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Selection of optical modes in a ribbon fibre with a modulated gain

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***Abstract*. The gain spectrum of optical modes is theoretically analysed in an antiwaveguide ribbon fibre in which the amplifying regions are made coincident with periodically arranged regions of a reduced refractive index. A class of resonance periodic structures is selected in which the gain of one mode with the field uniformly distributed over channels exceeds the gain of all other modes by the value independent of the number of channels up to their critical value, which was also found analytically.**

Keywords: *fibre laser, phase-locking, resonator.*

1. Introduction

In Ref. [1], a new design of an active optical fibre was proposed which seems promising for scaling the output power of a fibre laser by increasing the width of a ribbon fibre retaining single-mode lasing at the same time. The gain in such a fibre is assumed to be localised in periodically arranged regions with a reduced refractive index, which gives grounds to call it an antiwaveguide fibre.

The authors of paper [1] concluded that a fibre in which the modulation of the refractive index does not exceed 0.001 is most efficient. However, it is difficult in practice to modulate the refractive index with such a small amplitude. In addition, a weak modulation of the refractive index results in an enhanced sensitivity of the characteristics of optical modes to random variations in the refractive index.

The laser structure of a similar geometry was considered earlier and realised in diode laser arrays (see Ref. [2] and references therein). The numerical study [3] of such structures performed in the approximation of the effective refractive index revealed the so-called resonance structures in which the width of the active element and the distance between the elements are multiples of the corresponding side wavelengths (recall that waves in an antiwaveguide grating propagate over the entire grating, undergoing reflection and refraction from boundaries of the elements). It was pointed out that, to achieve single-mode lasing, it is necessary to introduce selecting losses between the elements (it is difficult to perform the spatial modulation of the pump current in diode lasers). The analytic study in the approximation of the effective refractive index [4, 5] gave a deeper qualitative understanding of the requirements imposed on the structure geometry. The explicit expressions were obtained for the side radiative losses of the resonance mode [4] and the loss discrimination of the nearest mode [5].

Because the refractive-index nonlinearity in semiconductors is substantially greater than in glass fibres, the characteristic modulation amplitude of the refractive index in laser diodes is greater than that in the fibre. Usually, the active structure (laser diode itself) is placed on a substrate so that the side boundaries of the chip are located far from the pumped region and are made mat to avoid the influence of poorly controlled reflection of laser radiation from the boundaries. Because the active core of a fibre laser is surrounded by a cladding with lower refractive index, total internal reflection of light takes place at the core – cladding boundary, i.e., side radiative losses are absent. These differences in the physics of processes make actual a more general formulation of the problem.

In this paper, we analysed theoretically the possibilities of maintaining mode discrimination in an antiwaveguide grating with increasing its size. General criteria are formulated which allow the selection of the grating parameters so that the fundamental mode will be distributed uniformly over all the elements. This excludes effects caused by the inhomogeneous gain saturation. The relation is obtained between the step in the refractive index at the boundary of the active element and the geometrical parameters of the grating, which provides the maximum discrimination of adjacent modes preserved with increasing the grating size.

2. Band structure of the spectrum of optical modes

The structure under study is shown schematically in Fig. 1. Radiation propagates along the z axis. We assume that the reflection of light from flat boundaries in the transverse direction (along the y axis) is perfect. The thickness of a ribbon waveguide is assumed small, so that we may restrict our consideration to one transverse mode. Then, the field of an irradiated mode can be averaged with the profile of the refractive index in the transverse direction (model of the effective refractive index). In the transverse direction (along the x axis), the ribbon fibre has a periodic modulation of the refractive index and gain (losses). This structure belongs

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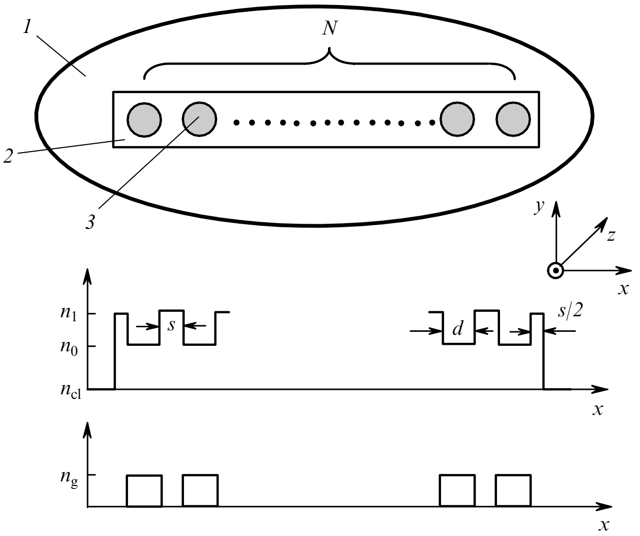

