/2}$, where n_0 and n_s are the refractive indices of the external medium and substrate, respectively. By changing the relation between the thickness of the layers, the value of n_s can be varied in the range $n_1^2 < n_s < n_2^2$, where n_1 and n_2 ($n_1 < n_2$) are the refractive indices of the TIMD layers [6]. The theoretical analysis of the interaction of a light pulse with the TIMD in this case can be reduced to the analysis of its interaction with an equivalent quarter-wave film. The pulse reflected from the TIMD is subjected to the most strong and interesting transformation, which we will study below.

By neglecting losses and dispersion, we consider the normal incidence of a plane wave packet on a single-layer structure. Our analysis is based on the known expression for the reflection coefficient for a single-layer film

$$r = \frac{r_0 + r_1 \exp(-i2\varphi_0)}{1 + r_0 r_1 \exp(-i2\varphi_0)}, \quad (1)$$

where $r_0 = (n_0 - n)/(n_0 + n)$ and $r_1 = (n - n_s)/(n + n_s)$ are the reflection coefficients for the external medium – film and film – substrate interfaces, respectively; $\varphi_0 = 2\pi n d/\lambda$ is the phase thickness of the film; and d is its geometrical thickness. We assume that the incident-wave amplitude is normalised to its maximum. Expression (1), which is valid for monochromatic waves, can be generalised, under certain conditions considered below, for the waves with time-dependent amplitudes and phases. Such a generalisation was performed in paper [5] by representing expression (1) as a sum of two waves reflected from a film and a substrate with the phase difference equal to $2\varphi_0 = \omega_0 \Delta t$, where ω_0 is the average frequency of the wave (carrier frequency); $\Delta t = 2nd/c = T/2$ is the time delay of the second wave with respect to the first one; and T is the period of oscillations at the carrier frequency.

By representing the complex amplitude of the incident wave in the form $A(t)\exp[-i\varphi(t)]$, where $A(t)$ is the real amplitude and $\varphi(t)$ is the phase, we obtain, for $r_0 = r_1$ and $\omega_0 \Delta t = \pi$ (the matching condition at the frequency ω_0) [5], the expression for the complex amplitude of the reflected wave

Yu.A. Bobrovnikov, P.N. Gorokhov, A.V. Kozar' Department of Physics, M.V. Lomonosov Moscow State University, Vorob'evy gory, 119992 Moscow, Russia; e-mail: avk@phys.msu.su; gorokhov@phys.msu.su; web-site: http:\\www.phys.msu.su

Received 6 March 2003
Kvantovaya Elektronika **33** (11) 1019 – 1021 (2003)
Translated by M.N. Sapozhnikov

$$E_0(t) = r_0 \frac{A(t)\exp[-i\varphi(t)] - A(t-\Delta t)\exp[-i\varphi(t-\Delta t)]}{1 - r_0^2 \exp[i\phi(t)]}, \quad (2)$$

where $\phi(t) = \varphi(t) - \varphi(t - \Delta t)$ is the additional phase shift caused by phase modulation. The real part (envelope) of the reflected wave or PM pulse $[E_0(t)]$ has the form

$$E_{0r}(t) = |r_0| \left[\frac{A^2(t) - 2A(t)A(t-\Delta t)\cos\phi(t) + A^2(t-\Delta t)}{1 - 2r_0^2\cos\phi(t) + r_0^4} \right]^{1/2}. \quad (3)$$

In the absence of phase modulation $[\varphi(t) = \mathrm{const}]$, by expanding $A(t - \Delta t)$ in a series in powers of Δt (assuming that such an expansion is possible), we obtain the expression for the envelope

$$E_{0r}(t) = \left| \frac{r}{1 - r_0^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \frac{d^n A}{dt^n} (\Delta t)^n \right|, \quad (4)$$

which was analysed in detail in Ref. [5]. Recall that r_0 in (2)–(4) is the coefficient of reflection not from a film but from the external medium–equivalent layer interface (with the refractive index defined above). For a TIMD, it can be conveniently written in the form $r_0 = (1 - \sqrt{\chi})/(1 + \sqrt{\chi})$, where $\chi = n_s/n_0$ [5]. We will assume below that $n_0 = 1$. Note also that expressions (2)–(4) are also valid for a half-wave filter, for which $n_0 = n_s$ and $\Delta t = T$.