Figure 1. Scheme of the cross section of a ribbon fibre and profiles of the effective refractive index and the gain; (*1*) external cladding with the refractive index n_{cl}; (*2*) optical fibre; (*3*) active cores.

to the class of photonic crystals, which are actively discussed in the last years [6]. Photonic crystals attract great interest because they allow one to change substantially the dispersion properties of waves propagating in them due to the presence of forbidden bands. In particular, doping photonic crystals with defects makes it possible to localise the field near these defects and form waveguides confining the field in a broad wavelength range. The appearance of forbidden bands in the spectrum of waves propagating in the side direction in a ribbon laser is undesirable because it results in the division of the structure into independent domains. Our aim is to find the parameters of structures at which forbidden bands are absent.

Radiative losses through resonator mirrors can be replaced by losses uniformly distributed over the volume. Under above assumptions, the field can written in the form $U(x)\exp(\pm i\beta z)$, where $U(x)$ satisfies the Helmholtz equation

$$\frac{d^2U}{dx^2} + U\left[k^2\varepsilon(x) - \beta^2\right] = 0, \tag{1}$$

where β is the propagation constant; $k = \omega/c$ is the wave number in vacuum; $\varepsilon(x)$ is the dielectric constant $[\varepsilon(x) = (n_1 + in'')^2$ in passive regions and $\varepsilon(x) = (n_0 + in'' - in_g)^2$ in active regions]; n'' corresponds to distributed losses; n_g corresponds to the gain; n_0 and n_1 are the refractive indices in the active and passive regions, respectively. The propagation constant β in the model of a laser with distributed losses is real and is quantised in accordance with the boundary conditions on mirrors, resulting in a discrete spectrum of longitudinal modes. We will assume below that the spectrum of longitudinal modes is sufficiently dense and will neglect the discreteness of β.

The method of solution of Eqn (1) is well known. Let us introduce the basis functions f and g, which are linearly independent solutions of Eqn (1) in a cell. Then, any solution can be written as a linear combination of these functions $U = a_j f + b_j g$, where the coefficients a_j and b_j depend on the cell number j. The condition of smoothness of the solutions on the boundaries of the grating period leads to the recurrent relation $\boldsymbol{a}_{j+1} = T\boldsymbol{a}_j$, where T is the unimodular ($\det T = 1$) coupling matrix, and the vector $\boldsymbol{a}_j$ is composed of the coefficients a_j and b_j.

We can study the band structure of optical modes of a ribbon fibre by neglecting effects of the field gain. The spectrum of optical modes is determined by the eigenvalues of the matrix T, which can be written in the form $\exp(\pm iS)$. A convenient characteristic is the spur of matrix T, $\mathrm{Sp}T = 2\cos S$, which is expressed in terms of the structure parameters. In particular, the allowed band is determined by the condition $|\mathrm{Sp}T| \leqslant 2$. The opposite inequality corresponds to the forbidden band. The equality $|\mathrm{Sp}T| = 2$ (i.e. $S = K\pi$, where $K = 0, 1,...$) determined the boundaries of the bands. By using the known expression for the spur of the coupling matrix, we obtain the equation

$$\cos S = \cos(qd)\cos(ps) - \frac{p^2 + q^2}{2pq}\sin(qd)\sin(ps) \tag{2}$$

for the ribbon structure with the step modulation functions of the gain and refraction, where d and s are the lengths of the active and passive regions of a cell; q and p are the corresponding transverse components of the wave vector.