Consider now the applicability of the expressions obtained above. As shown in Ref. [5], expression (4) for the waves without phase modulation is valid for $t \gg \Delta t$ (the time t is measured from the instant of the pulse incidence on the structure), i.e., for the established interference regime. It is obvious that the appearance of the discontinuity of the amplitude $A(t)$ or its derivative at any instant t will result in the destruction of this regime and its new establishment during the same time interval. It follows from the calculation that for really used dielectrics with $n_s < 3.5$, the inequality $t \gg \Delta t$ can be replaced with high accuracy by a less strict condition $t > 3\Delta t$, which is equivalent to the condition of a slowly varying amplitude. Similar arguments are also valid for the time-dependent phase $\varphi(t)$. Therefore, expressions (2)–(4) are valid for comparatively smooth pulses without sharp variations in phases and amplitudes.

Consider the interaction of PM pulses having Gaussian $\{A(t) = \exp[-(t/\tau)^2/2]\}$ and super-Gaussian $\{A(t) = \exp[-(t/\tau)^{2p}]\}$ shapes of envelopes with a TIMD, assuming that the phase modulation is quadratic $\varphi(t) = \alpha t^2/2$, which is most interesting for applications [9, 10]. Figure 1 shows the shapes of PM pulses reflected from a film deposited on a substrate with $n_s = 3.42$, which were calculated by the spectral method. In both cases, the pulse duration τ was $7T$, the carrier wavelength was $\lambda = 1.5$ μm; $\alpha\tau^2 = 1$ for the Gaussian pulse, and $\alpha\tau^4 = 4$ for the super-Gaussian pulse, and the parameter $p = 3$. The envelope E_{0r} of the reflected pulses is plotted using expression (3). The results presented in Fig. 1 are obtained with the help of the Fourier transform of the incident pulse written in the form $E(t) = A(t)\exp[-i(\omega_0 t + \varphi(t))]$. The temporal profile of the reflected pulse was written in the form

$$E_r(t) = \int_{-\infty}^{+\infty} f(\omega) r(\omega) \exp(-i\omega t) d\omega, \quad (5)$$

where $f(\omega)$ is the Fourier spectrum of the incident pulse and $r(\omega)$ is the coefficient of reflection from the TIMD defined

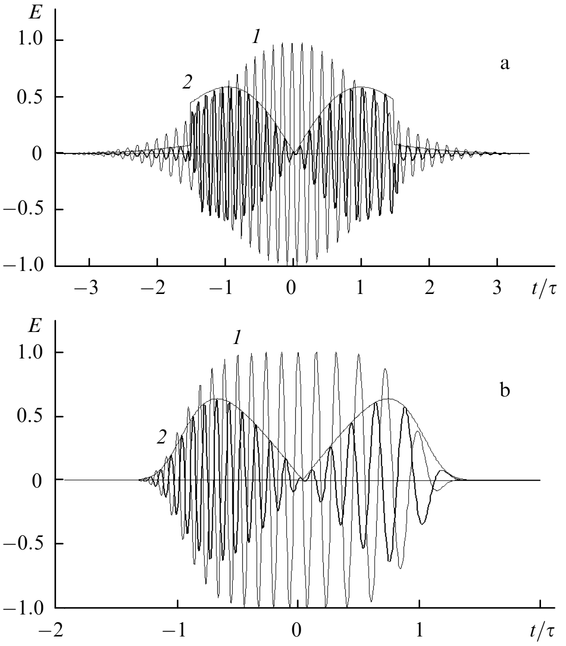

Figure 1. Phase-modulated incident (*1*) and reflected (*2*) pulses with the Gaussian (a) and super-Gaussian (b) envelopes. Reflected pulses are magnified by factor of 7 (a) and 2 (b).

by expression (1). Figure 1 also shows the profiles of reflected pulses plotted directly by using the expression $E(t) = E_0(t)\exp(-i\omega_0 t)$. One can see that the temporal profiles of both pulses almost completely coincide with those calculated by the spectral method from (5), which demonstrates the high accuracy of their description by expressions (3) and (4).

Phase modulation in Fig. 1a takes place for $-1.5\tau < t < 1.5\tau$ (the origin of coordinates is made coincident with the momentum centre). The envelope is described within this interval by expression (3) and outside both by expression (3) for $\phi(t) = 0$ and expression (4), in which it is sufficient to take only the first term of the expansion.

The analytic expressions obtained above allow one to determine the development of the phase modulation of PM pulses. This follows from the fact that information on the phase modulation for the amplitude (and intensity) in (3) is preserved in the form of the function $\phi(t)$. If the envelopes of the incident and reflected pulses $A(t)$ and $E_{0r}(t)$, respectively, are obtained by correlation methods, then the phase function $\phi(t)$ can be determined from (3).

In practice the situations are often encountered when PM pulses have a flat or almost flat top [10]. In the region of a flat top, $A(t) = A(t - \Delta t) = A_0$ (in our case, $A_0 = 1$), and it follows from (3) that the shape of the envelope of the reflected pulse is determined only by the phase function $\phi(t)$. This explains an almost linear dependence $E_{0r}(t)$ in the central part of the reflected pulse (see Fig. 1b). Indeed, for small values of $\phi(t)$, we can simplify (3), assuming that $\cos\phi(t) \simeq 1$:

$$E_{0r}(t) = \left| \frac{r_0}{1 - r_0^2} \phi(t) \right|. \quad (6)$$

In the case of quadratic phase modulation, $\phi(t) = \alpha t \Delta t - 0.5\alpha(\Delta t)^2$, which gives a linear dependence $E_{0r}(t)$. The time in these expressions is measured with respect to the pulse centre. When phase modulation is weak, we can use the expansion of $\phi(t)$ in a series in powers of Δt. Then, we obtain from (6) the expression for the envelope of the PM pulse in the region of a constant amplitude

$$E_{0r}(t) = \left| \frac{r_0}{1 - r_0^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \frac{d\varphi}{dt} (\Delta t)^n \right|, \tag{7}$$

which is similar to expression (4) for pulses with a constant phase. When $\phi(t)$ is small, we can set $\phi(t) = (d\varphi/dt)\Delta t = \delta\omega(t)\Delta t$, and, using (6) or (7), obtain at once the time dependence of the frequency modulation $\delta\omega(t)$ in the region of the flat top of the pulse. In the general case, when the envelopes $A(t)$ and $E_{0r}(t)$ are known, we can obtain directly from (3) the equation $\phi(t) = \varphi(t) - \varphi(t - \Delta t) = F[A(t), E_{0r}(t)]$ as a function of envelopes, and by solving numerically this equation, to determine the function $\varphi(t)$. This equation can be reduced to the differential equation, which is analogous to expression (7), by representing $\phi(t)$ as a power series, in which we can retain only the first expansion terms.

Let us point out also the possibility of practical applications of pulses transformed in TIMDs. One can see from Fig. 1 that a pulse reflected from a TIMD is split in fact into two pulses, one of which mainly contains high-frequency and another low-frequency components of the spectrum (Fig. 2a). This allows TIMDs to be used in compression schemes to obtain compressed pulses with different carrier frequencies. Figure 2b presents the calculated shapes of reflected pulses shown in Fig. 1b after their propagation in a medium with the normal dispersion of the refractive index $n(\omega)$. We used the Lorentzian profile of dispersion, as in paper [4], and the parabolic approximation of the wave vector $k(\omega)$ in the vicinity of ω_0 [9]. The evolution of a pulse propagating in a dispersion medium is described by the expression [9, 10]

$$E(t, z) = \int_{-\infty}^{+\infty} f(\omega) r(\omega) \exp[-i(\omega t - k(\omega) z)] d\omega,$$

where $f(\omega)$ and $r(\omega)$ are defined in (5). The value of z was determined near the point of the maximum pulse compression [10]. Therefore, we used in fact in our calculations the model of a perfect quadratic compressor.