The equation for the band boundaries $\cos S = -1$ can be simply reduced to the form

$$\left[\left(\frac{p}{q} + 1\right)\cos\frac{ps + qd}{2}\right]^2 - \left[\left(\frac{p}{q} - 1\right)\cos\frac{ps - qd}{2}\right]^2 = 0. \tag{3}$$

The expression in the left can be represented as a product of two factors, the vanishing of each of them corresponding to the forbidden band boundary. The requirement that both these factors vanish simultaneously determines the parameters of the structure in which the forbidden band is absent. This requirement is equivalent to the simultaneous vanishing of expressions in the square brackets in Eqn (3), from which it follows, first, that $p_0 s + q_0 d = \pi(2m + 1)$, where m is an integer, i.e., the total phase shift of the wave after propagation through the cell is equal to an odd number of π. The second condition for $p \neq q$ has the form $p_0 s - q_0 d = \pi(2l + 1)$, where l is an integer. The simultaneous fulfilment of these two conditions means that half-integers of side wavelengths fit within the active element and the gap between the elements. Such structures were studied in diode laser arrays [2].

The condition $p = q$ corresponding to the absence of modulation of the refractive index also leads to the disappearance of forbidden bands. It is this variant, i.e., the limit corresponding to a homogeneous fibre that was considered numerically in Ref. [1]. In this case, the confined modes have a simple form $\sin(\pi m_0 x/L)$, where m_0 is the mode number and L is the width of the fibre ribbon. If the gain is periodically modulated over the width with the period $\Lambda = L/N$, then the integral of the overlap of the mode intensity with the gain grating proves to be maximal for the mode with the same period. Because of the orthogonality of trigonometric functions with different periods, the difference in the overlap integrals is preserved when N tends to infinity despite the condensation of the mode spectrum. For example, for the gain modulated by the function $[1 - \cos(\pi x/\Lambda)]/2$, the overlap integral is 0.5 for all the modes except the mode $m_0 = N/2$, for which it is equal to 0.75. It is this effect that lies in the basis of the construction analysed in Ref. [1]. For a finite number of active channels, the difference in the overlap integrals is preserved for small $\Delta n = n_1 - n_0$.

The question arises of whether the effect of a finite gain discrimination of optical modes is preserved in constructions where the step in the refractive index on the active region boundary is not small but the forbidden band is also absent? This question was not considered in Ref. [5] that was devoted to the analysis of the discrimination of adjacent modes over losses in finite antiwaveguide resonance gratings. Note also that radiation incident on the side external faces in diode laser arrays is scattered and lost, resulting in additional losses.

3. The gain spectrum of optical modes

The parameters of the structure should be selected bearing in mind that it is necessary to suppress the inhomogeneous saturation of the gain in the medium, which is the main mechanism destroying single-mode lasing. It is obvious that the most favourable situation appears when the field of the generated mode is distributed uniformly over the structure cross section. In addition, the gain degeneracy of the modes should be avoided. Both these conditions are fulfilled in resonance structures mentioned above, which were studied earlier in the development of diode laser arrays [2–5].

Let us find the gain spectrum of different modes in the resonance structure. The dependence of the gain on the mode number appears due to different overlaps of mode fields with the gain concentrated in active elements.

Because the imaginary parts $\varepsilon(x)$ are usually small compared to the real ones, we can use for the wave vectors the approximate relations

$$q^2 \approx k^2 n_0^2 - \beta^2 + 2\mathrm{i}k^2 n_0 n'' - 2\mathrm{i}k^2 n_0 n_{\mathrm{g}},$$
$$p^2 \approx k^2 n_1^2 - \beta^2 + 2\mathrm{i}k^2 n_1 n''. \tag{4}$$

By excluding the propagation constant from these expressions, we can find the relation between the wave vectors. It is determined by the step of the real and imaginary parts of the refractive index

$$p^2 - q^2 \approx k^2(n_1^2 - n_0^2) + 2\mathrm{i}k^2 n_0 n_{\mathrm{g}} \approx 2k^2 n_0(\Delta n + \mathrm{i}n_{\mathrm{g}}). \tag{5}$$