The left pulse in Fig. 2b was obtained by equating to zero the low-frequency part of the spectrum of the reflected pulse shown in Fig. 2a, beginning from ω_0, the right pulse was obtained by equating to zero the high-frequency part of the reflected pulse. Figure 2b shows that such a transformation of the spectrum does not distort noticeably the shape of compressed pulses. The interesting feature of these pulses is the dependence of the filling frequency on α, i.e., on the phase-modulation rate. For example, for the value of α and characteristics of the incident pulse used in calculations, the frequency shift $\Delta\omega$ with respect to ω_0 for both pulses was $\pm 0.4\omega_0$. The duration of the pulses obtained in this case did not exceed the duration of the compressed incident pulse.

Therefore, our analysis have shown that TIMDs can be used both to study the time dependence of the PM pulse phase and to obtain ultrashort pulses with the properties mentioned above.

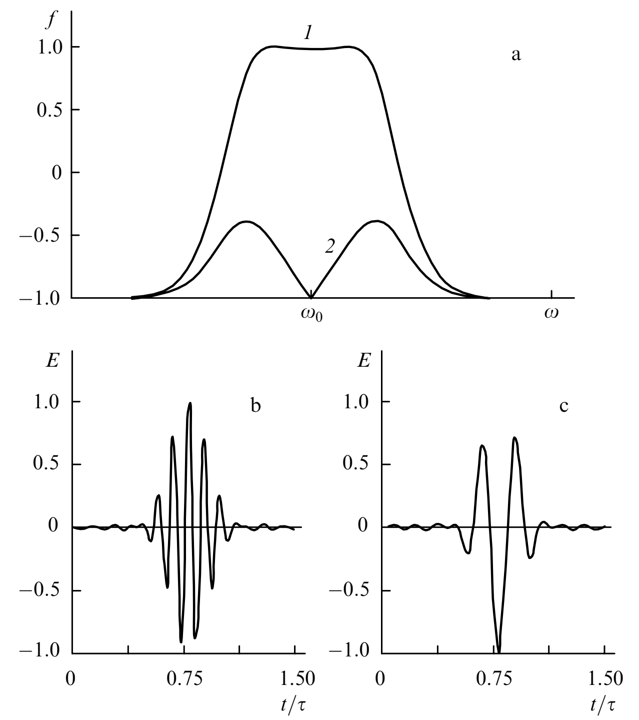

Figure 2. Spectra of the incident (*1*) and reflected (*2*) pulses shown in Fig. 1b (a), and transformed reflected pulses after compression (doubled) (b, c).

References

1. Szipöcs R., Ferencz K., Spielmann C., Krausz F. *Opt. Lett.*, **19** (3), 201 (1994).
2. Szipöcs R., Kőhazi-Kis A., Lakó S., Apai P., Kovacs A.P., DeBell G., Mott L., Louderback A.W., Tikhonravov A.V., Trubetskov M.K. *Appl. Phys. B*, **70**, 1 (2000).
3. Koroteev N.I., Magnitskii S.A., Tarasishin A.V., Zheltikov A.M. *Opt. Comm.*, **159**, 191 (1999).
4. Tarasishin A.V., Magnitskii S.A., Zheltikov A.M. *Laser Phys.*, **11** (1), 31 (2001).
5. Kozar' A.V., Bobrovnikov Yu.A., Gorokhov P.N. *Izv. Ross. Akad. Nauk, Ser. Fiz.*, **12**, 1823 (2002).
6. Kozar' A.V. *Opt. Spektrosk.*, **59**, 1132 (1985).
7. Kozar' A.V. *Opt. Spektrosk.*, **64**, 1130 (1988).
8. Kozar' A.V., Putrina E.V., Fionova O.V. *Vestn. Mosk. Gos. Univ., Ser. Fiz. Astron.*, **36**, 39 (1995).
9. Vinogradova M.B., Rudenko O.V., Sukhorukov A.P. *Teoriya voln* (Theory of Waves) (Moscow: Nauka, 1990).
10. Akhmanov S.A., Vysloukh V.A., Chirkin A.S. *Optika femtosekundnykh lazernykh impul'sov* (Optics of Femtosecond Laser Pulses) (Moscow: Nauka, 1988).