We took into account above that the step Δn in the refractive index between the active and passive regions is usually small. The fundamental antiphase mode in the resonance structure has the form shown in Fig. 2. It consists of the parts of cosine curves in active regions and intervals between them (see Ref. [7]). The angular distribution of radiation from one active element has the lowest divergence when one half-wave fits within the element. Then, the field inside the element is proportional to $\cos(\pi x/2d)$ (x is measured from the element centre), i.e., $q_0 = \pi/d$. In the region between the elements, $p_0 = 2\pi m/s$; then, the sizes of the active and passive regions and the step in the refractive index are related by the expression

$$\frac{4m^2}{s^2} - \frac{1}{d^2} = \frac{8n_0(\Delta n)}{\lambda^2}. \tag{6}$$

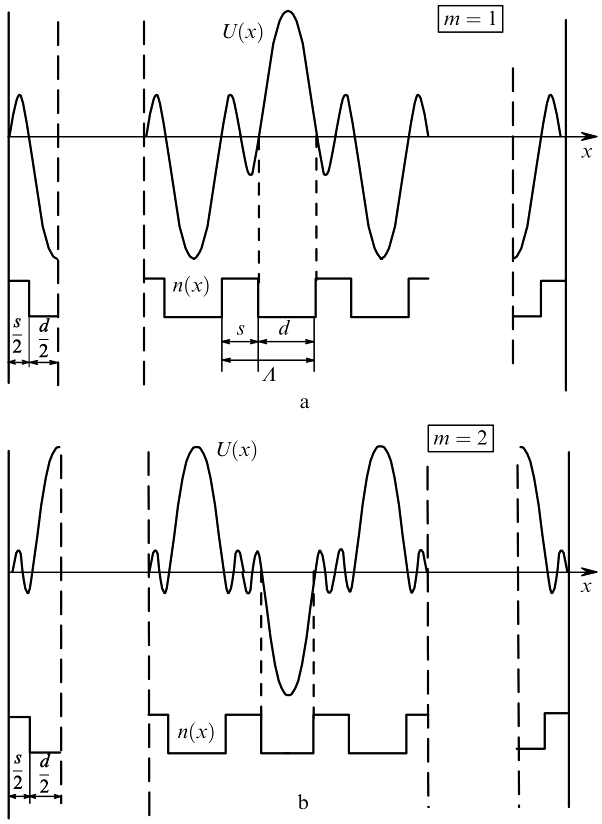

Figure 2. Distributions of the field $U(x)$ of the antiphase mode and the refractive index $n(x)$ in cells of resonance structures with $m = 1$ (a) and 2 (b).

Condition (6) determines resonance structures in which the forbidden band disappears. The integral of overlap of the gain in the form of a step of unit height and width d with the antiphase mode is

$$\Gamma_0 = \left[1 + \frac{(s/d)^3}{4m^2}\right]^{-1} \tag{7}$$

(a similar expression was obtained earlier in Ref. [7]). To find the gain discrimination for the adjacent modes, it is necessary to calculate the integral of overlap of the intensity of the corresponding mode with the gain profile, which requires the calculation of the intensity profiles for adjacent mode in the explicit form. The discrimination can be also obtained by solving the dispersion equations (see Appendix) if the spectrum of optical modes is known. It is expected for a grating containing many elements that the integral of overlap of the nearest-frequency modes with the gain profile will be close to that for the fundamental mode. The detunings of the transverse wave vectors $\tilde{p}$, $\tilde{q}$ from their resonance values p_0, and q_0 for the nearest modes are small: $|\tilde{p}s|$, $|\tilde{q}d| \ll 1$. In this case, Eqn (2) can be written in the form

$$F^2 = \left(\tilde{p}s + \frac{\tilde{q}d}{t}\right)(\tilde{p}s + t\tilde{q}d), \tag{8}$$

where $F = S - (2m+1)\pi$; and $t = p_0/q_0 = 2md/s$. By substituting the values F for the adjacent modes into (8) (see Appendix) and taking into account relation (5) between the wave vectors in the active and passive regions, we can find the real and imaginary detunings of the wave vectors as functions of the grating parameters and the mode number.

Equation (5) for modes near the resonance can be rewritten in the form

$$p_0\tilde{p} - q_0\tilde{q} = \mathrm{i}k^2 n_0 n_\mathrm{g},$$

from which the detuning of one of the wave vectors from the resonance value is expressed in terms of the detuning of another wave vector. The detuning of the wave vector $\tilde{p}$ can be found from Eqn (4) as

$$\tilde{p}s = \mathrm{Re}(\tilde{p})s + \frac{\mathrm{i}k^2 s^2 n_0 n''}{2m\pi}.$$

It follows from this expression that the imaginary part of the wave vector $\tilde{p}$ is proportional to n''. By substituting these relations into (8), we obtain the equation for the quantity n''/n_g equal to the integral of overlap of the optical mode intensity with the gain profile normalised in height to unity.

By introducing the notation $Z = \tilde{p}s$, we obtain from (8) the complex phase shift between the elements

$$(Z - \mathrm{i}Z_1)(Z - \mathrm{i}Z_2) = B, \tag{9}$$

where

$$Z_1 = \frac{k^2 n_0 n_\mathrm{g} sd}{\pi(2m+t)};\ Z_2 = \frac{k^2 n_0 n_\mathrm{g} sdt^2}{\pi(2m+t^3)};\ B = \frac{4m^2F^2}{(2m+t)(2m+t^3)}.$$

It is convenient to show the motion of the roots of Eqn (9) with changing $F = \pi l/N$ in the complex plane (Fig. 3). For $l = 0$ (and, therefore, $B = 0$), which corresponds to the resonance antiphase mode, Eqn (9) has two imaginary roots. By using the root Z_2 and taking into account that the imaginary part $Z = \tilde{p}s$ is proportional to n'', we find the expression for the overlap integral $(n''/n_\mathrm{g})_\mathrm{out} = [1 + (s/d)^3/4m^2]^{-1} = \Gamma_0$, which coincides with the overlap integral (7) calculated above. The root Z_1 corresponds to the solution of Eqn (1), which does not satisfy the boundary conditions, and therefore can be omitted.

The rest of the modes correspond to $l \neq 0$. For the given value of N, the value of B changes discretely and proportionally to the square of the mode number. The roots of the quadratic equation (9) can be conveniently written in the form

$$Z = \mathrm{i}\frac{Z_1 + Z_2}{2} \pm (B - B_\mathrm{c})^{1/2}, \tag{10}$$

where

$$B_\mathrm{c} = \frac{(Z_1 - Z_2)^2}{4}. \tag{11}$$

For $B > B_\mathrm{c}$, the imaginary part of the root of Eqn (9) is equal to the same value for all the modes:

$$\mathrm{Im}\,(\tilde{p}s) = \frac{k^2 s^2 n_0 n''}{2m\pi} = \frac{Z_1 + Z_2}{2}.$$

This gives the expression for the integral of overlap of the gain with the field of optical modes:

$$\Gamma_\mathrm{c} = \frac{t}{2(2m+t)} + \frac{t^3}{2(2m+t^3)}. \tag{12}$$

For a ribbon fibre with N channels, the maximum value of B for the nearest mode with $l = 1$ remains finite. As long as this value is larger than B_c, the overlap integrals for the resonance or any other modes differ by the same number, which is independent of N. The condition $B(l = 1) = B_\mathrm{c}$ gives the critical number N_c of elements in the ribbon up to which the found difference in the overlap integrals is preserved:

$$N_\mathrm{c} = \frac{\lambda^2}{n_0 n_\mathrm{g} sd}\left[\frac{(2m+t)(2m+t^3)}{(t^2-1)^2}\right]^{1/2}. \tag{13}$$

The criterion (13) for the number of channels in the construction depends substantially on material constants and the size of the ribbon cell.

Therefore, as long as the number of channels in ribbon fibre is less than the critical value determined by expression (13), the overlap integrals for the fundamental mode (Γ_0) and the rest of the modes (Γ_c) differ by a finite value depending only on the ribbon geometry. Recall that the size of the elements and the distance between them in the resonance structure are related to the step in the refractive index according to (6).

The dependence of the overlap integrals on the parameer $A = (s\mathrm{NA}/m\lambda)^2 = 1 - 1/t^2$, where NA is the numerical aperture of a waveguide (see, for example, Ref. [8]) is presented in Fig. 4 for $m = 1$ and 2. The limit $A = 0$ corresponds to a homogeneous fibre ($p_0 = q_0$). The overlap integral in this case can be written from general considerations as $\Gamma = d/(d+s)$ (where $s = 2md$). One can easily see that this expression gives the same values of Γ for $A = 0$ as in Fig. 4. The second limit corresponds to the disappearance of gaps between the elements ($s \to 0$, $p \to \infty$). It is obvious in this case that both overlap integrals become unity. For the intermediate values of the parameter, the overlap integral for the fundamental mode is greater than that for the rest of the modes.

The analysis performed above can be also applied to the structures generating in-phase modes. For this purpose, it is sufficient to add on the outside to the structure under study, which consists of N identical cells where the region with the

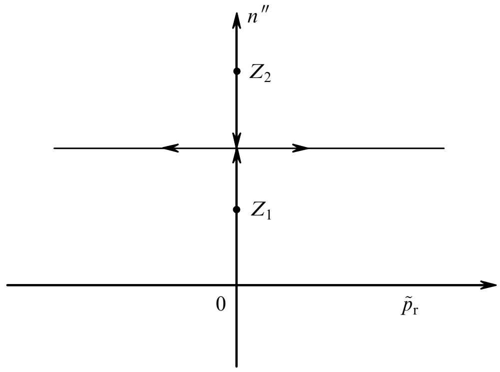

Figure 3. Motion of the roots of Eqn (9) in the complex plane upon variation of its right-hand part.

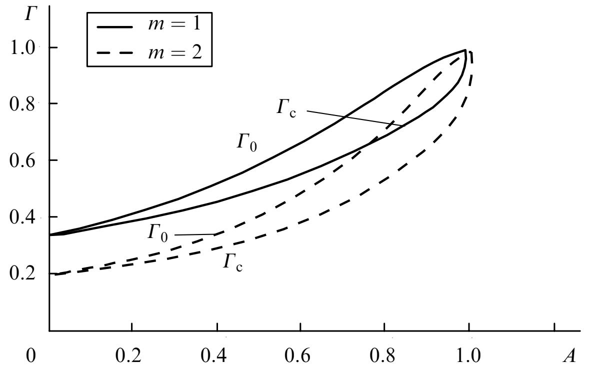

Figure 4. Dependences of the overlap integrals for the resonance (Γ_0) and the rest of antiphase modes (Γ_c) on the parameter A for $m = 1$ and 2.

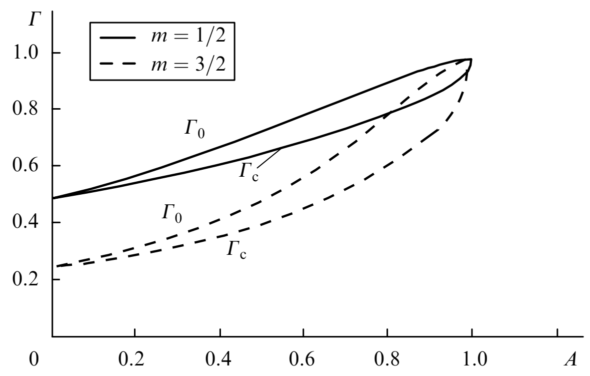

Figure 6. Dependences of the overlap integrals for the resonance (Γ_0) and the rest of in-phase modes (Γ_c) on the parameter A for $m = 1/2$, 3/2.

lower refractive index n_0 is surrounded by halves the regions with the higher refractive index, half the layer with $n = n_1$. The fundamental resonance mode in such a structure is an in-phase mode, as demonstrated in Fig. 5. We can show that the results of analysis are also valid for in-phase modes formally generalised for half-integers $m = 1/2$, 3/2, etc. The dependences of the overlap integrals for the in-phase resonance mode and the rest of the modes on the parameter A obtained under the condition $N < N_c$ are shown in Fig. 6. The degree of mode discrimination, defined as the difference $D = \Gamma_0 - \Gamma_c$ between the overlap integrals for the resonance mode (7) and the rest of the modes (12), as a function of the parameter A is presented in Fig. 7 for different values of m. One can see that the discrimination maximum D_{max} increases monotonically with m and its position approaches the point $A = 1$. The parameters of structures with the maximum mode discrimination are presented in Table 1. Below, we will analyse such structures.

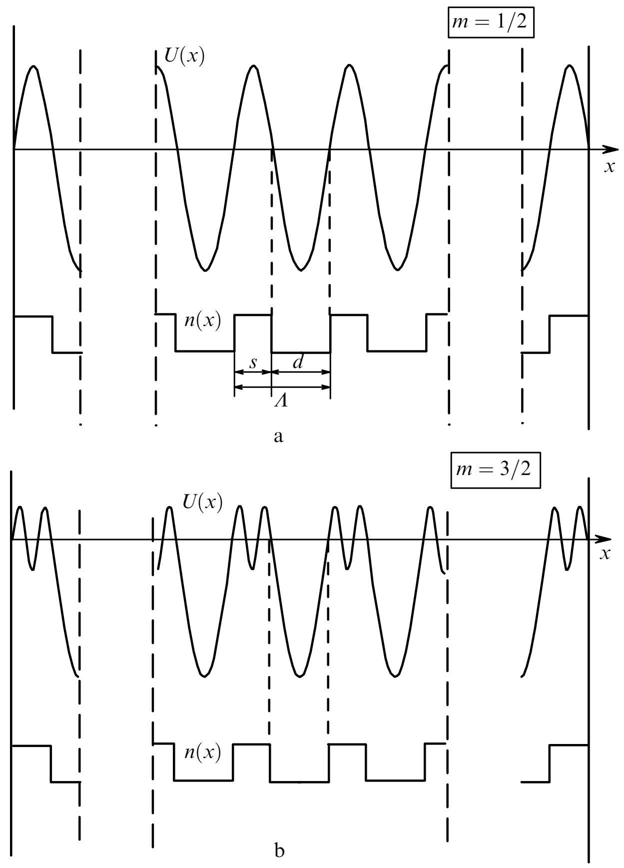

Figure 5. Distributions of the in-phase mode field $U(x)$ and the refractive index $n(x)$ in cells of resonance structures with $m = 1/2$ (a) and 3/2 (b).

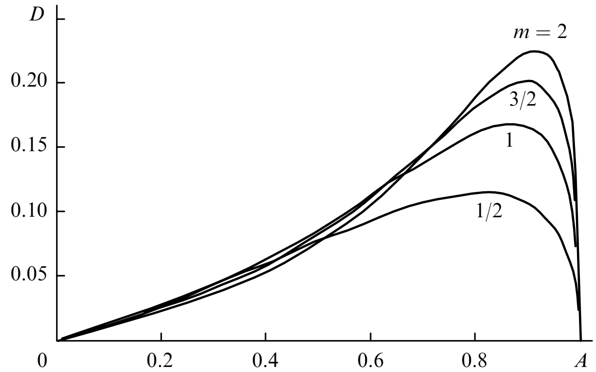

Figure 7. Dependences of the discrimination degree D of higher modes with respect to the resonance mode on the parameter A for $m = 2$, 3/2, 1, and 1/2.

Table 1.

m	D_{max}	d/s	A	φ
1	0.167	1.366	0.866	1.927
2	0.224	0.852	0.914	0.915
1/2	0.113	2.294	0.81	4.062
3/2	0.19	1.111	0.91	1.225

4. Analysis of structures with the maximum discrimination

One can see from Table 1 that the maximum discrimination degree D_{max} depends on the number m of side wavelengths fitting the waveguide gap. The condition of the maximum discrimination introduces additional restrictions on the structure parameters. The lengths d and s prove to be rigidly interrelated and the relation between the numerical aperture NA and the ratio s/λ also appears. It is also interesting to estimate the admissible number of channels in the assembly. For the maximum discrimination condition, the relation

$$N_c = \frac{\Delta n}{n_g A}\varphi(m)$$

can be obtained, where

$$\varphi(m) = \frac{3^{1/4}2^{3/2}(d/s)^{1/2}}{\sqrt{m}[\sqrt{3}(2md/s) - 1]}.$$

The values of $\varphi(m)$ are also presented in Table 1. The admissible number of channels in the laser are determined within an order of magnitude by the ratio of the step in the refractive index to its imaginary part, which can be expressed in terms of the gain g as $n_g = g\lambda/4\pi$. By taking the typical parameters of fibre lasers, we estimate the admissible number of channels as $\sim 10^3$. This means that the size of the assembly is virtually not restricted. For the typical parameters of diode lasers (the wavelength is ~ 1 μm), the admissible number of channels is 10^2.

The value of the step in the refraction index at the boundaries of the elements gives the scale for random variations in the refractive index that do not cause a substantial change in the mode structure. For this reason, the larger the step Δn, the more rigid the mode structure provided by the produced distribution of the refractive index. The estimate of the admissible amplitude of random variations in the refractive index is a separate problem. The presence of a finite mode discrimination, which does not depend on the number of elements, ensures that admissible fluctuations are not too small.

By selecting the parameter m, its influence on the axial brightness of the output radiation should be taken into account. The antiphase mode gives a split peak in the far-field zone. To avoid this, 'a rectifying' phase plate having the phase difference π between adjacent periods Λ can be placed at the output. In this case, the dependence of the axial brightness on m can be estimated by the square of the integral from the field amplitude over the structure period. A comparison of Figs 2a and 2b shows that the $m = 1$ antiphase mode has a lower axial brightness than the $m = 2$ mode because upon integration of the field inside the active channel and in the gap between the channels, the fields are subtracted, while the contribution from a passive gap for the $m = 2$ mode is zero. Such an effect is absent for the in-phase mode (Fig. 5). Therefore, structures with greater values of m are preferable, which can be obtained by increasing s or the numerical aperture. An increase in s is accompanied by a decrease in the filling of the output aperture by laser radiation, which is in turn accompanied by the field energy transfer from the central peak to the side peaks. The possibility of increasing the numerical aperture is determined by the manufacturing technology of an optical fibre. Therefore, special studies are required to select the proper value of the parameter m.

5. Conclusions

By using the approximation of the effective refractive index, we have analysed theoretically the gain spectrum of collective modes in a ribbon fibre laser with the antiwaveguide periodical structure. We have shown that for resonance antiwaveguide structures, in which the gain occurs in the regions with a lower refractive index, there exists a critical number of elements below which all the modes are discriminated with respect to the fundamental mode, the discrimination being independent of the assembly dimensions. We have found explicit conditions providing the discrimination maximum and obtained the explicit dependence of the admissible number of elements in a ribbon fibre on the material parameters and geometry of the assembly. Our estimates have shown that the restriction on the assembly size in the case of maximum discrimination proves to be weak. The restriction on the size of an assembly where phase synchronisation is possible can be much more severe because of the scatter in the parameters of individual channels.

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Appendix

Spectrum of the ribbon fibre modes with an arbitrary profile of the refractive index on the period

Let us show that, in the case of total internal reflection from the side boundaries of a fibre, the spectrum of optical modes can be found without specifying the distribution of the field and profile of the refractive index in a cell. In the general case, the elements of the matrix T are described by the expressions

$$\begin{aligned}
T_{11} &= \frac{1}{W}[f'(\Lambda/2)g(-\Lambda/2) - f(\Lambda/2)g'(-\Lambda/2)],\\
T_{12} &= \frac{1}{W}[g'(\Lambda/2)g(-\Lambda/2) - g'(-\Lambda/2)g(\Lambda/2)],\\
T_{21} &= \frac{1}{W}[f'(\Lambda/2)f(-\Lambda/2) - f'(-\Lambda/2)f(\Lambda/2)],\\
T_{22} &= \frac{1}{W}[g'(\Lambda/2)f(-\Lambda/2) - g(\Lambda/2)f'(-\Lambda/2)],
\end{aligned} \qquad \text{(A1)}$$

where Λ is the cell length and $W = [g(x)f'(x) - f(x)g'(x)]$ is the Wronskian of Eqn (1).

Because the core of a ribbon fibre is surrounded by a cladding with a lower refractive index (Fig. 1), light experiences total internal reflection from the side boundary of the ribbon for the angles of incidence lower than the critical one. If the light field penetration depth into the cladding is neglected, the field vanishes at the boundary. It is convenient to select the independent solutions of Eqn (1) in a cell so that one of them coincides with the field in the extreme right cell (in this case, $f(\Lambda/2) = 0$), while another coincides with the field in the extreme left cell, so that $g(-\Lambda/2) = 0$. In this case, it follows from (A1) that $T_{11} = 0$, and $T_{22} = \mathrm{Sp}T$. Such a choice of the functions leads to the boundary conditions $a_1 = 0$, $b_N = 0$ imposed on the coefficients for a grating consisting of N cells.

By substituting these conditions to the solution of a linear recurrent relation, we can derive the dispersion equation of the form $\sin(NS) = 0$. The spectrum of S

obtained from the dispersion equation is purely real because of the absence of the side radiative losses. The mode eigenfrequencies linearly depend on the mode number l and the band number n as

$$S = K\pi + \frac{\pi l}{N}, \quad l = 0, 1..., N-1; \quad K = 0, 1... \qquad \text{(A2)}$$

In accordance with the general concepts, it follows from (A2) that each band contains N modes. The $l = 0$ mode for the resonance structure under study is either the antiphase (for odd K) or in-phase (for even K) mode.

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